LDT 17/4/2020

$$\bar{p}(D) = \sum_{\sigma} A^{\ell(\sigma)} \cdot A^{-r(\sigma)} \cdot \left(-A^2-A^{-2}\right)^{|\sigma|-1}$$

$$[\times] = A\cdot[\ldots] + A^{-1}[\ldots]$$
$$[\bigcirc \, |\,] = (-A^2-A^{-2})\cdot[\,|\,]$$
$$[\bigcirc] = 1$$

$$p(D) = \bar{p}(D)\cdot\left(-A^2-A^{-2}\right)$$

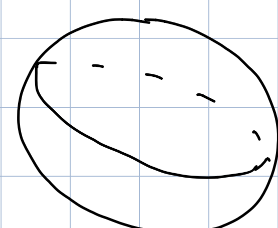
Visto $\mathrm{Span}(\bar{p}(D)) = 4\cdot c(D)$

<u>Cor</u>: $\mathrm{Span}(p(D)) = 4\cdot(c(D)+1)$

<u>Teo</u> (Jordan-Schönflies 3D): $\Sigma \subset S^3$, $\Sigma \cong S^2$ (PL o DIFF)

$\Rightarrow S^3 = B_1 \cup B_2$, $\partial B_1 = \partial B_2 = \Sigma$, $\bar{B}_1 \cong \bar{B}_2 \cong D^3$.

FALSO TOP



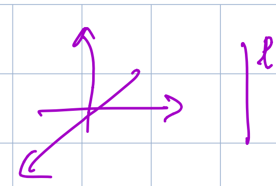
<u>Dim</u> (DIFF): $\Sigma \neq S^3$: wlog $\infty \notin \Sigma$.

Basta vedere che $\Sigma \cong S^2_{\text{DIFF}} \subset \mathbb{R}^3$ è ∂B, $\bar{B} \cong D^3$.

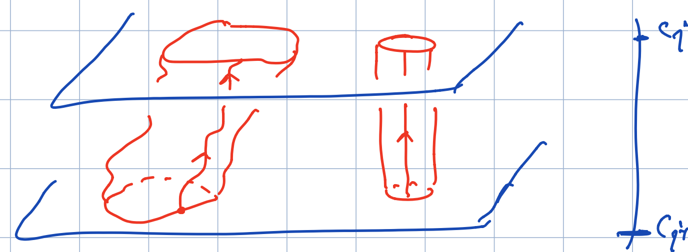
<u>Fatto</u>: scegliendo ℓ a caso (a meno di perturbazione) ho $h = \pi_\ell|_\Sigma$ soddisfa quanto: $h: \Sigma \to \mathbb{R}$

- ha # finito pti con differenziale nullo (Σ piano tg. orizzontale)

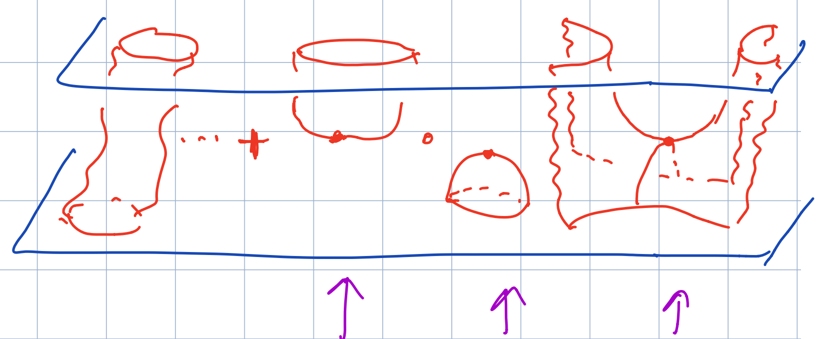
- in tali punti la matrice hessiana è non degenere
- tali pti hanno valori di h diversi.



Chiamo $c_0 < c_1 < \dots < c_N$ tali valori: su ogni $h^{-1}([c_{j-1}+\varepsilon, c_j-\varepsilon])$

