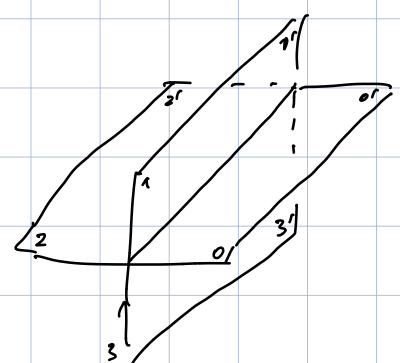
LDT - 23/4/2020

$$\begin{tikzcd} & M_{j+1} \arrow[d, "2:1"] \\ D^2 \arrow[ur, hook] \arrow[r, dashed] \arrow[r, bend left=90] & D_j \end{tikzcd}$$

crea linee doppie interne
(anche doppi: più eliminati)

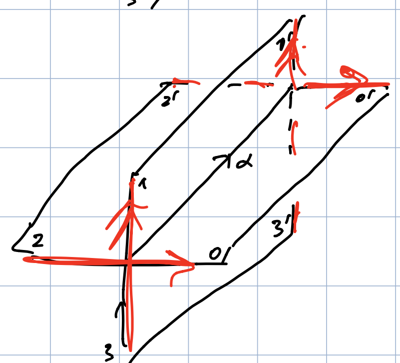
le eliminiamo:



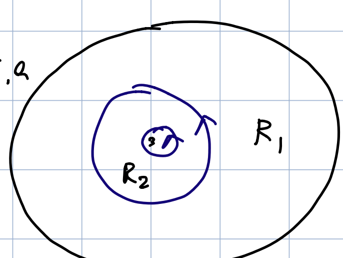
II 0123 → 0'1'2'3'

III 0123 → 1'0'3'2'

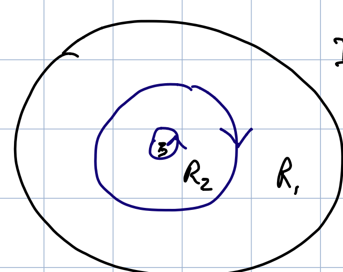
II



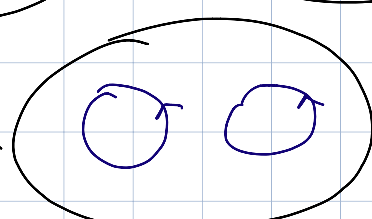
II.1.a



II.1.b



II.2.a



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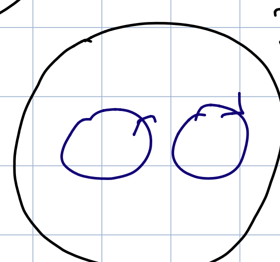
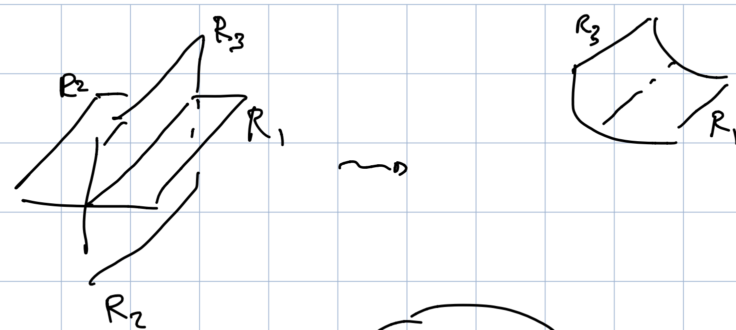
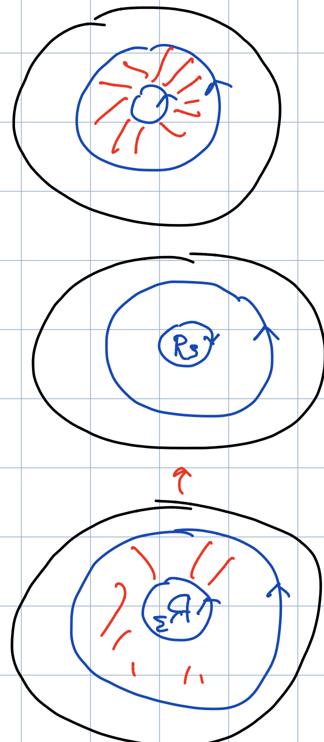


