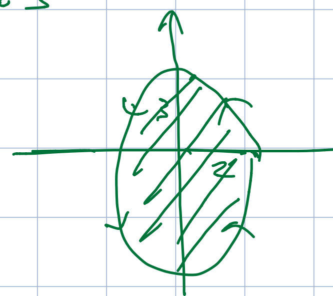
25/6/19 - n.7

$$\int_\alpha \left(\left(e^{x^2}-y\right)dx + \left(x - e^{-y^3}\right)dy\right)$$

$\alpha : [0, 2\pi] \to \mathbb{R}^2$

$\alpha(t) = \begin{pmatrix} 2\cos(t) \\ 3\sin(t) \end{pmatrix}$

$\alpha = \partial E$ E = ellisse di semiassi 2 e 3



$$\int_\alpha \omega = \int_{\partial E} \omega = \int_E d\omega$$

$$= \int_E -dy\,dx + dx\,dy = 2\int_E dx\,dy$$

$$= 2A(E) = 2\pi \cdot 2 \cdot 3 = 12\pi.$$

$\boxed{\text{24.2 libro}}$ Per quali $k \in \mathbb{R}$ le $\begin{pmatrix} 2+k^3 & 1-3k \\ 2+k & 18 \end{pmatrix}$

hanno basi ortonormali di autovettori

$\iff$ simm $\iff$ $2+k = 1-3k$ $k = -1/4$

$\boxed{\text{7.6 libro}}$ In $\mathbb{P}^2(\mathbb{R})$ trovare

$\{[3t-4 : -6 : 2t] : t \in \mathbb{R}\} \cap \{[t : t^2+2 : -2] : t \in \mathbb{R}\}$

$[3t-4 : -6 : 2t] \overset{?}{=} [s : s^2+2 : -2]$

$$\text{rank}\begin{pmatrix} 3t-4 & -6 & 2t \\ s & s^2+2 & -2 \end{pmatrix} \overset{?}{=} 1$$

$3t-4 = -\lambda t \qquad t = \dfrac{4}{3+\lambda}$

$$\frac{\cancel{8}^{\,2}(\lambda^2+2)}{3+\lambda} = \cancel{12}^{\,3}$$

$$2\lambda^2+4 = 9+3\lambda$$

$$2\lambda^2-3\lambda-5=0$$

$$(2\lambda-5)(\lambda+1)$$

$\lambda=-1 \qquad \lambda = 5/2$

$t=2 \qquad t=\dfrac{8}{11}$

$[2 : -6 : 4] \qquad [-20 : -66 : 16]$

$[-1 : 3 : -2] \qquad [10 : 33 : -8]$

OK $\qquad$ OK

$$\cap = \{ [1 : -3 : 2],\ [10 : 33 : -8] \}$$

$\boxed{8/1/20 \text{ n.}3}$

Per quali $t\in\mathbb{R}$ la retta di $\mathbb{P}^2(\mathbb{R})$ per $[2 : -1 : t]$ e $[3 : t-1 : -2]$ contiene $[1 : -6 : 12]$.

$$\{[3t-4 : -6 : 2t] : t\in\mathbb{R}\} \cap \{[t : t^2+2 : -2] : t\in\mathbb{R}\}$$

$$= \{[3u-4 : -6 : 2u] : u\in\mathbb{R}\} \cap \{[\varphi : \varphi^2+2 : -2] : \varphi\in\mathbb{R}\}$$

$$= \{[3u-4 : -6 : 2w] : u,w\in\mathbb{R}\} \cap \dots$$

NO NO

$\boxed{5.7 \text{ libro}}$

$X = \{ 2x_1 + 5x_2 - 3x_3 = 0 \}$

$Y = \mathrm{Span} \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$

matrice proiez. su X per $\mathbb{R}^3 = X \oplus Y$.

$$(2, 5, -3) \cdot \left(\underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}} - t \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \right) = 0$$

$$2x_1 + 5x_2 - 3x_3 = t \cdot (6 - 10 - 3) \qquad t = - \frac{2x_1 + 5x_2 - 3x_3}{7}$$

$$p_X(x) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \frac{2x_1 + 5x_2 - 3x_3}{7} \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} =$$

$$= \frac{1}{7} \underbrace{\begin{pmatrix} 13 & 15 & -9 \\ -4 & -3 & 6 \\ 2 & 5 & 4 \end{pmatrix}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$\boxed{30/1/18 - 4}$

Descrivere azione A ortog. 3×3 con $\det(A) = -1$ e $\mathrm{tr}(A) = 0$

In generale

$$\begin{pmatrix} \pm 1 & & \\ & \pm 1 & \\ & & \pm 1 \end{pmatrix} \qquad \begin{pmatrix} \cos\vartheta & -\sin\vartheta & 0 \\ \sin\vartheta & \cos\vartheta & 0 \\ 0 & 0 & \pm 1 \end{pmatrix}$$