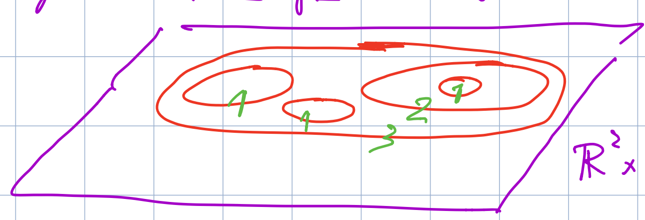


$h^{-1}([c_j-\varepsilon, c_j+\varepsilon])$

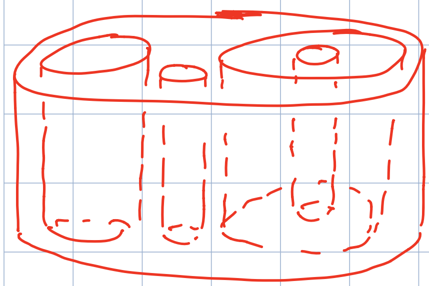


Prendo $c_0 < a_0 < c_1 < a_1 < \dots < c_N < a_N$

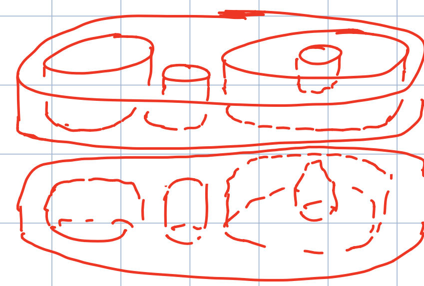
Chiamo Σ' la superficie ottenuta da Σ facendo le seguenti chirurgie su ogni $h^{-1}(a_j)$:



$\mathbb{R}^2 \times \{a_j\} = h^{-1}(a_j)$



Tagli a livello a_j + aggiungo dischi/ quelli più interni sono più alti sopra e più bassi sotto



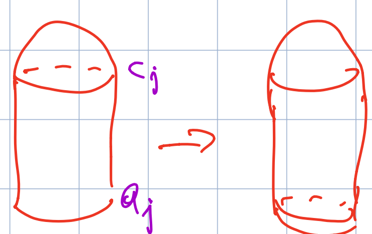
Tale Σ' è una superficie con molte componenti:

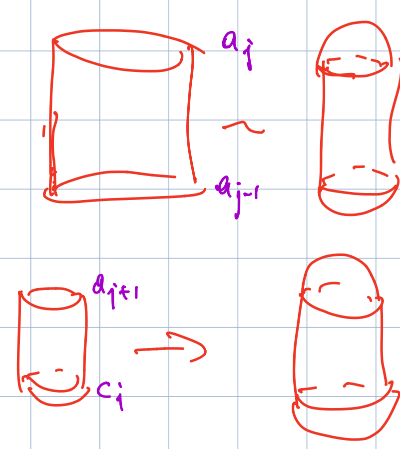
Ma ogni componente contiene solo min o max + tubi altri min o max + selle.

Claim: ogni componente di Σ' è bordo di un D^3. (ignorando le altre componenti).

Infatti:

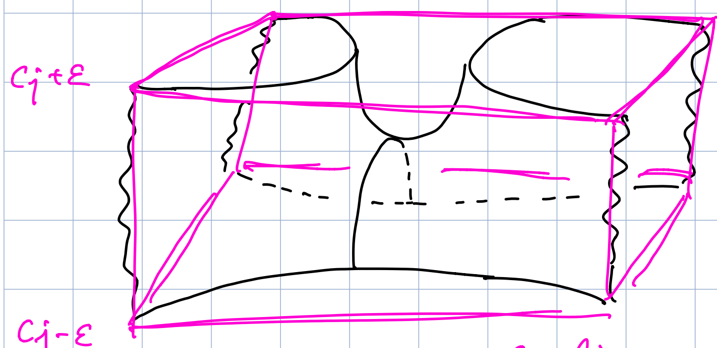
- se non contiene selle





$\to$ facile

• Se contiene selle



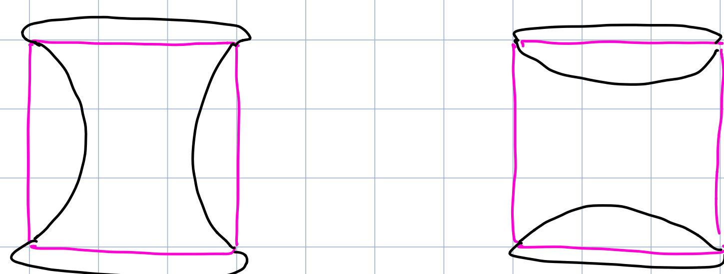
+ il resto verticale.

sopra $c_j+\varepsilon = a_j$

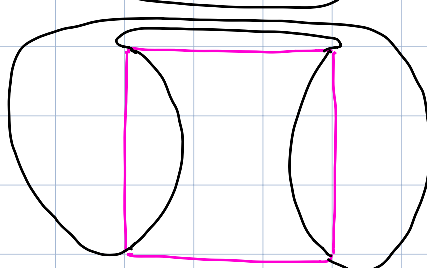
sotto $c_j-\varepsilon = a_j$

Cosa può accadere fuori dal cubo?

