Geom - Circ - 14/5/21

9/6/20 (8)

$$\left(\begin{array}{ccc|c} -1 & 3 & 0 & -1 \\ 3 & -5 & 2 & 1 \\ 0 & 2 & 1 & 0 \\ \hline -1 & 1 & 0 & 0 \end{array}\right)$$

$d_1 < 0$ $\boxed{d_2 < 0}$ $\boxed{d_3 = 5 - 9 + 4 = 0}$ $\Longrightarrow$ parab. iperb. perché $d_4 \neq 0$

$$d_4 = \left|\begin{array}{ccc|c} -1 & 3 & 0 & -1 \\ 2 & -2 & 2 & 0 \\ 0 & 2 & 1 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right| = \dots \neq 0$$

30/6/20 (1) $tr(A) = 5$ $det(A) = -7$ $p_A(2) = -11$

ATT: $p_A(t) = det(t \cdot I_n - A) = t^n - tr(A) \cdot t^{n-1} + \dots + (-1)^n \cdot det(A)$

$p_A(t) = t^3 - 5t^2 + c \cdot t + 7$

$p_A(2) = -11 \Longrightarrow 8 - 20 + 2c + 7 = -11$ $c = -3$

$p_A(t) = t^3 - 5t^2 - 3t + 7 = (t-1)(t^2 - 4t - 7)$

$\lambda_1 = 1$ $\lambda_{2,3} = 2 \pm \sqrt{4+7} = 2 \pm \sqrt{11}$

(2) $\begin{pmatrix} k^2+k-10 & 3-k \\ k^2-9 & 2 \end{pmatrix}$

$tr = \boxed{k^2+k-8} = \lambda_1 + \lambda_2$

$\det = 2k^2+2k-20 - 3k^2 + 27 + k^3 - 9k$

$= k^3-k^2-7k+7 = \lambda_1 \cdot \lambda_2$

$= k^2(k-1)-7(k-1)$

$= \boxed{(k^2-7)(k-1)}$

$\lambda_1 = k^2-7 \qquad \lambda_2 = k-1$

$\lambda_1 = \lambda_2$ per $k^2-7 = k-1$ cioè $k^2-k-6=0$

$(k-3)(k+2)=0$

$k=3 \qquad k=-2$

Per $k \neq 3,-2$ diago.

$k=3 \qquad \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ sì

$k=-2 \qquad \begin{pmatrix} \cdot & 5 \\ \cdot & \cdot \end{pmatrix}$ no (dovrebbe essere già diagonale)

(3) $v \qquad \|v\|=1 \qquad v_1+v_2+v_3=0 \qquad v \perp \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$

Pongo $v = \begin{pmatrix} x \\ y \\ -x-y \end{pmatrix}$ $\qquad \begin{cases} x^2+y^2+(x+y)^2=1 \\ 5x-3y+4(-x-y)=0 \end{cases}$ ...

Invece: impongo II + III e poi normalizzo

II : $v \perp \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

II + III : $w = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \wedge \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \\ -8 \end{pmatrix}$

$v = \pm \frac{1}{\sqrt{114}} \cdot \begin{pmatrix} 7 \\ 1 \\ -8 \end{pmatrix}$

④ $M \in \mathcal{M}_{3\times 3}(\mathbb{R})$ ortog

M è coniugata (tramite ortog) a

$$\boxed{\left(\begin{array}{c|c} R_\vartheta & \begin{matrix} 0 \\ 0 \end{matrix} \\ \hline \begin{matrix} 0 & 0 \end{matrix} & \pm 1 \end{array}\right)}$$

$\begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \begin{pmatrix} -1 & \\ & -1 \end{pmatrix}$

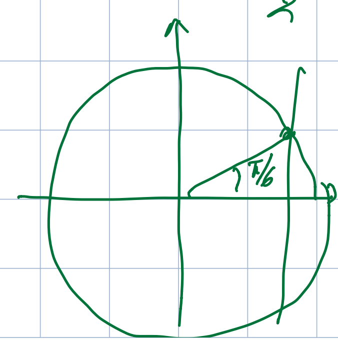
$\begin{pmatrix} 1 & & \\ & -1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & -1 & \\ & & -1 \end{pmatrix}$

$\det = -1 \qquad tr = \sqrt{3} - 1 \implies 2\cos\vartheta - 1 = \sqrt{3} - 1$

$\cos\vartheta = \frac{\sqrt{3}}{2}$

$\Downarrow$

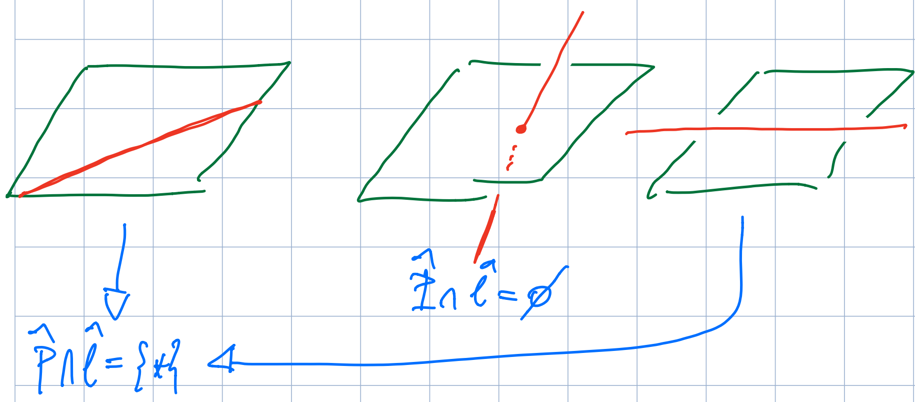
$$M = \begin{pmatrix} \cos\vartheta & -\sin\vartheta & 0 \\ \sin\vartheta & \cos\vartheta & 0 \\ 0 & 0 & -1 \end{pmatrix}$$



rotaz. intorno a l retta di $\vartheta = \pi/6$

composta con rifl. nel piano $l^\perp$

(5) $P, l \subset \mathbb{R}^3$ $\quad \hat{P}, \hat{l} \subset \mathbb{P}^3(\mathbb{R})$ $\quad \hat{P} \cap \hat{l} = ?$