II.1

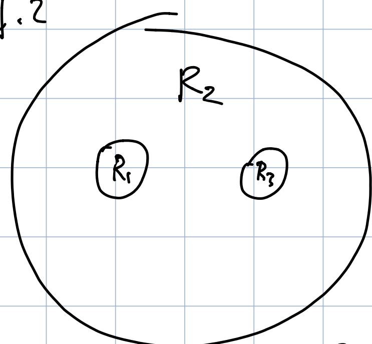


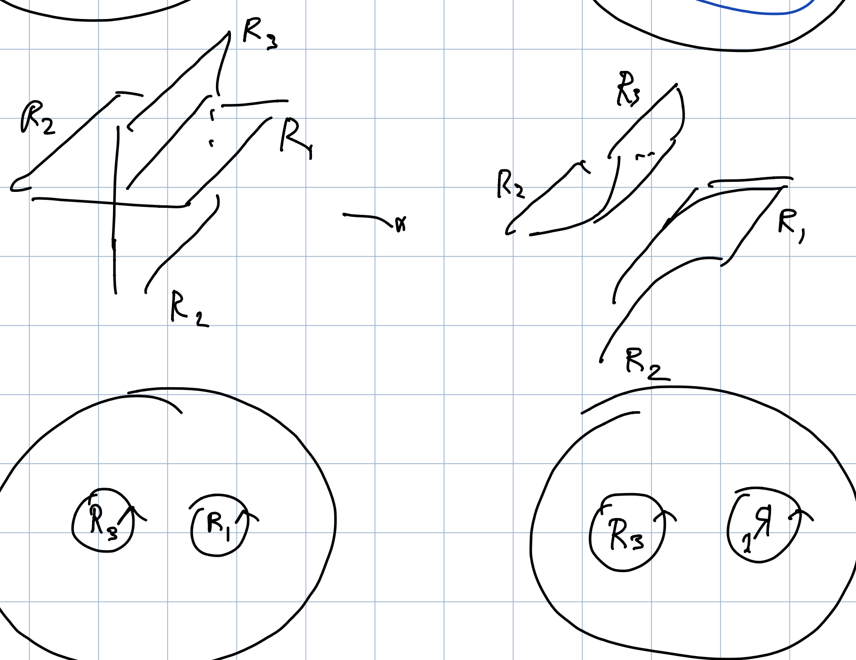
a

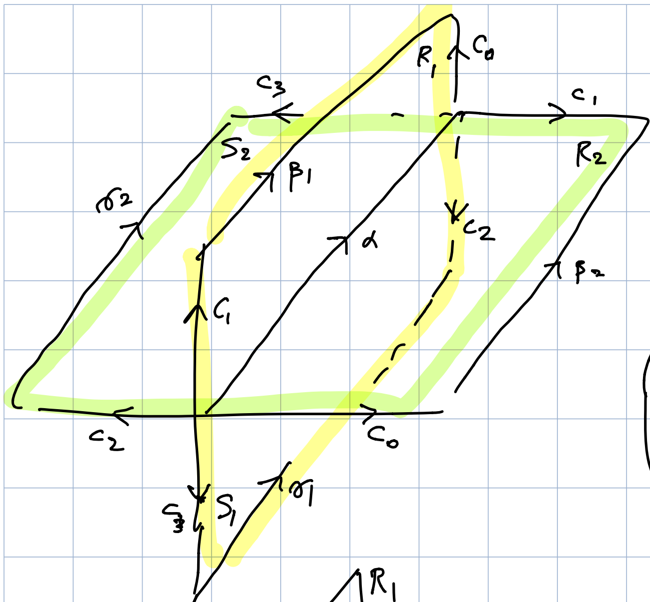
b

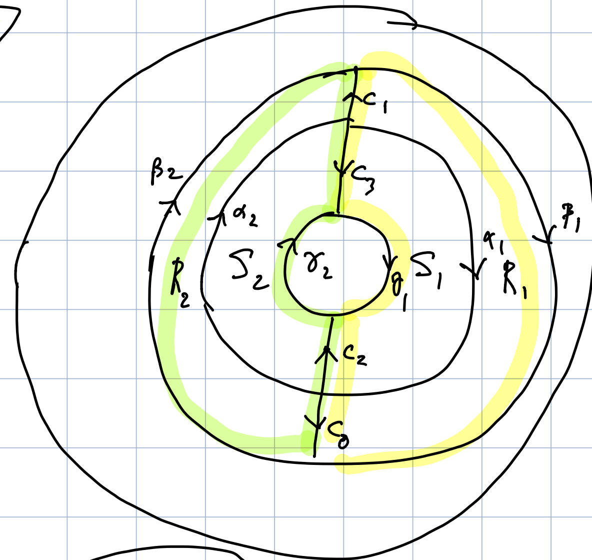


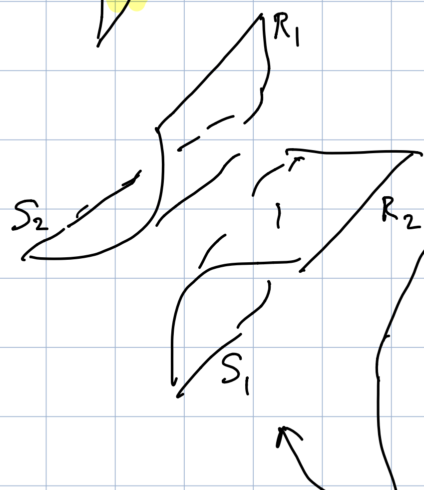
II.2

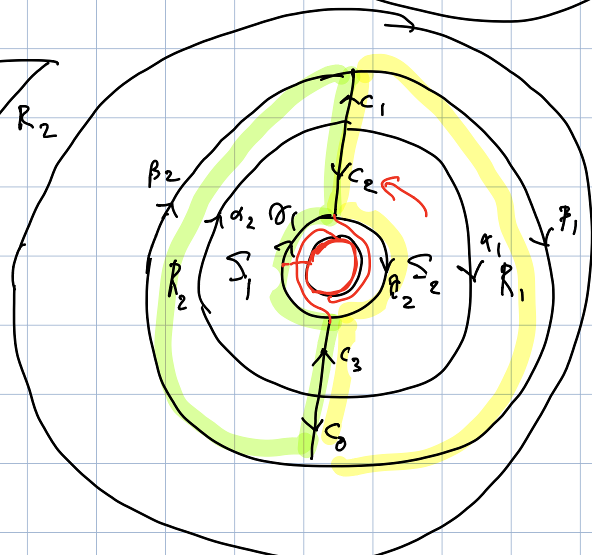












$\blacksquare$

— o —

Classificaz. di $\mathcal{M}_3 = \{$ 3-var. chiuse / con bordo / orientate... $\}$

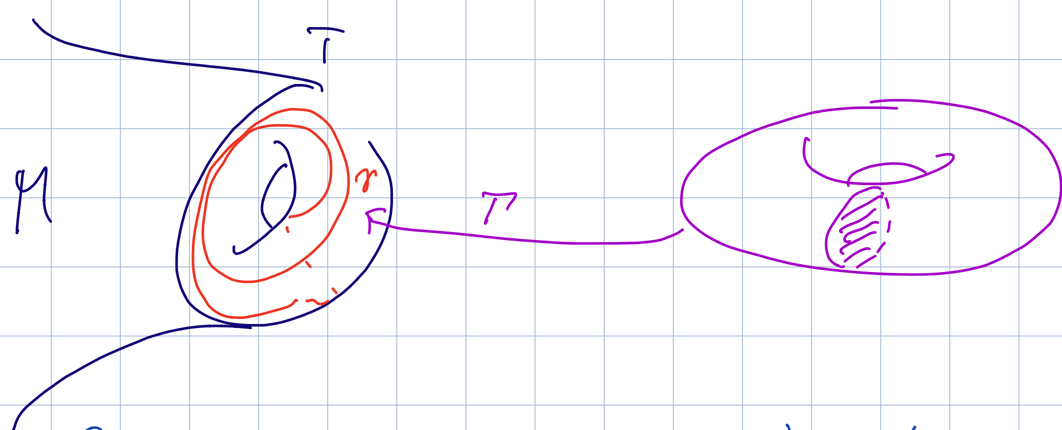
- metodo combinatorio per rappresentarle (oggetti / mosse)
- possibilità di stabilire che $M_1 = M_2$
- invarianti con cui dire che $M_1 \neq M_2$

Dehn filling / surgery

<u>Oss 1</u>: su T toro c'è un solo tipo di curve essenziali a meno di omeo di T; parametrizzate da $\mathbb{Q} \cup \{\infty\}$ se fisso base μ, λ di $H_1(T)$
$\gamma \longleftrightarrow p/q$ se $[\gamma] = [p\mu + q\lambda] \in H_1(T)$.

M 3-var cpt; $\partial M \supset T$; $\gamma \subset T$ essenziale

$M_\gamma = M \cup_\Gamma D^2 \times S^1$ $\qquad \Gamma : \partial(D^2 \times S^1) \to T$
$\qquad\qquad\qquad\qquad\qquad \Gamma : \partial D^2 \times \{*\} \to \gamma$



<u>Oss</u>: M_γ è ben def (indip. da Γ); infatti
1) Si attacca $\mu \to \gamma$
2) Resta da attaccare D^3 lungo la S^2 creata: si fa in solo modo poiché ogni automorfismo di S^2 si estende a D^3.