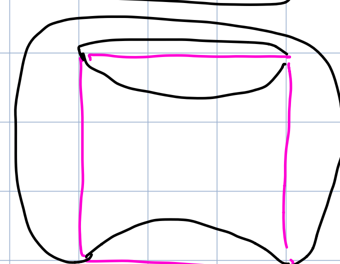
(1)



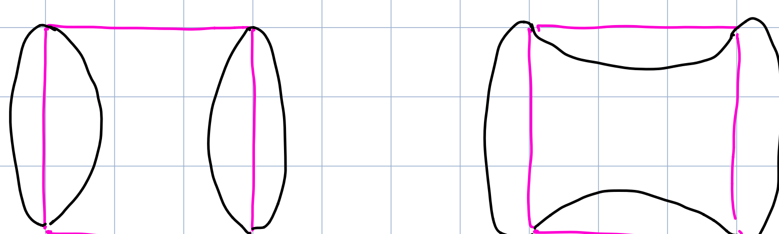
(2)



o (2)'

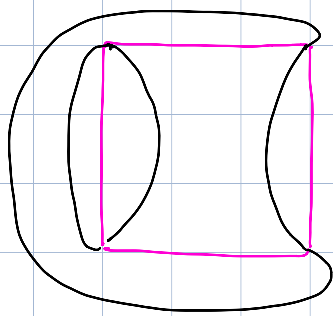


(3)

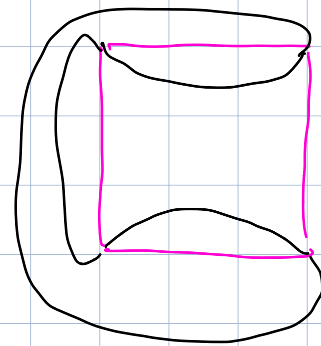


$\Rightarrow$ (1) capovolto

(4)



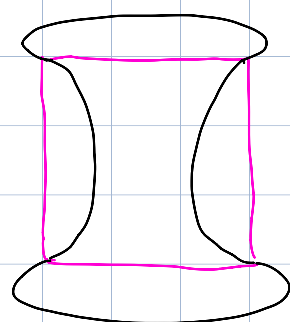
o (4)'



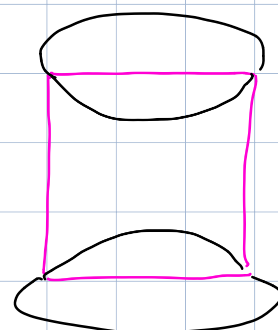
$\Rightarrow$ (2) capovolta

Tratto (1) e (2):

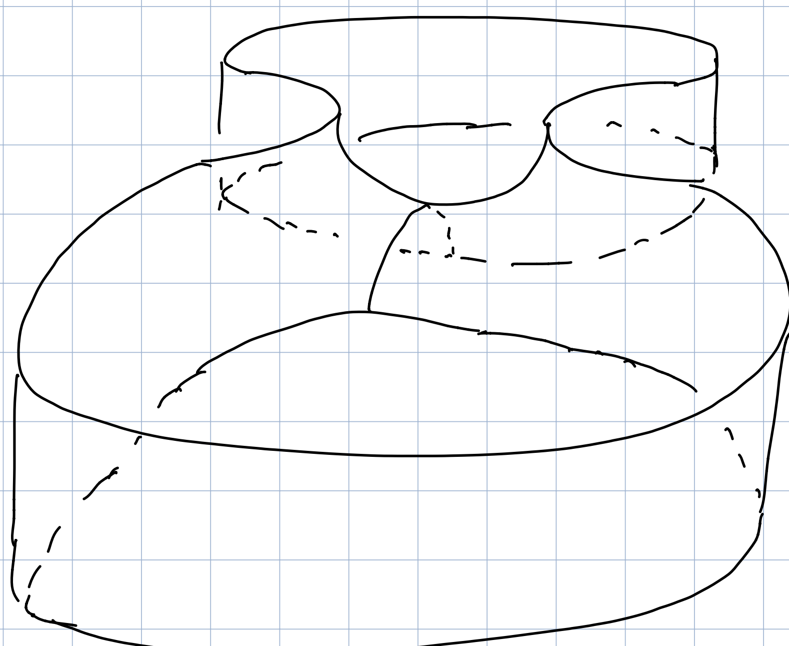
(1)