(6)

$$\begin{pmatrix} 1 & -2 & 3 & 0 \\ -2 & 5 & 2 & 0 \\ 3 & 2 & 13 & 5 \\ 0 & 0 & 5 & 0 \end{pmatrix}$$

$d_1 > 0 \quad d_2 > 0 \quad d_3 = 65 - 12 - 12 - 45 - 52 - 4 < 0$

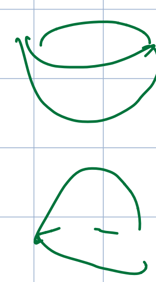
Q : $+, +, -$

A : $+, +, -, \pm$

$+, +, -, +$

$d_4 = -5 \cdot 5 \cdot \begin{vmatrix} 1 & -2 \\ -2 & 5 \end{vmatrix} < 0$

$x^2 + y^2 - z^2 + 1 = 0 \qquad z^2 = 1 + x^2 + y^2$



iperboloide a 2 falde (ellittico)

(7) $\begin{pmatrix} 1 & 6 \\ 6 & -4 \end{pmatrix}$

$\lambda_1 \neq \lambda_2$ (diagonalizz. ma non diag.)

$\Rightarrow v_1 \perp v_2$

$$t^2+3t-40 = (t+8)(t-5)$$

$$\lambda_{1,2} = \frac{-3 \pm \sqrt{9+160}}{2} = \frac{-3 \pm 13}{2} = \begin{cases} 5 \\ -8 \end{cases}$$

v_1: $\underline{x+6y} = 5x \qquad -4x+6y=0 \qquad \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} 1 & 6 \\ 6 & -4 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 15 \\ 10 \end{pmatrix} = 5 \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

v_2: $\cancel{x+6y=-8x} \qquad \cancel{9x+6y=0}$

$$\begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$u_1 = \frac{1}{\sqrt{13}}\begin{pmatrix} 3 \\ 2 \end{pmatrix} \qquad u_2 = \frac{1}{\sqrt{13}}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

(8) $f(x,y) = (5x-4y+1)\cos(2x-3y)$

$$\frac{\partial f}{\partial x} = 5 \cdot \cos(\cdot) + (5x-4y+1) \cdot 2 \cdot \sin(\cdot)$$

$$\frac{\partial f}{\partial y} = -4 \cdot \cos(\cdot) + (5x-4y+1) \cdot (-3) \cdot \sin(\cdot)$$

$$\frac{\partial^2 f}{\partial x^2}(0,0) = \cancel{5 \cdot 2 \cdot \sin(0)} + \cancel{5 \cdot 2 \cdot \sin(0)} + 1 \cdot 2 \cdot 2 \cdot \cos(0) = 4$$

$$\frac{\partial^2 f}{\partial x \partial y}(0,0) = \cancel{5\cdot(-3)\sin(0)} + \cancel{(-4)\cdot 2\cdot \sin(0)} + 1\cdot 2\cdot(-3)\cdot\cos(0) = -6$$

$$\frac{\partial^2 f}{\partial y^2}(0,0) = \cancel{(-4)\cdot(-3)\cdot\sin(0)} + \cancel{(-4)\cdot(-3)\cdot\sin(0)} + 1\cdot(-3)\cdot(-3)\cos(0)$$

$$= 9$$

$$Hf(0,0) = \begin{pmatrix} 4 & -6 \\ -6 & 9 \end{pmatrix} \qquad d_1 \neq 0 \qquad d_2 = 0$$

$$\lambda_{1,2} : \; +,\; 0$$

21/7/20 ①
$$\begin{pmatrix} 9-k^2 & k-2 & 0 \\ 0 & k+3 & 0 \\ k^2-1 & k+1 & k^2+1 \end{pmatrix}$$

$$\det(t\cdot I - A) = \begin{pmatrix} t-(9-k^2) & * & 0 \\ 0 & t-(k+3) & 0 \\ * & * & t-(k^2+1) \end{pmatrix}$$

$$\lambda_1 = 9-k^2 \qquad \lambda_2 = k+3 \qquad \lambda_3 = k^2+1$$

$$\lambda_1 = \lambda_2 \qquad 9-k^2 = k+3 \qquad k^2+k-6=0 \qquad k=2,\; k=-3$$

$$\lambda_1 = \lambda_3 \qquad 9-k^2 = k^2+1 \qquad 2k^2 = 8 \qquad k = \pm 2$$

$$\lambda_2 = \lambda_3 \qquad k+3 = k^2+1 \qquad k^2-k-2=0 \qquad k=2,\; k=-1$$

$k \neq -3, -1, -2, 2,$ diago.

Per $k = -3, -1, -2, 2$ sostituire [...]