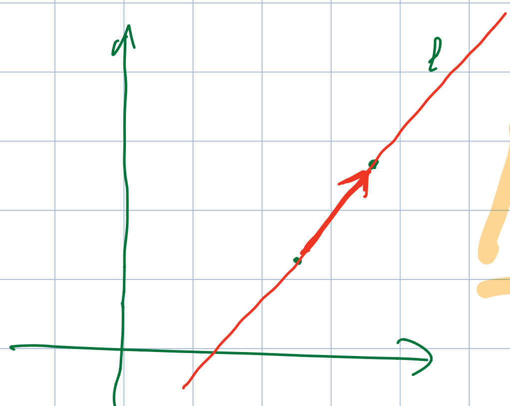
Geom 29/5/20

• Trovare il pto a ∞ della retta di $\mathbb{R}^3$ per $\begin{pmatrix}2\\1\\-3\end{pmatrix}$ $\begin{pmatrix}5\\4\\1\end{pmatrix}$.



Risposta:

$[3:3:4]$

$$l = \left\{ \begin{pmatrix}2\\1\\3\end{pmatrix} + t\cdot\left(\begin{pmatrix}5\\4\\1\end{pmatrix} - \begin{pmatrix}2\\1\\-3\end{pmatrix}\right) : t\in\mathbb{R} \right\}$$

$$= \left\{ \begin{pmatrix}2\\1\\3\end{pmatrix} + t\begin{pmatrix}3\\3\\4\end{pmatrix} : t\in\mathbb{R} \right\}$$

$$l = \left\{ [2:1:3:1] + t\,[3:3:4:0] : t\in\mathbb{R} \right\}$$

$$= \left\{ [2+3t : 1+3t : 3+4t : 1] : t\in\mathbb{R} \right\}$$

$$= \left\{ \left[\tfrac{2}{t}+3 : \tfrac{1}{t}+3 : \tfrac{3}{t}+4 : \tfrac{1}{t}\right] : t\in\mathbb{R} \right\} \cup \left\{[2:1:3:1]\right\}$$

$$t\to\infty \longrightarrow [3:3:4:0]$$

• Trovare $a\in\mathbb{R}$ t.c. $\begin{pmatrix}0 & 3 & -5\\ -3 & 0 & 7\\ 5 & -7 & 0\end{pmatrix}$ sia congruente a

$$\begin{pmatrix}0 & 0 & 0\\ 0 & 0 & a\\ 0 & -a & 0\end{pmatrix}.$$

Antisimm congruente a diag. a blocchi $(0) \oplus \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}$
con a t.c. $\pm ia$ sono autovalori

$$p_A(t) = \det\begin{pmatrix} t & -3 & 5 \\ 3 & t & -7 \\ -5 & 7 & t \end{pmatrix} = t^3 - 3\cdot 5\cdot 7 + 3\cdot 5\cdot 7 + t\,(25+9+49)$$

$$= t\,(t^2+83)$$

autoval $\pm i\sqrt{83}$

$a = \pm\sqrt{83}$

• $A = \begin{pmatrix} 0 & 2 & \sqrt5 \\ -2 & 0 & -6 \\ -\sqrt5 & 6 & 0 \end{pmatrix}$ congruente $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & k \\ 0 & -k & 0 \end{pmatrix}$

$k = \pm\sqrt{4+5+36}$

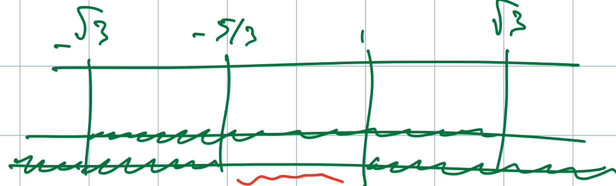
• dire per quali $t \in \mathbb{R}$ la conica $x^2+2kxy+3y^2-2x+2y+3=0$ è ellisse. N.66 Q.7

$$\begin{pmatrix} 1 & k & -1 \\ k & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$$