sopra

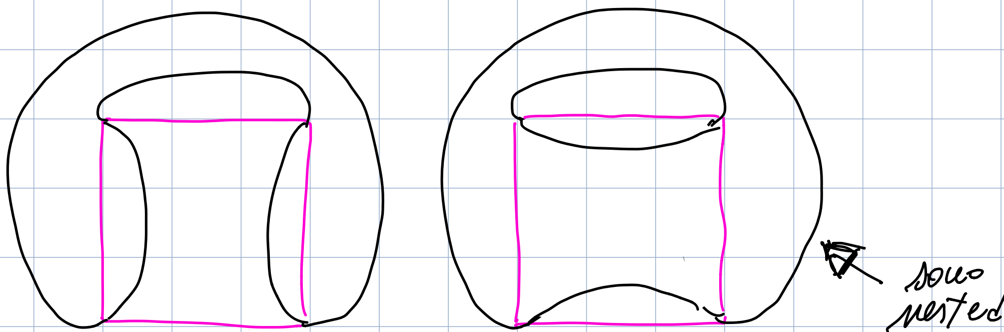


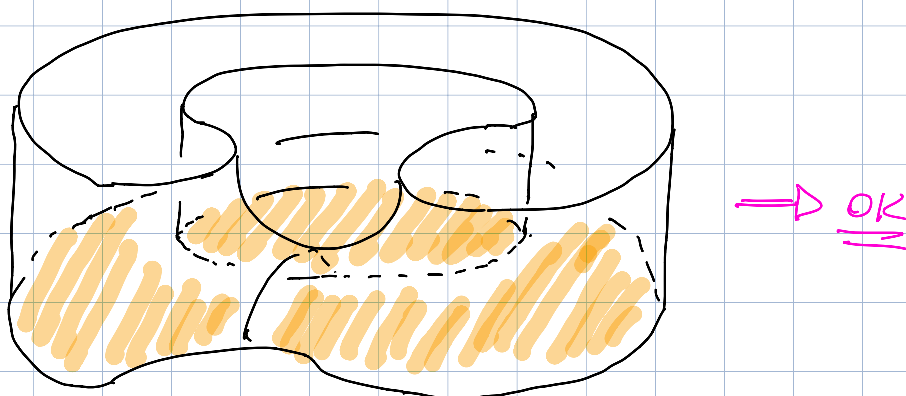
sotto



$\Rightarrow$ OK

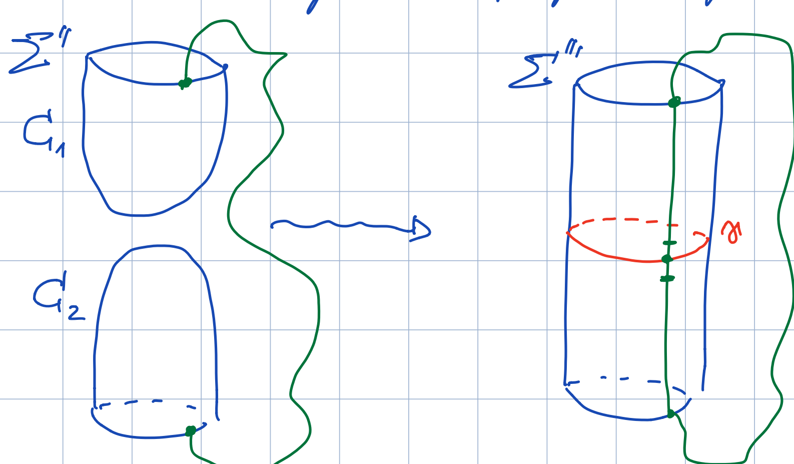
(2)





Claim provato: ogni corpo di Σ' è ∂D^3.

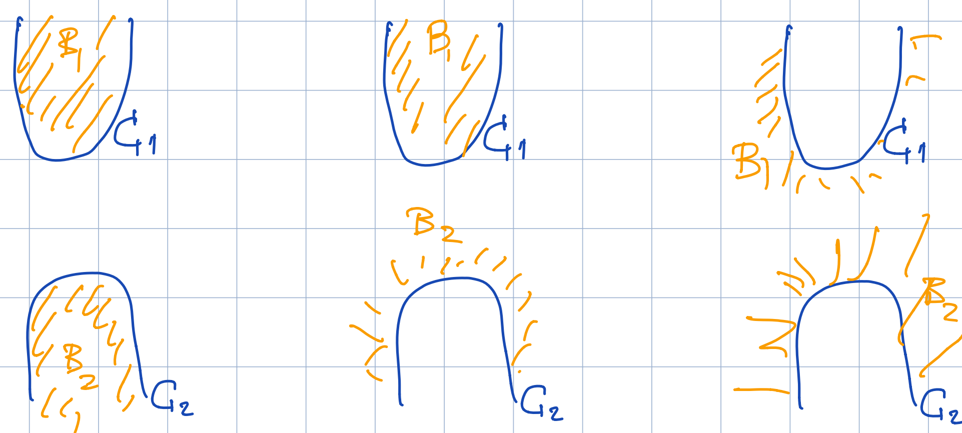
Conclusione: affermo che disfacendo una alla volta le operazioni di chirurgia le proprietà permane.