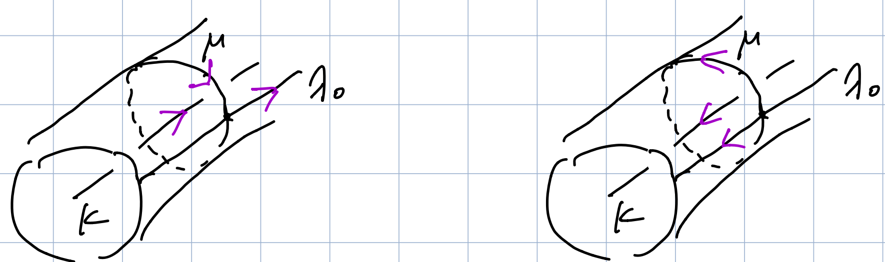
Dehn filling.

$L \subset S^3$ link ; $E(L) = S^3 \setminus \mathring{U}(L)$

$\partial E(L) = T_1 \cup \dots \cup T_k$

Chirurgia di Dehn lungo L = riempimento di $E(L)$.

<u>Oss</u>: su ogni $\partial E(L)$ ho base canonica $\pm(\mu, \lambda_0)$



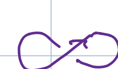
Dunque le chirurgie lungo L sono parametrizzate da $\alpha_1, \dots, \alpha_k \in \mathbb{Q} \cup \{\infty\}$.

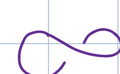
<u>Teo</u> (Lickorish-Wallace): ogni $M^{(3)}$ chiusa orientata è ottenuta da qualche link in S^3 con chirurgia <u>intera</u>.

Dunque posso usare diagrammi con framing diagrammatico.

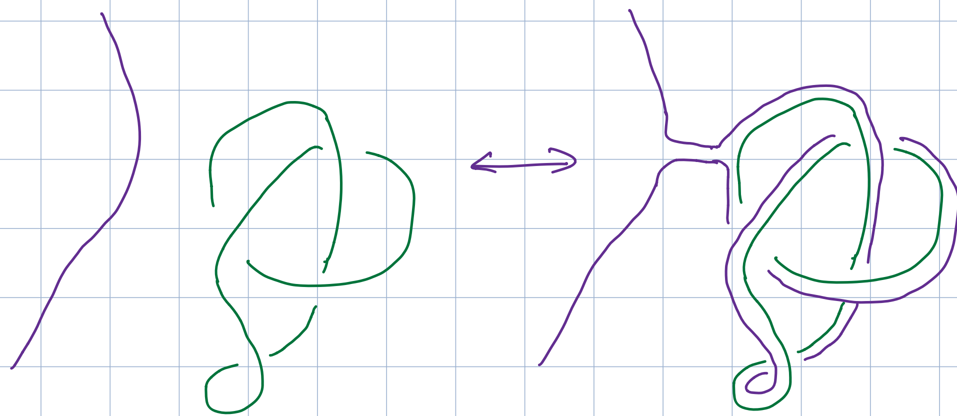
<u>Teo</u> (Kirby): $L_1, L_2 \subset S^3$ link con framing intero;
$M(L_1) \cong M(L_2) \iff L_1$ connessi da mosse:

K_1: $L \longleftrightarrow L \sqcup \bigcirc_{\pm 1}$

 $= \bigcirc_{+1}$

 $= \bigcirc_{-1}$

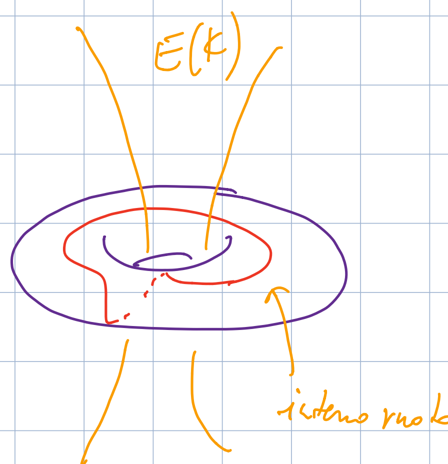
K_2 (diagr.):