Sapendo $\det = -1$

$$\begin{pmatrix} \pm 1 & & \\ & \pm 1 & \\ & & -1 \end{pmatrix} \qquad \begin{pmatrix} \cos\vartheta & -\sin\vartheta & 0 \\ \sin\vartheta & \cos\vartheta & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

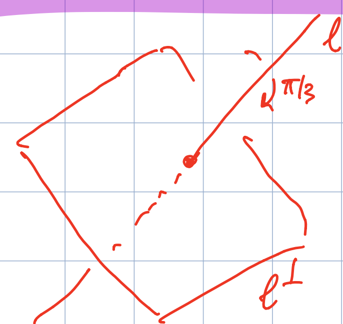
mai $t_1 = 0$

$$\begin{pmatrix} \boxed{\begin{matrix} 1/2 & \mp \sqrt{3}/2 \\ \pm\sqrt{3}/2 & 1/2 \end{matrix}} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & -1 \end{pmatrix}$$

∃ base ortonormale di $\mathbb{R}^3$ per cui A $\bar{e}$

rotaz intorno a l di angolo $\cancel{\pm}\,\pi/3$

riflessione rispetto al piano $l^\perp$



$\boxed{\text{T55.6 Geom}}$ E' ortog. la

$$\frac{1}{8}\begin{pmatrix} 6+\sqrt{2} & 2\sqrt{2} & 2\sqrt{3}-\sqrt{6} \\ 2\sqrt{2} & -4\sqrt{2} & -2\sqrt{6} \\ 2\sqrt{3}-\sqrt{6} & -2\sqrt{6} & 2+3\sqrt{2} \end{pmatrix}$$

col. unitarie?

$$I = \frac{1}{8^2}\cdot\left(36 \cancel{+12\sqrt{2}} + 2 + 8 + 12 \cancel{-12\sqrt{2}} + 6\right)$$ OK

$$II: \frac{1}{8^2}\left(8 + 32 + 24\right)$$ OK

$$III: \frac{1}{8^2}\left(12 \cancel{-12\sqrt{2}} + 6 + 24 + 4 \cancel{+12\sqrt{2}} + 18\right)$$ OK

ortog. fra loro?

$\mathrm{I} \perp \mathrm{II}$ $\quad \cancel{12\sqrt{2}} + 4 - 16 - \cancel{12\sqrt{2}} + 12 \quad 0 \leftarrow \quad \dots$

$\mathrm{I} \perp \mathrm{III} \quad \dots$

$\mathrm{II} \perp \mathrm{III} \quad \dots$

18/7/17 - n.6 $\qquad t$ è sempre lui

Determinare i pti di intersezione tra $\{[t-1 : t+2 : t] : t \in \mathbb{R}\}$ e l'insieme dei pti a ∞ di $\boxed{x^2 + 3xy - 2yz} + 7x = 12$.

$$(t-1)^2 + 3(t-1)(t+2) - 2(t+2) \cdot t = 0$$

$$t^2 - 2t + 1$$
$$+ 3t^2 + 3t - 6$$
$$-2t^2 - 4t \quad = 0$$

$$2t^2 - 3t - 5 = 0 \qquad (t+1)(2t-5)$$

$t = -1 : \quad [-2 : 1 : -1] = [2 : -1 : 1]$

$t = 5/2 : \quad [3/2 : 9/2 : 5/2] = [3 : 9 : 5]$

28/1/20 n.7 AL $\qquad Z : \ 3x_1 - 5x_2 = 0$

$\qquad W : \ 4x_1 + 7x_2 = 0$

verificare che $\mathbb{R}^2 = Z \oplus W$ e trovare matr. proiez. su Z.

$\dim Z = \dim W = 1 \qquad Z \cap W = \{0\} \qquad \Rightarrow Z + W = \mathbb{R}^2 \Rightarrow Z \oplus W = \mathbb{R}^2$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = t \cdot \begin{pmatrix} 5 \\ 3 \end{pmatrix} + s \cdot \begin{pmatrix} 7 \\ -4 \end{pmatrix}$$

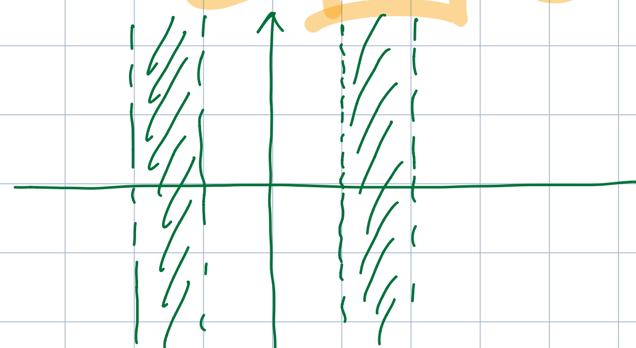
$$t = \frac{4x_1 + 7x_2}{41} \qquad \frac{4x_1 + 7x_2}{41} \begin{pmatrix} 5 \\ 3 \end{pmatrix} = \boxed{\frac{1}{41} \begin{pmatrix} 20 & 35 \\ 12 & 21 \end{pmatrix}} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