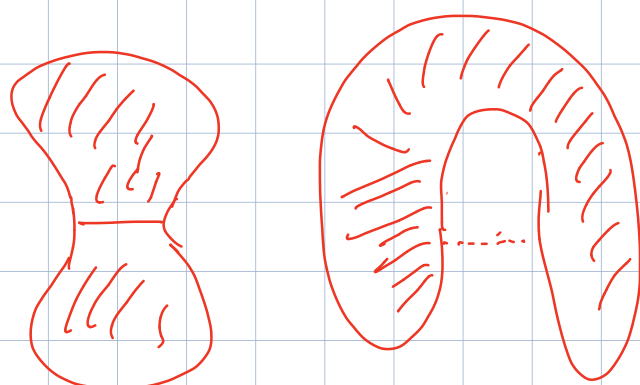


Se $C_1 = C_2$ come corpo di Σ'' $\qquad (\Sigma' \dashrightarrow \Sigma'' \dashrightarrow \Sigma)$

allora γ è non separante su Σ'';
tale proprietà sopravvive alla chirurgia
$\Rightarrow$ avrei γ non separante in Σ : falso $(\Sigma \cong S^2)$.

Dunque $C_1 \neq C_2$; $C_1 = \partial B_1$, $C_2 = \partial B_2$





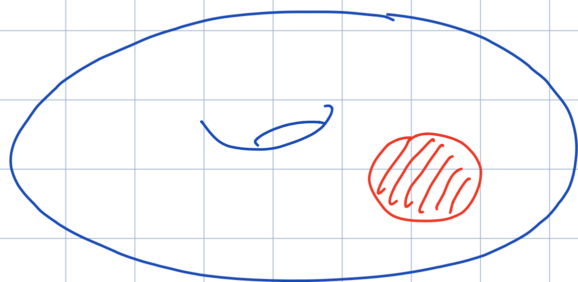
ASSURDO
$\overline{B_1} \cup \overline{B_2} = \mathbb{R}^3$ cpt.

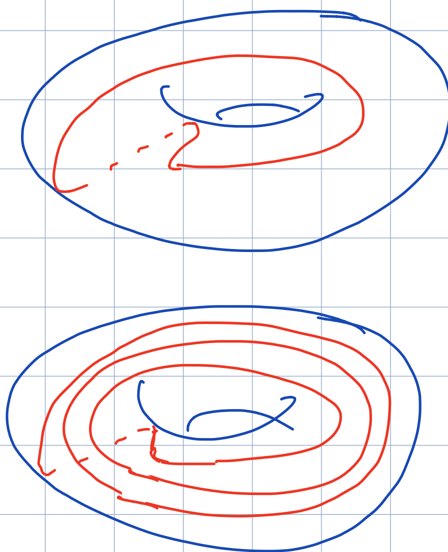
$B = \text{"}B_1 \cup B_2\text{"}$ $\qquad$ $B = B_2 \setminus B_1$

∎

<u>Oss</u> : T toro ; ci sono su T solo* 2 tipi di curve semplice chiusa α : bordante $\alpha = \partial D^2$ oppure non bordante.

* a meno di omeo di T





Ragiono: se taglio T lungo α ho 2 casi:

1) disconnessa

χ invariato

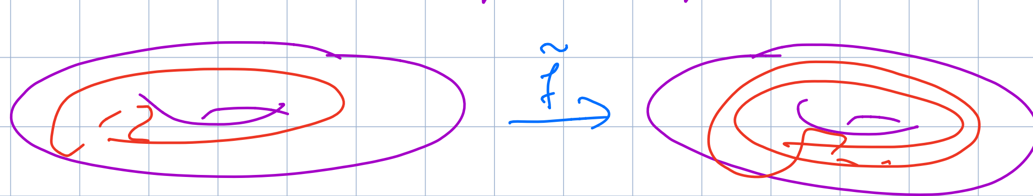
$\underset{\text{toro con buco}}{0 = 1} + \underbrace{(-1)}_{\text{disco}}$