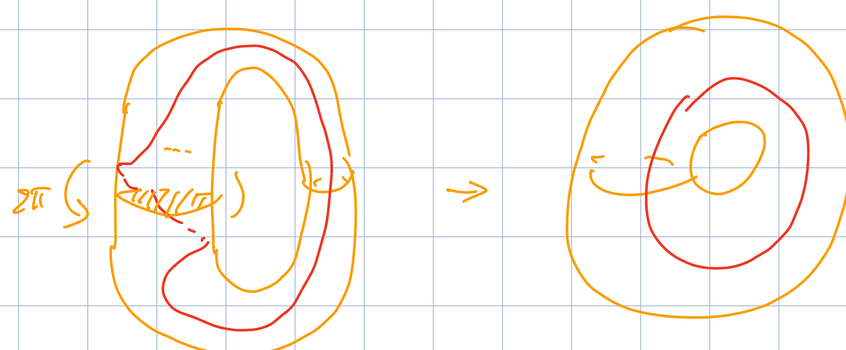


$\boxed{\text{Dimo} \Rightarrow}$

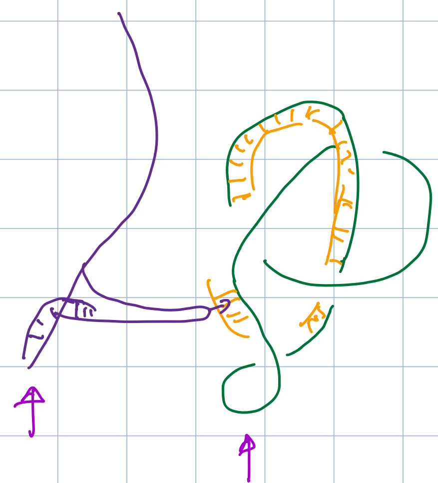
K1

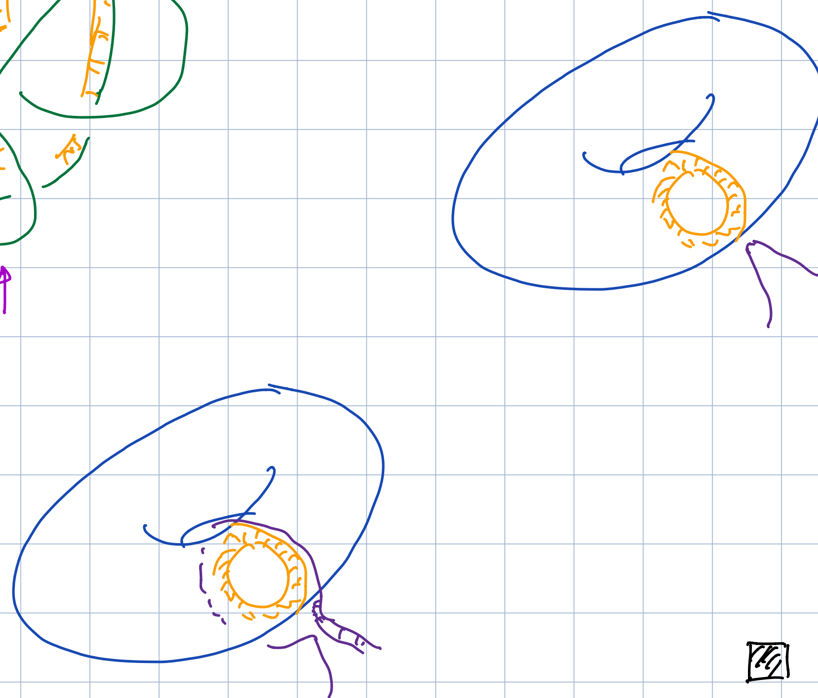
=





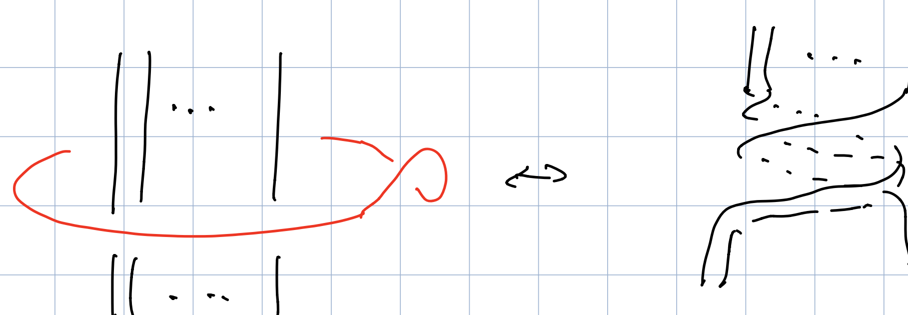
K2





Teo (Fenn-Rourke): $M(L_1) \cong M(L_2) \iff$ legati da mosse

FR :



Fatto: •) poco efficiente per riconoscimento.

•) dà buoni invarianti.

$[\cdot]$ inv. per link con framing $\in \mathbb{Z}[A^{\pm 1}]$.

A partire da questo e con $A = e^{2\pi i/q}$ si trovano invarianti Reshetikhin-Turaev-Witten.

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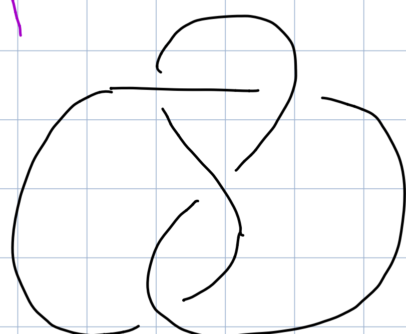
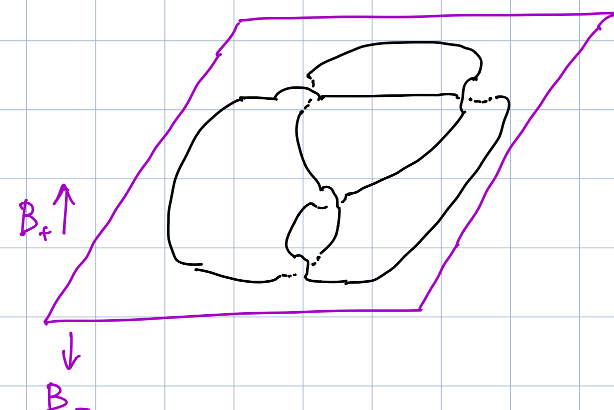
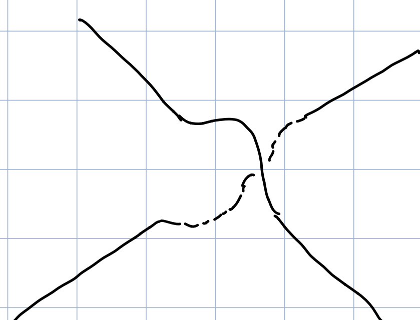
Triangolazioni ideali e spine speciali
(molto efficiente per riconoscimento)

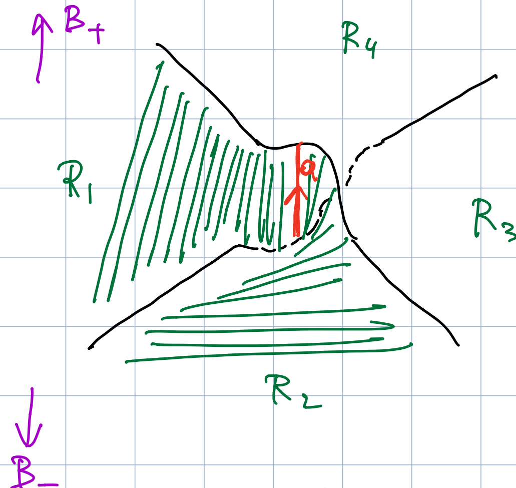
$M = S^3 \setminus$



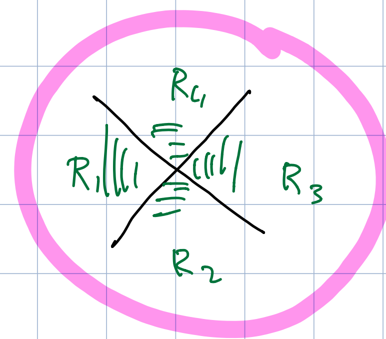
(varietà non cpt)



