Geom - Civ - 13/5/21

Scritto 9/6/20

(1) Sì poiché ha 3 autoval distinti e $\dim(V)=3$

(2) $$\begin{pmatrix} 7-t^2 & t+3 & t-3 \\ 0 & t+1 & t-4 \\ 0 & 0 & t^2-11 \end{pmatrix}$$

Poiché è triang. sup. autoval:

$$\lambda_1 = 7-t^2 \qquad \lambda_2 = t+1 \qquad \lambda_3 = t^2-11$$

Se distinti è diago; vedo quando non lo sono:

- $7-t^2 = t+1 \qquad t^2+t-6=0 \qquad (t+3)(t-2)=0$

  $t=-3,\ t=2.$

- $7-t^2=t^2-11 \qquad 2t^2=18 \qquad t=\pm 3$

- $t+1 = t^2-11 \qquad t^2-t-12=0 \qquad (t-4)(t+3)=0$

  $t=-3,\ t=4.$

Sicuramente diago per

$$t \neq -3, 2, 3, 4.$$

$t=-3$ $$\begin{pmatrix} -2 & 0 & -6 \\ 0 & -2 & -7 \\ 0 & 0 & -2 \end{pmatrix}$$ No

$$\begin{pmatrix} 7-t^2 & t+3 & t-3 \\ 0 & t+1 & t-4 \\ 0 & 0 & t^2-11 \end{pmatrix}$$

Oss: se A ha un solo autoval. è diagonalizzabile solo se è già diagonale.

$$t=2 \quad \begin{pmatrix} 3 & 5 & -1 \\ 0 & 3 & -2 \\ 0 & 0 & -7 \end{pmatrix} \qquad m.a.(3)=2$$

$$m.g.(3) = 3 - \operatorname{rank}(3\cdot I - A) = 3 - \operatorname{rank}\begin{pmatrix} 0 & -5 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & .. \end{pmatrix}$$

$$= 3-2 = 1 \qquad \text{NO}$$

$$\begin{pmatrix} 7-t^2 & t+3 & t-3 \\ 0 & t+1 & t-4 \\ 0 & 0 & t^2-11 \end{pmatrix}$$

$$t=3 \quad \begin{pmatrix} -2 & 6 & 0 \\ 0 & 4 & -1 \\ 0 & 0 & -2 \end{pmatrix}$$

$$m.a.(-2)=2 \qquad m.g.(-2) = 3 - \operatorname{rank}(A+2I)$$

$$= 3 - \operatorname{rank}\begin{pmatrix} 0 & 6 & 0 \\ 0 & 6 & -1 \\ 0 & 0 & 0 \end{pmatrix} = 3-2 = 1 \qquad \text{NO}$$

$$t=4 \quad \begin{pmatrix} -9 & 7 & 1 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix} \qquad m.a.(5)=2$$

$$m.g.(5) = 3 - \operatorname{rank}(A-5I) = 3 - \operatorname{rank}\begin{pmatrix} -14 & 7 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$= 3 - 2 = 1 \qquad \text{SI}$$

(3) $v = \begin{pmatrix} 3-2i \\ 4+i \end{pmatrix}$ w t.c. unitario, $\perp v$, I compo $\in i\cdot\mathbb{R}$

potrei cercare $w = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ t.c.

$$\begin{cases} |\alpha|^2 + |\beta|^2 = 1 \\ \mathrm{Re}(\alpha) = 0 \\ \alpha \cdot (3+2i) + \beta \cdot (4-i) = 0 \end{cases}$$

$\alpha = x \cdot i$
$\beta = y + z \cdot i$
$x, y, z \in \mathbb{R}$

$$\begin{cases} x^2 + y^2 + z^2 = 1 \\ \dots \end{cases}$$

Invece: cerco dapprima $u = \begin{pmatrix} i \\ \beta \end{pmatrix}$ con $u \perp v$
e poi andranno bene $w = \pm \frac{u}{\|u\|}$
poiché le condiz. II e III
sono preservate normalizzando.

$$i(3+2i) + \beta \cdot (4-i) = 0$$

$$\beta = \frac{2-3i}{4-i} = \frac{(2-3i)(4+i)}{17} = \frac{1}{17}(11-10i)$$

$\tilde{u} = \begin{pmatrix} 17i \\ 11-10i \end{pmatrix}$ $w = \pm \frac{1}{\sqrt{17^2+11^2+10^2}} \begin{pmatrix} 17i \\ 11-10i \end{pmatrix}$

(4) $\langle \cdot | \cdot \rangle_A$ $A = \begin{pmatrix} 3 & 2 & t^2-3t \\ 2 & t+1 & -1 \\ 4t-10 & -1 & t+3 \end{pmatrix}$ prod. scal.?