2) connessa

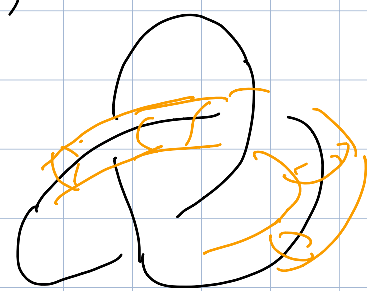
$\underset{\text{toro}}{0} = 0$

anello perché è sfera $\setminus$ 2 dischi

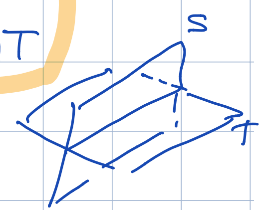
$\chi(\Sigma \setminus \text{dischi}) = \chi(\Sigma) - \#\text{dischi}$



Teo: $T \subset S^3$ toro $\Rightarrow$ $T = \partial U(K)$
DIFF

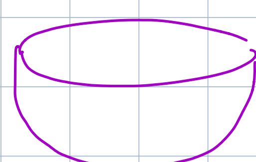


Dim: Claim: $\exists\, S$ sfera in S^3 t.c. $S \pitchfork T$ e $S \cap T$ contiene γ non banale su T.

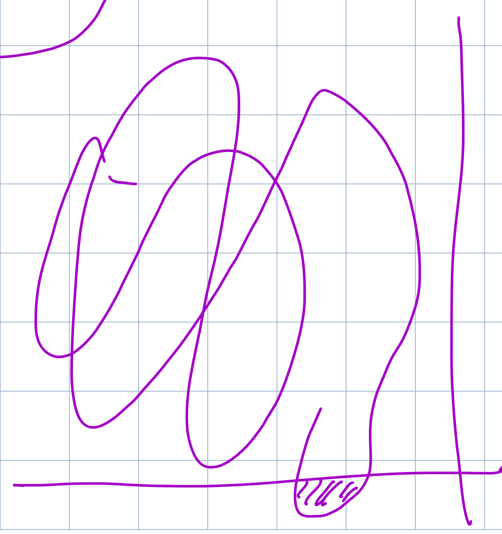


Infatti: $T \neq S^3 \Rightarrow T \subset \mathbb{R}^3$; prendo h funzione altezza come sopra; valori critici $c_1 < \dots < c_N$; prendo $c_1 < a_1 < c_2 < a_2 < \dots < c_N < a_N$. Pongo $T_j = h^{-1}((-\infty, a_j])$

Q: T_j è una unione di sfere con buchi?