Devo imporre
$d_2>0 \quad d_3<0$

$$\begin{cases} 3-k^2>0 \\ 9-k-k-3-3k^2-1<0 \end{cases} \qquad \begin{cases} -\sqrt3<k<\sqrt3 \\ 3k^2+2k-5>0 \end{cases}$$

$$\begin{cases} -\sqrt3<k<\sqrt3 \\ (3k+5)(k-1)>0 \end{cases}$$



$-\sqrt3<t<-5/3 \quad , \quad 1<t<\sqrt3$

risp. errata sul libro

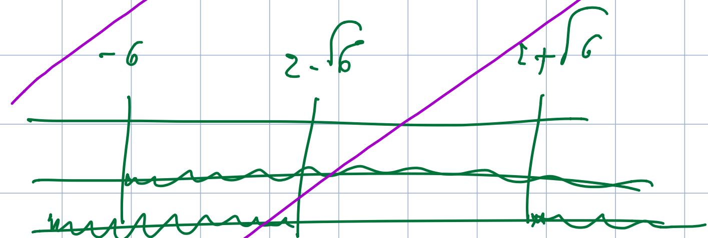
• Per quali $t \in \mathbb{R}$ la forma bil. associata a

$$\begin{pmatrix} t+6 & t^2-7 \\ t+5 & t^2-\frac{t}{2}-6 \end{pmatrix}$$

è prod. scal.?

Se e solo se $d_1 > 0$ e $d_2 > 0$. $\qquad t > -6$

$$d_2 = t^3 - \frac{t^2}{2} - 6t$$
$$+6t^2 - 3t - 36$$
$$-t^3 - 5t^2 + 7t + 35 = \frac{1}{2}t^2 - 2t - 1 = \frac{1}{2}(t^2 - 4t - 2)$$
$$2 \pm \sqrt{4+2} = 2 \pm \sqrt{6}$$



$$-6 < t < 2-\sqrt{6} \quad \text{o} \quad t > 2+\sqrt{6}$$

NO

$\boxed{5/6/18 - \text{II} - \text{Es } 1}$

$$M = \begin{pmatrix} iz & (i-2)z-5 \\ (1+z)^2 & 13 \end{pmatrix}$$

(A) Per quali $z \in \mathbb{C}$ la $\left\{ \begin{array}{l} M \text{ ha autoval. reali ed esiste base} \\ \text{ortonormale di autovettori?} \end{array} \right\} \Longleftrightarrow M$ hermitiana.

$$\Longleftrightarrow M^* = M \Longleftrightarrow z = it \quad t \in \mathbb{R}$$

$$(i-2) \cdot it - 5 = \overline{(1+it)^2}$$

$\Longleftrightarrow \quad z = it \quad t \in \mathbb{R}$

$$-t - \cancel{2it} - 5 = 1 - \cancel{2it} - t^2$$

$$t^2 - t - 5 = 0 \qquad (t-3)(t+2) = 0$$

$$z = 3i \qquad z = -2i$$

(B) Per z t.c. $\langle \cdot | \cdot \rangle_M$ non è prod. scal. hermitiano trovano segni autoval.

$$z = 3i \longrightarrow M = \begin{pmatrix} -3 & \bar{\alpha} \\ \alpha & 13 \end{pmatrix}$$ discordi

Verifico che invece per $z = -2i$ è un prod. scal. hermitiano

$$(1-2i)^2 = 1 - 4i - 4 = -3 - 4i$$

$$(i-2)(-2i) - 5 = 2 + 4i - 5 = -3 + 4i$$

$$M = \begin{pmatrix} 2 & -3+4i \\ -3-4i & 13 \end{pmatrix} \qquad \begin{matrix} d_1 = 2 > 0 \\ d_2 = 26 - (9+16) = 1 > 0 \end{matrix}$$

ok

(C) per $z = -2i$ ortonormalizzare $\underset{w_1}{\begin{pmatrix} 1 \\ i \end{pmatrix}} \underset{w_2}{\begin{pmatrix} i \\ 2 \end{pmatrix}}$ rispetto a $\langle \cdot | \cdot \rangle_M$.