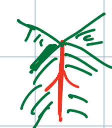


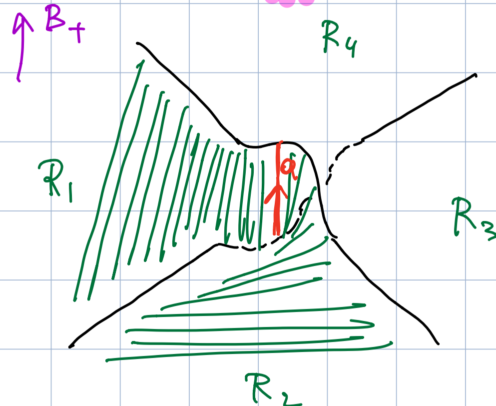


B_+
R_4
R_1
a
R_3
R_2
B_-

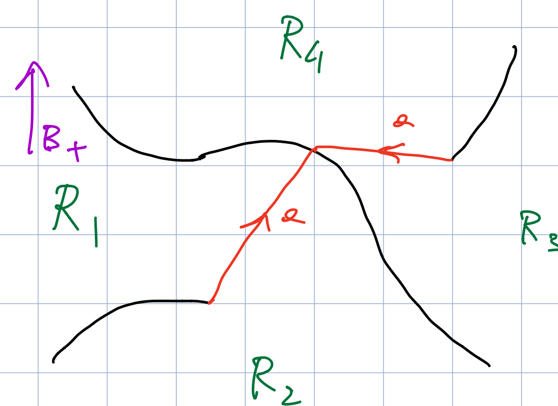


R_4
R_1
R_3
R_2

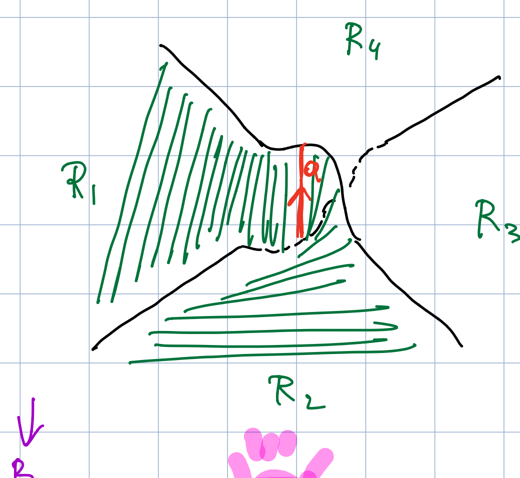




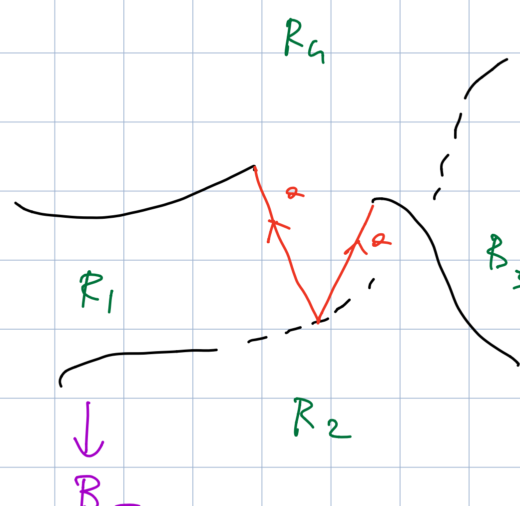
B_+
R_4
R_1
a
R_3
R_2



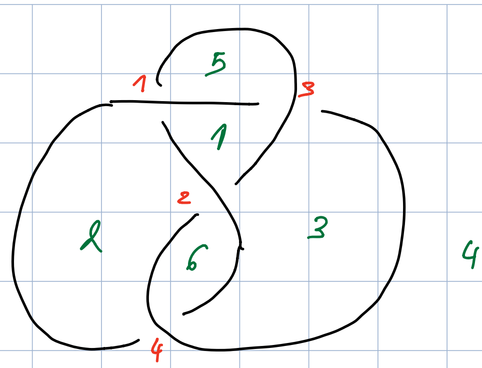
R_4
B_+
a
a
R_1
R_3
R_2



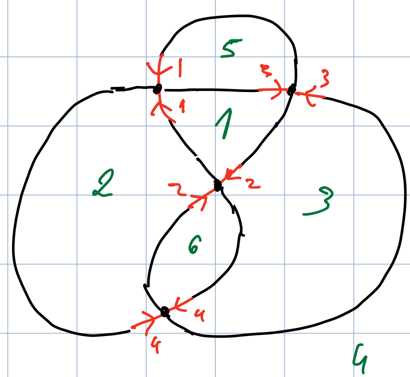
R_4
R_1
a
R_3
R_2
B_-

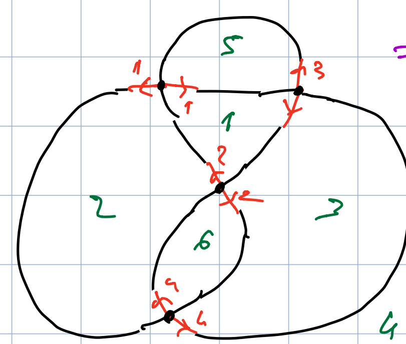


R_4
a
a
R_1
R_3
R_2
B_-



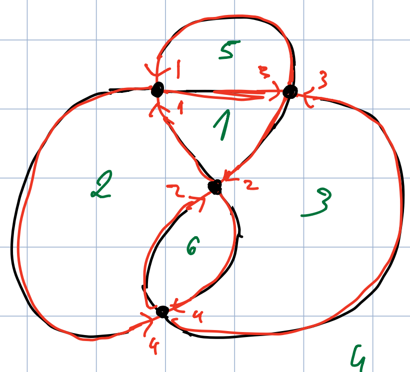
∂B_+

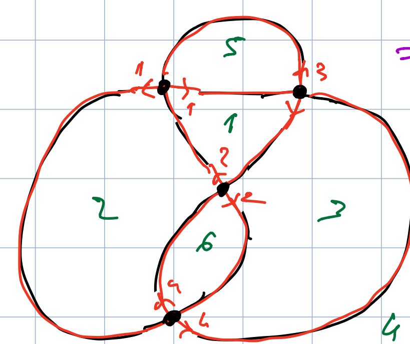




$= \partial B_-$

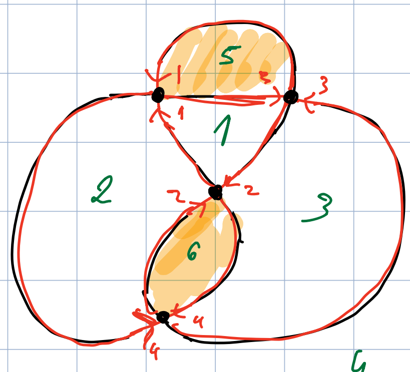
∂B_+

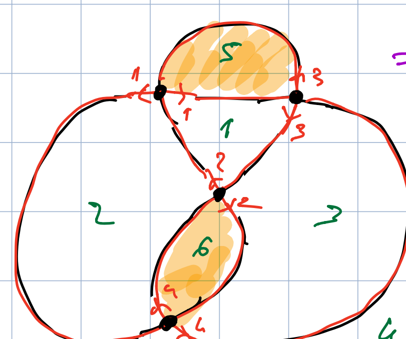




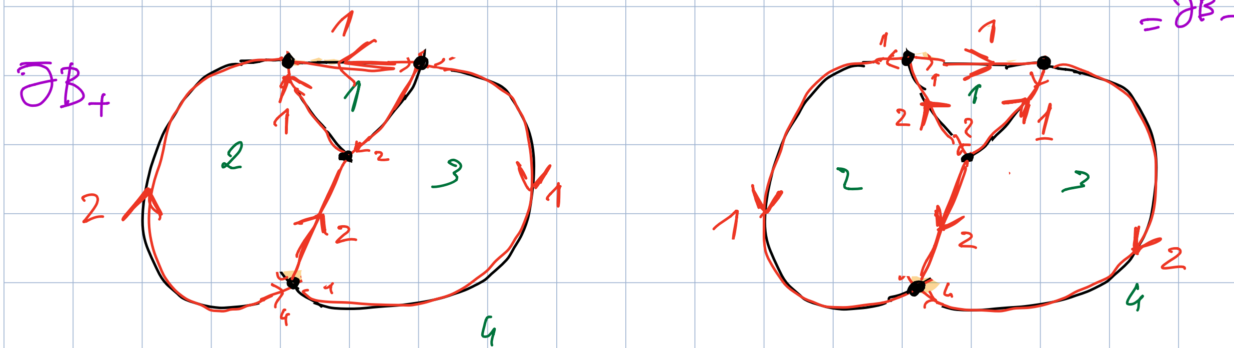
$= \partial B_-$

∂B_+

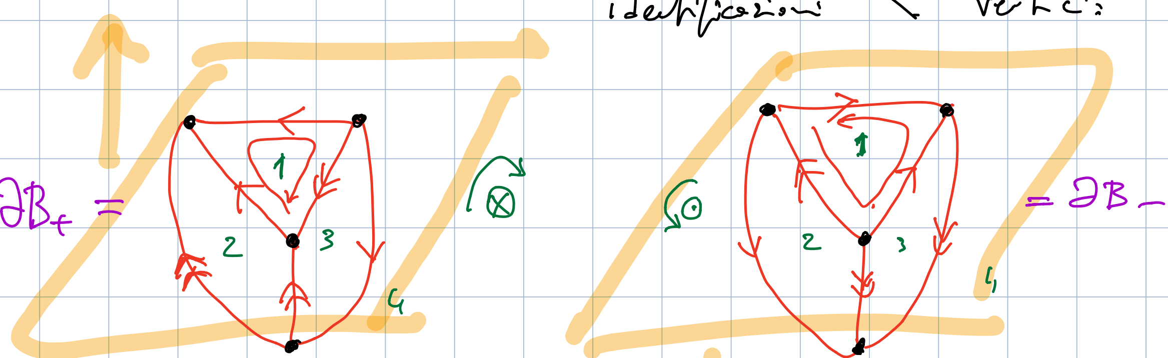