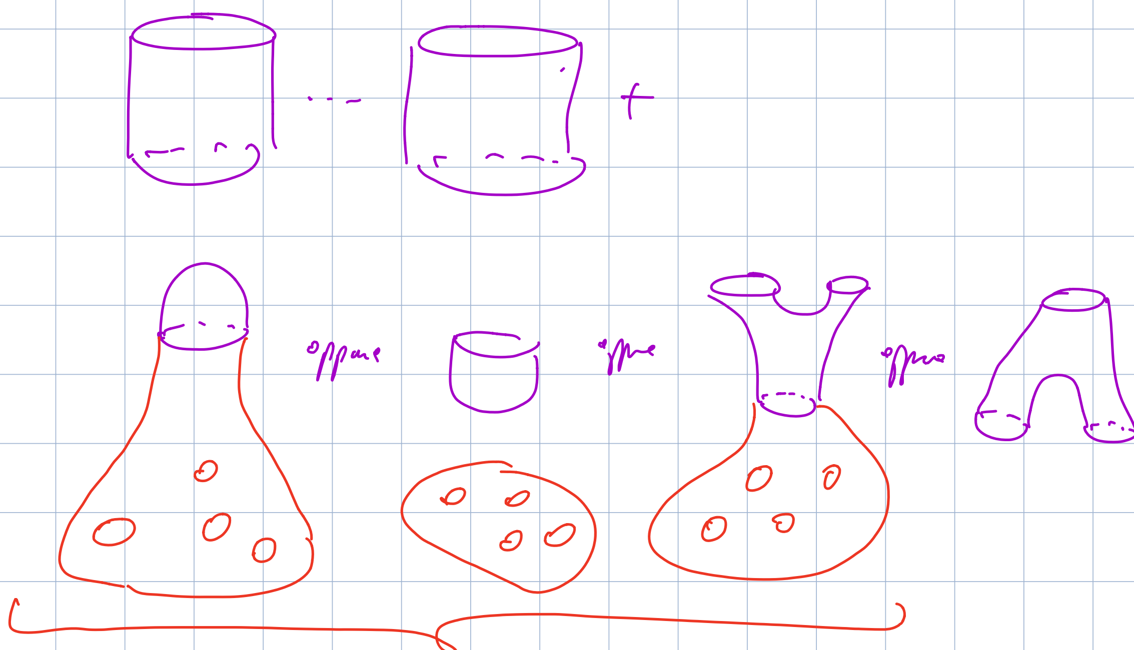
A_1: T_1 sì:  = disco

A_N: $T_N = T$ no

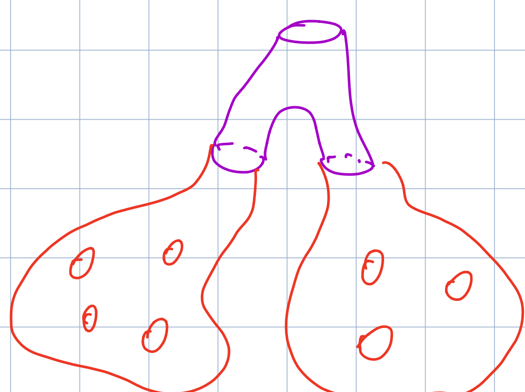


⇒ ∃ j t.c. T_j è unione di sfere con buchi
T_{j+1} non lo è.

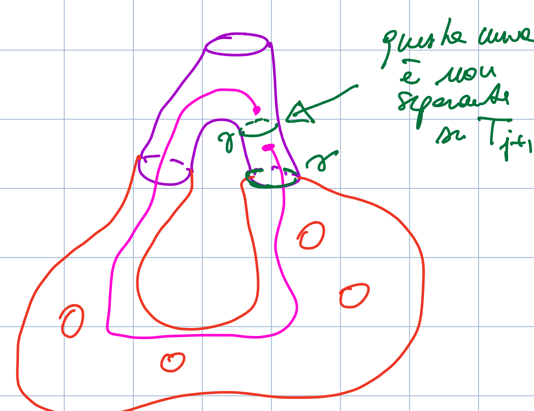
passaggio da T_j a T_{j+1} è attaccamento di:



la risposta rimarrebbe sì



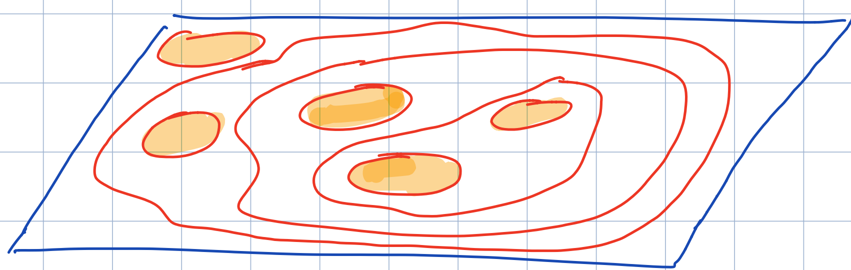
rimane sì

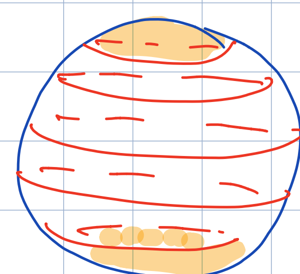


γ rimane non separante in T

γ è una delle componenti di $T \cap \underbrace{\left(\mathbb{R}^2 \times \{a_j\} \cup \{\infty\}\right)}_{S}$

Esamino $S \cap T$ su S:





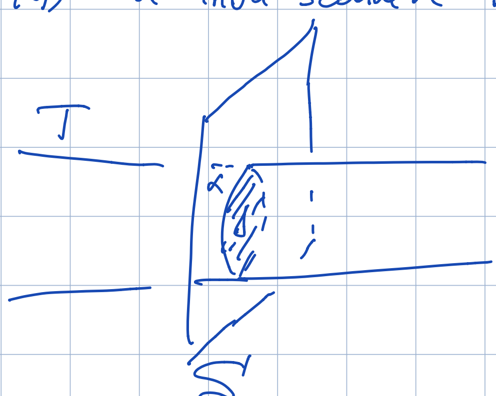
Prendo α innermost in S:

$\alpha = \partial \Delta \qquad \Delta \subset S, \quad \Delta \cap T = \alpha$

Due casi: (1) α non banale in T ($\Rightarrow$ OK)

(2) α banale in T ($\Rightarrow$ posso eliminarla)