$$v_1 = \frac{w_1}{\|w_1\|_M} \qquad \|w_1\|_M^2 = (1, i) \cdot \begin{pmatrix} 2 & -3+4i \\ -3-4i & 13 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$= (1, i) \begin{pmatrix} 6+3i \\ -3-17i \end{pmatrix} = 6 + 3i - 3i + 17 = 23$$

$$u_1 = \frac{1}{\sqrt{23}} \cdot \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$\tilde{u}_2 = w_2 - \langle w_2 | u_1 \rangle_M \cdot u_1 = w_2 - \frac{1}{\|w_1\|_M^2} \cdot \langle w_2 | w_1 \rangle \cdot w_1$$

$$= \begin{pmatrix} i \\ 2 \end{pmatrix} - \frac{1}{23} \cdot \Bigg(\underbrace{(i, 2) \cdot \begin{pmatrix} 2 & -3+4i \\ -3-4i & 13 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -i \end{pmatrix}}_{\in \mathbb{C}} \Bigg) \cdot \begin{pmatrix} 1 \\ -i \end{pmatrix} = \dots$$

$$u_2 = \frac{\tilde{u}_2}{\|\tilde{u}_2\|_M} = \dots$$

- Per quali $t \in \mathbb{R}$ la forma bil. associata a
$$\begin{pmatrix} t+6 & t^2-7 \\ t+5 & t^2-\frac{t}{2}-6 \end{pmatrix}$$
è prod. scal.?

Deve essere simmetrica: $t^2 - 7 = t + 5$

$$t^2 - t - 12 = 0$$

$$(t-4)(t+3) = 0$$

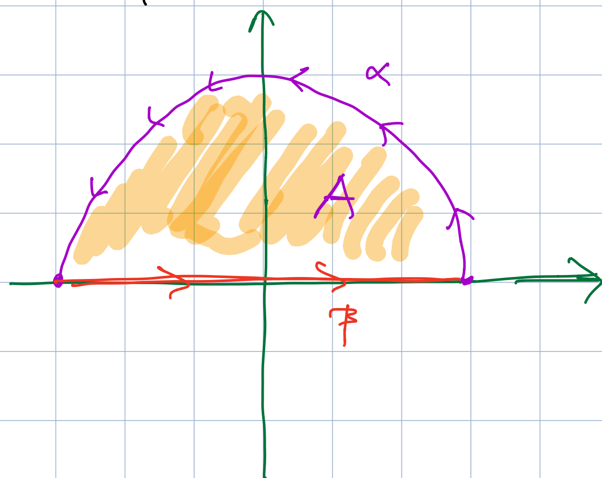
$$t = 4 \quad t = -3$$

$t = 4$: $\begin{pmatrix} 10 & 9 \\ 9 & 8 \end{pmatrix}$ $\quad d_1 > 0$, $d_2 = 80 - 81$ $\quad$ No

$t = -3$: $\begin{pmatrix} 3 & 2 \\ 2 & 9/2 \end{pmatrix}$ $\quad d_1 > 0$, $d_2 = \frac{27}{2} - 4 > 0$ $\quad$ Sì

- $\displaystyle\int_\alpha \left(\left(x + \log\left(\frac{1}{2+\cos(y)}\right)\right)\right) dy = I$

$\alpha : [0,\pi] \to \mathbb{R}^2$
$\alpha(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \end{pmatrix}$



$$\int_\beta (\ldots)\, dy = 0$$

$$I = \int_{\alpha \cup \beta} (x + F(y))\, dy = \int_A d\left((x+F(y))\, dy\right)$$
$$= \int_A dx\, dy + F'(y) \cdot \underbrace{dy\, dy}_{0}$$
$$= \int_A dx\, dy = \text{Area}(A) = \pi$$