$= \partial B_-$

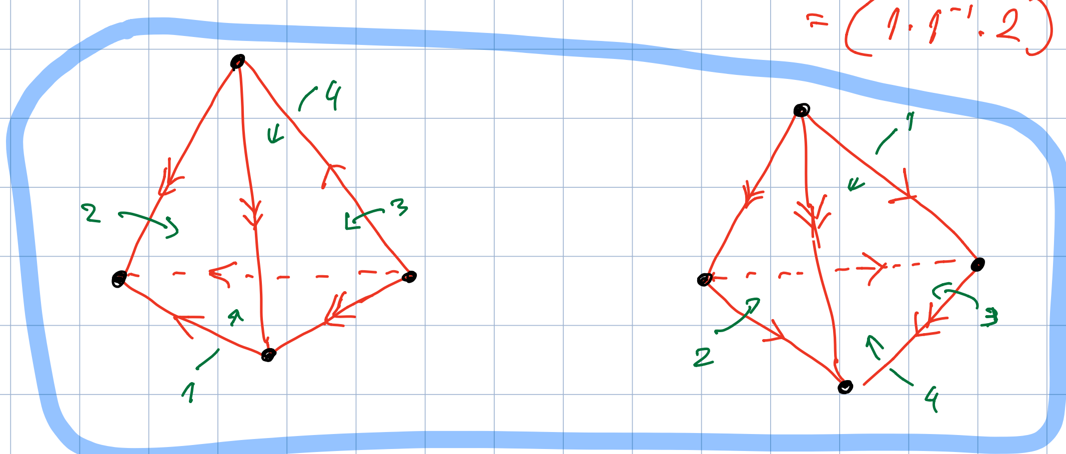


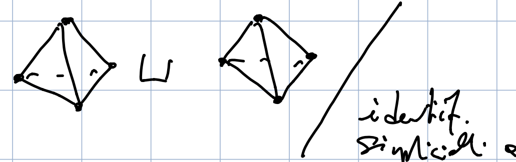
Morale: $S^3 \setminus$ $=$ $B_+ \cup B_-$ con le opportune identificazioni \ vertici



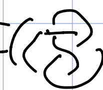
$\partial 1 = 1 \cdot 1^{-1} \cdot 2$

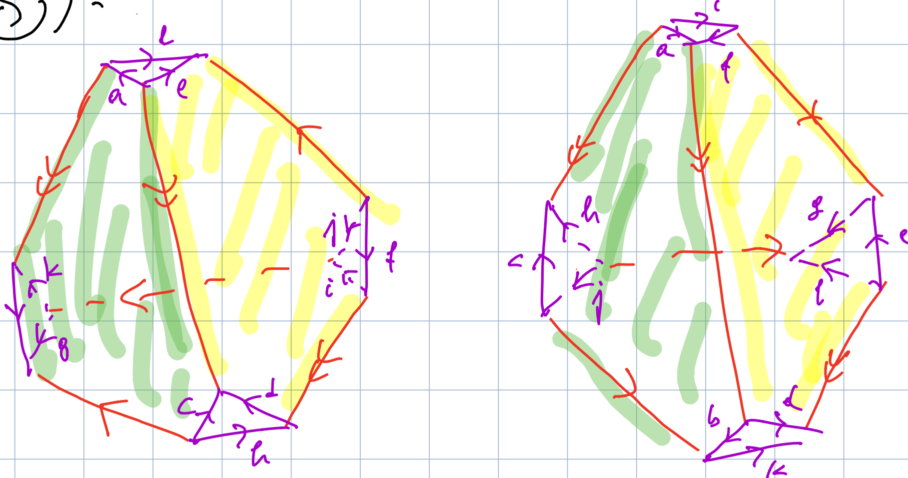
$\partial 1 = 2^{-1} \cdot 1 \cdot 1^{-1}$
$= (1 \cdot 1^{-1} \cdot 2)^{-1}$

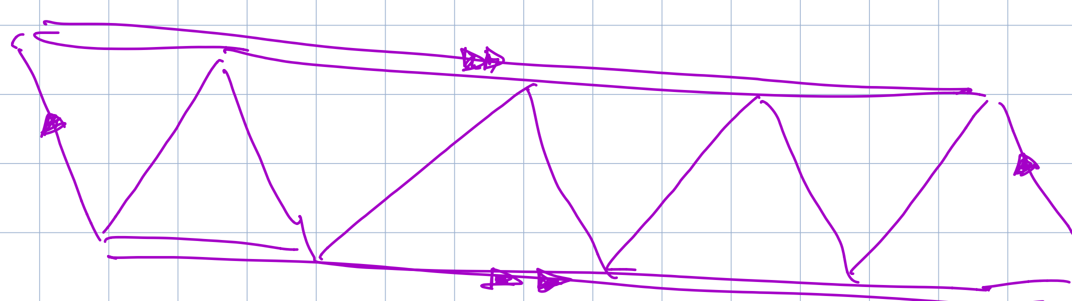


Dunque $(S^3 \setminus$  $) =$  / identif. simpliciali a coppie tra facce \ vertici

(Thurston)

$\Rightarrow E($  $) =$





<u>Oss</u>: se M è chiusa è ben def $M' = M \setminus B^3$
viceversa se $\partial M' = S^2$ è ben def $M = M' \cup_{S^2} D^3$

Nel resto della lezione ogni M è cpt. connessa con $\partial M \neq \emptyset$.