4/6/19 h.7 Geom

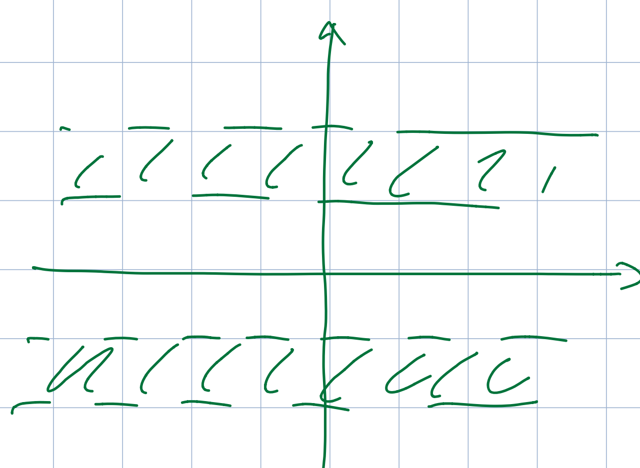
Esercizio fare disegno non esatto a

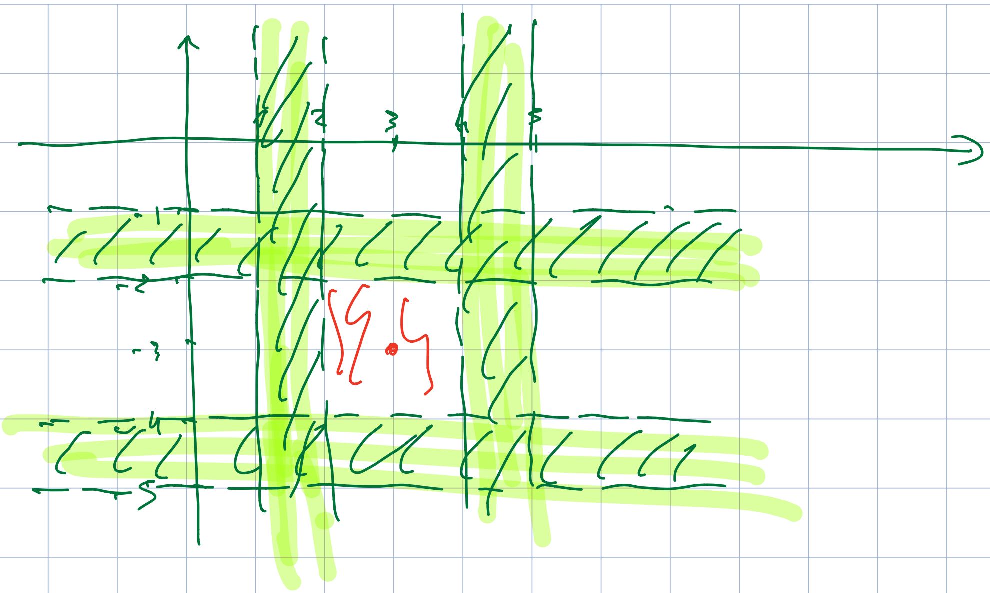
$A = \{(x,y) \in \mathbb{R}^2 : 1 < |x-3| < 2$ $\boxed{\text{oppure}}$ $1 < |y+3| < 2\}$?

$\{(x,y) : 1 < |x| < 2\} =$



$\{(x,y) : 1 < |y| < 2\} =$



$A =$ 

Sì, ad es. $\dfrac{-(y+3)\,dx + (x-3)\,dy}{(x-3)^2 + (y+3)^2}$

$\boxed{\text{8/1/19 Es. 2 (A)}}$ $\qquad \alpha : \mathbb{R} \to \mathbb{R}^2 \qquad \alpha(t) = \begin{pmatrix} t \cdot e^{2t} \\ t^2 + e^{-t} \end{pmatrix}$

Provare che è regolare.

$$\alpha'(t) = \begin{pmatrix} e^{2t} + 2t \cdot e^{2t} \\ 2t - e^{-t} \end{pmatrix} = \begin{pmatrix} e^{2t}(1+2t) \\ 2t - e^{-t} \end{pmatrix}$$

$X'(t)$ nulla solo a $t = -1/2$ $\Longrightarrow$ $\alpha'(t)$ mai nulla.

$Y'(-1/2) = -1 - e^{-1/2} \neq 0$

$\boxed{\text{44.47 AL}}$ $\qquad$ Posto $g(x) = \begin{pmatrix} 2x_2 - 2x_1 \\ 2x_1 - x_3 \\ -3x_2 - x_3 \end{pmatrix}$

$W : 3x_1 + 6x_2 - 2x_3 = 0$

provare che $g(W) \subset W$ e $[g]_{\mathcal{B}_2}^{\mathcal{B}_2}$ $\mathcal{B}_2 = \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$

Verifico che $g(W) \subset W$ provando che $g\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$ e $g\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$ stanno in W

$$g\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \\ -3 \end{pmatrix} = -4 \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} - 1 \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$$

$$g\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix} = 1 \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} - 1 \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} -4 & 1 \\ -1 & -1 \end{pmatrix}$$