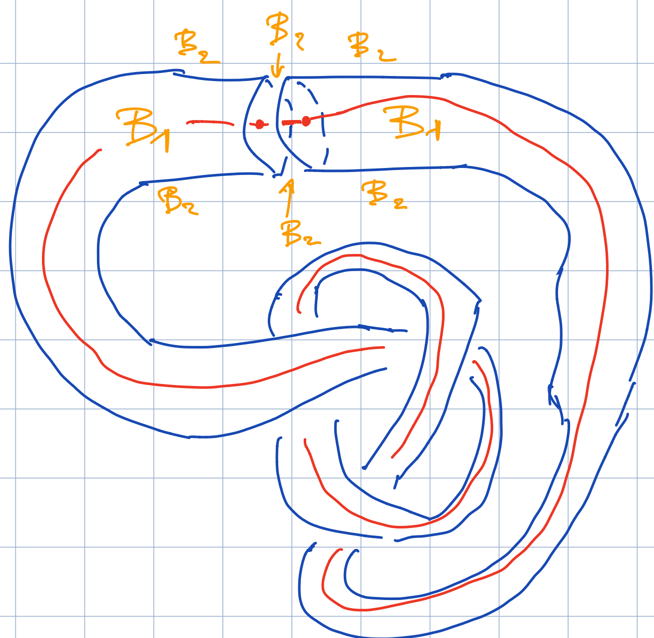
(1) α non sconnette T:



taglio T lungo α e tappo con $\Delta \times \{\pm\varepsilon\}$

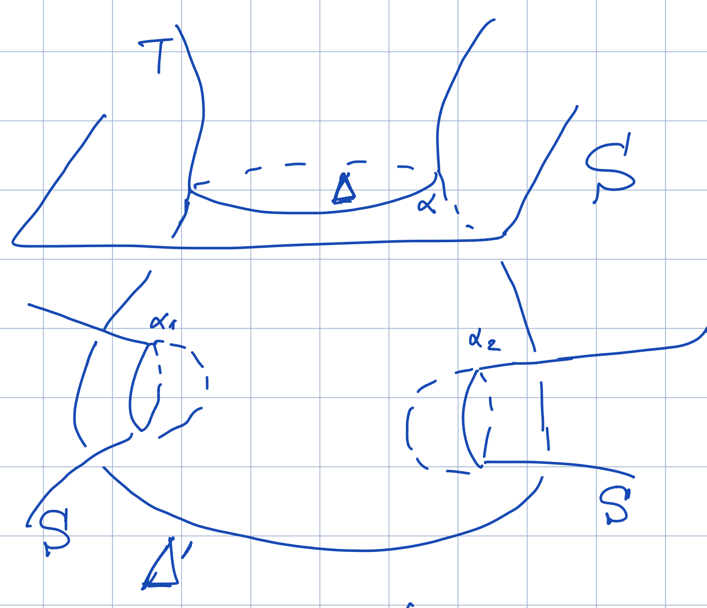
Σ sfera

$$\Sigma = \partial B_1 = \partial B_2 \qquad B_1 \cong B_2 \cong D^3$$



$\Longrightarrow$ <u>ok</u>

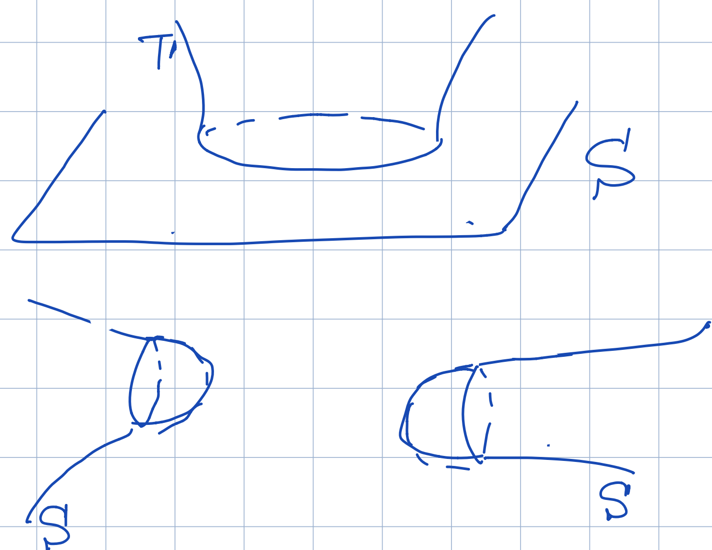
(2) α sconnette T $\Longrightarrow$ $\alpha = \partial \Delta'$ $\quad \Delta' \subset T$



$$\Delta' \cap S' = \underbrace{\alpha \cup \alpha_1 \cup \alpha_2 \cup \dots}_{\text{tutte banali in } T}$$

Ora $\Delta \cup \Delta' = \Sigma$ sfera S^2 $\Rightarrow$ bordo D^3 da entrambe le parti

Modifico posizione di T tramite isotopia



$T' \cap S = T \cap S \setminus (\underbrace{\alpha \cup \alpha_1 \cup \alpha_2 \dots}_{\text{banali in } T})$

$\Downarrow$

$T' \cap S$ contiene γ non banale in T

$\blacksquare$