Oss: $N = S^3 \setminus$ $M = E(\ )$

- $M \subset N$ $M = \bar{N} \setminus U(\text{vertice})$
- $N = M \setminus \partial M = S^3 \setminus \overline{U(\ )}$.

Def: chiamo triangolazione ideale di M una delle seguenti equivalenti:

1) $M \setminus \partial M = \left(\sqcup \dots \sqcup \right) \big/ \text{identif. simpl. tra facce}$ $\setminus$ vertici

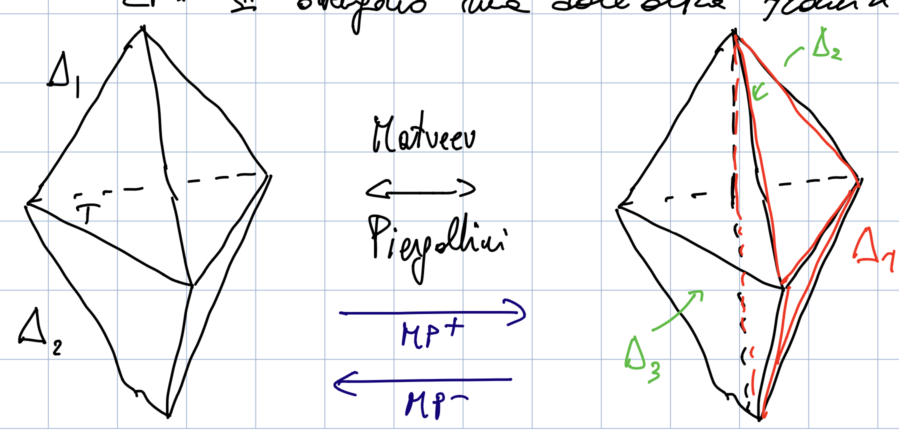
2) $(M, \partial M) = \left(\sqcup \dots \sqcup \right) \big/ \text{identif. indotte da simpl. tra facce}$

3) $\hat{M} = M \big/ \text{ogni comp. di } \partial M \text{ collassata a un pto} = \left(\sqcup \dots \sqcup \right) \big/ \sim$

∂M collassate $\longleftrightarrow$ vertici

Teo: 1) Ogni 3-varietà ha triang. ideali (con ≥ 2 tetraedri)

2) Due triang. ideali danno la stessa 3-varietà $\iff$ si ottengono una dall'altra tramite mosse



Date M_j con $\mathcal{T}_j$ triang. ideale $j=1,2$: $M_1 \overset{?}{\cong} M_2$

$$\begin{array}{ccccc} & & \mathcal{S}_1^{(j)} & \cdots\cdots & \mathcal{S}_{m(j)}^{(j)} \\ & \nearrow^{(MP^+)^k} & \downarrow MP^- & & \downarrow MP^- \\ \mathcal{T}_j & & \mathcal{R}_1^{(j)} & \cdots\cdots & \mathcal{R}_{N(j)}^{(j)} \end{array} \qquad j=1,2$$

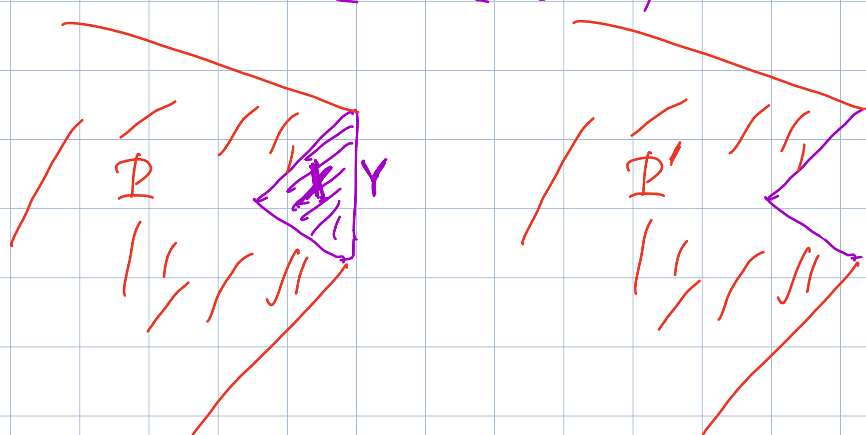
confronto ogni $\mathcal{R}_i^{(1)}$ con $\mathcal{R}_t^{(2)}$: se due sono combin. equivalenti <u>OK</u> : sì

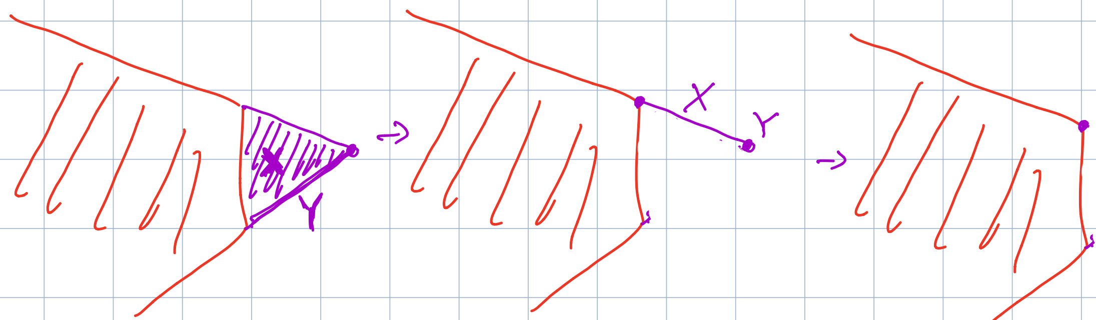
—— o ——

<u>Nozione PL</u> : collasso elementare

P poliedro, $X \subset P$, Y faccia di X t.c. Y è faccia <u>solo</u> di X

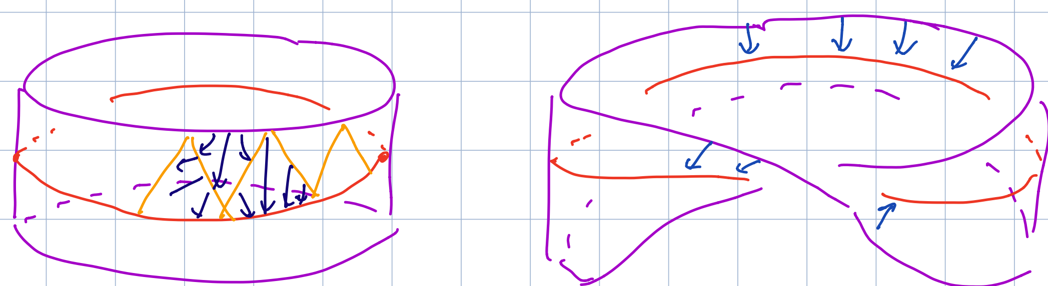
$P \rightsquigarrow P \setminus (\mathring{Y} \cup \mathring{X})$