- simmetrica $4t-10 = t^2-3t$ $t^2-7t+10=0$
  $t=2$ $t=5$.

• def. pos. $d_1, d_2, d_3 > 0$.

$t=2$ $\begin{pmatrix} 3 & 2 & -2 \\ 2 & 3 & -1 \\ -2 & -1 & 5 \end{pmatrix}$ $d_1 = 3 > 0$

$d_2 = 5 > 0$

$d_3 = 45 + 4 + 4 - 12 - 20 - 3 > 0$ Sì

$t=5$ $\begin{pmatrix} 3 & 2 & 10 \\ 2 & 6 & -1 \\ 10 & -1 & 8 \end{pmatrix}$ $d_1 = 3 > 0$

$d_2 = 14 > 0$

$d_3 = 144 - 20 - 20 - 600 - 32 - 3 < 0$ No

5 $A = \begin{pmatrix} 3 & 1+2i \\ -1+2i & 3 \end{pmatrix}$

normale: $A \cdot A^* = A^* \cdot A$.

$$\begin{pmatrix} 3 & 1+2i \\ -1+2i & 3 \end{pmatrix}\begin{pmatrix} 3 & -1-2i \\ 1-2i & 3 \end{pmatrix} = \begin{pmatrix} 3 & -1-2i \\ 1-2i & 3 \end{pmatrix}\begin{pmatrix} 3 & 1+2i \\ -1+2i & 3 \end{pmatrix}$$

$$\begin{pmatrix} 14 & 0 \\ 0 & 14 \end{pmatrix} \qquad \begin{pmatrix} 14 & 0 \\ 0 & 14 \end{pmatrix}$$

Oss: $\frac{1}{\sqrt{14}} \cdot A$ è unitaria

$\begin{pmatrix} 3 & 1+2i \\ -1+2i & 3 \end{pmatrix}$ $t^2 - 6t + (9+5) = 0$

$t^2 - 6t + 14 = 0$

$$\lambda_{1,2} = 3 \pm \sqrt{9-14} = 3 \pm i\sqrt{5}$$

$$u_{1,2} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \qquad A \cdot \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = (3 \pm i\sqrt{5}) \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

<u>Oss</u>: $\lambda_1 \neq \lambda_2 \Rightarrow$ automat. $u_1 \perp u_2$

$$\begin{pmatrix} 3 & 1+2i \\ \cdot & \cdot \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = (3 \pm i\sqrt{5}) \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$3\alpha + (1+2i)\beta = (3 \pm i\sqrt{5})\alpha$$

$$(1+2i)\beta = \pm i\sqrt{5}\,\alpha$$

$$u_{1,2} = \begin{pmatrix} 1+2i \\ \pm i\sqrt{5} \end{pmatrix}$$

(6) $P = [t : 2 : t^2+1] \in$ retta per $[-2:3:1], [4:-1:13] \subset \mathbb{P}^2(\mathbb{R})$

ℓ = proiez. in $\mathbb{P}^2(\mathbb{R})$ di $W = \mathrm{Span}\left(\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ -1 \\ 13 \end{pmatrix} \right)$

$$P \in \ell \Leftrightarrow \begin{pmatrix} t \\ 2 \\ t^2+1 \end{pmatrix} \in W$$

$$\Leftrightarrow \det \begin{pmatrix} t & -2 & 4 \\ 2 & 3 & -1 \\ t^2+1 & 1 & 13 \end{pmatrix} = 0$$

$$\underline{26t} \; \underline{+t^2} \; \underline{+1} \; \underline{-8} \; \underline{-4t^2} \; \underline{-4} \; \underline{+52} \; \underline{+t} = 0$$

$$3t^2 - 27t + 41 = 0$$

$$t_{1,2} = \frac{1}{6}\left(27 \pm \sqrt{\dots}\right)$$

7 Conica... parabola?

$$\begin{pmatrix} 2 & 1-t & 1 \\ 1-t & t+11 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$

$d_2 = 0$

$\boxed{d_3 \neq 0}$

$2t+22-1+2t-t^2=0 \qquad t^2-4t-21=0$

$(t-7)(t+3)=0$

$t=7 \qquad t=-3$

$t=7 \qquad \begin{pmatrix} 2 & -6 & 1 \\ -6 & 18 & 2 \\ 1 & 2 & 2 \end{pmatrix} \qquad d_3 = 72-12-12-18-72-8 \neq 0$

Parab.

$t=-3 \qquad \begin{pmatrix} 2 & 4 & 1 \\ 4 & 8 & 2 \\ 1 & 2 & 2 \end{pmatrix} \qquad d_3 = 32+8+8-8-32-8=0$

Degenere

Provate da soli a fare i quesiti d'esame (tutti quelli sul sito): se non tornano, scrivete e li faccio.