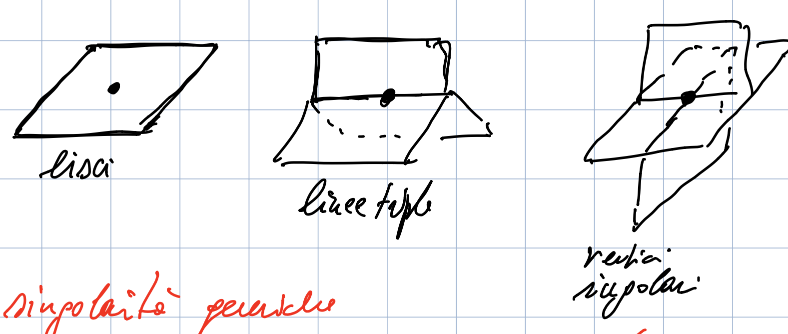
Chiamo collasso una successione di collassi elementari.

Def: data M 3-var (cpt con $\partial M \neq \emptyset$) chiamo spina di M un $P \subset \mathring{M}$ t.c. M collassa su P.

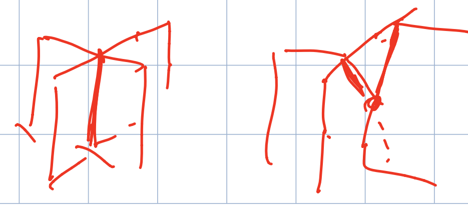


Def: dico che P è speciale se:

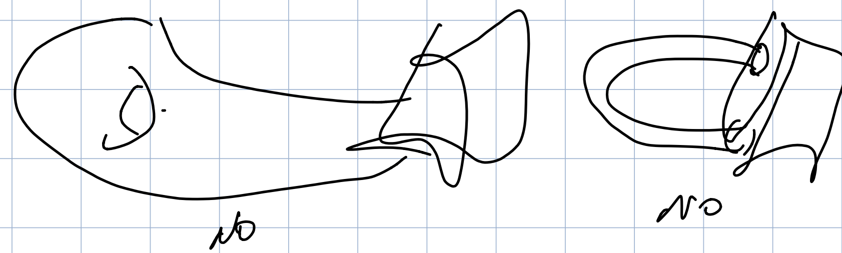
• loc. è così:



(oss sono le singolarità generiche per una bolla)

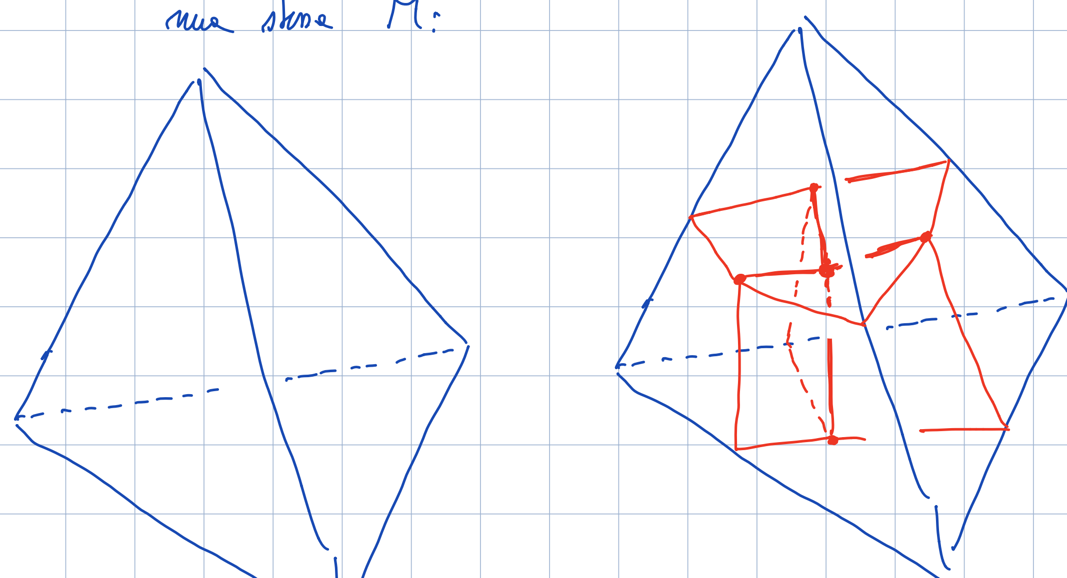


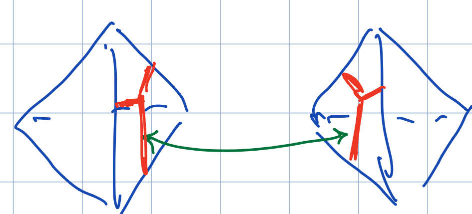
• le componenti dei pti lisci sono dischi

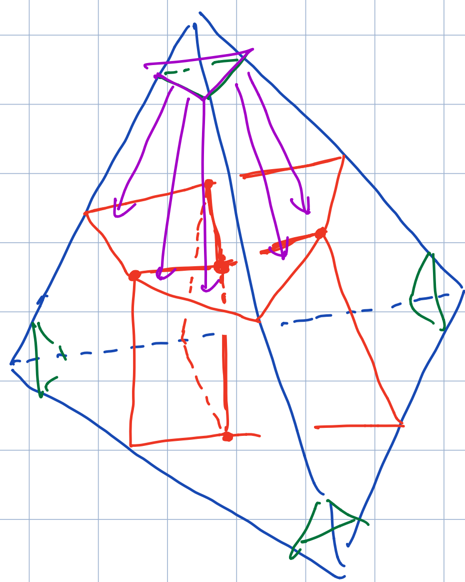


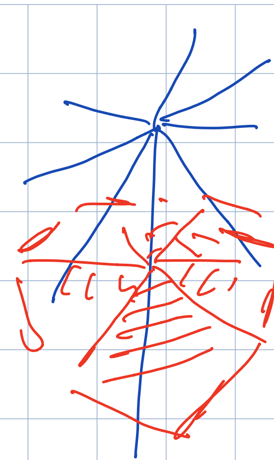
Prop: se due varietà hanno spine speciali omeomorfe sono omeomorfe.

Prop: esiste una corrispondenza biunivoca (dualità) tra le spine speciali e le triang. ideali di una stessa M:









Version per spine mosse MP.