Algebra Lineare 18/12/19

$\boxed{36}$ $\boxed{2}$ Dati 13 generat. di $\{p(t) \in \mathbb{R}_{\leq 9}[t] : p(1)=0, p''(-7)=0\}$ quanti ne scarto per avere base

$$13-(10-2)=13-8=5$$

$\boxed{3}$ $f: \mathbb{C}^8 \to \mathbb{C}^3$ lin. surj. $W \subset \mathbb{C}^8$, $\dim(W)=6$; $\dim\underline{(W \cap \ker f)}=?$

$\dim \ker(f)=5$

$$(W \cap \ker f)\,(W + \ker f) = \underline{W \quad + \quad \ker f}$$

$W \cap \ker f$	$W + \ker f$	W	$\ker f$
5	6	6	5
4	7		
3	8		

$$\boxed{3 \leq \cdot \leq 5}$$

$\boxed{5}$
$$\begin{cases} 2x-3y+5z=4 \\ 3x+2y-4z=1 \\ 5x-14y+24z=15 \end{cases}$$

$$\det\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 5 & -14 & 24 \end{pmatrix} = 96+60+216-210-50-112=0$$

$$\begin{cases} 2\alpha+3\beta=5 \\ -3\alpha+2\beta=-14 \end{cases} \qquad \alpha=\frac{10+42}{4+9}=\frac{52}{13}=4$$

$\beta=-1$

Verifica (non nec.) per coeff z: $\quad 20+4 = 24$ ✓

Verifica per termine noto: $\quad 16-1=15 \Rightarrow \infty$ soluz.

$$\boxed{\begin{pmatrix}1\\1\\1\end{pmatrix} + \mathrm{Span}\begin{pmatrix}2\\23\\13\end{pmatrix}}$$

verifica:

$4-69+65$ ✓

$6+46-52$ ✓

$\boxed{7}$ Dato $\mathbb{R}^4 = W \oplus Z \qquad W = \mathrm{Span}\left(\begin{pmatrix}2\\1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\-1\\1\end{pmatrix}\right) \quad Z = \mathrm{Span}\left(\begin{pmatrix}3\\0\\-1\\0\end{pmatrix}, \begin{pmatrix}0\\2\\0\\-1\end{pmatrix}\right)$

calcolare proiez. su W di e_1.

$$\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \underset{\substack{\cup\\ w\\ \|\\ \boxed{r\begin{pmatrix}2\\1\\1\\0\end{pmatrix}+s\begin{pmatrix}0\\1\\-1\\1\end{pmatrix}}}}{W} + \underset{\substack{\cup\\ z\\ \\ t\begin{pmatrix}3\\0\\-1\\0\end{pmatrix}+u\begin{pmatrix}0\\2\\0\\-1\end{pmatrix}}}{Z}$$

$$\begin{cases} 2r+3t=1 \\ r+s+2u=0 \\ r-s-t=0 \ ✓ \\ s-u=0 \ ✓ \end{cases} \qquad \begin{cases} u=s \\ t=r-s \\ 2r+3r-3s=1 \\ r+s+2s=0 \end{cases} \qquad \begin{cases} 5r-3s=1 \\ r+3s=0 \end{cases}$$

$$r=\frac{1}{6} \qquad s=-\frac{1}{18}$$

$$\frac{1}{6}\begin{pmatrix}2\\1\\1\\0\end{pmatrix} - \frac{1}{18}\begin{pmatrix}0\\1\\-1\\1\end{pmatrix} = \frac{1}{18}\begin{pmatrix}6-0\\3-1\\3+1\\0-1\end{pmatrix} = \frac{1}{18}\begin{pmatrix}6\\2\\4\\-1\end{pmatrix}$$

$\boxed{40}$ $\boxed{4}$ $\quad A=(v_1,v_2,v_3) \in \mathcal{M}_{3\times3}(\mathbb{C})$, $\det(A) = -i$

$B = (v_1+iv_2,\ v_1-iv_3,\ 2iv_2+v_3)$; $\det(B) = ?$

$\det(B) = (2i)\cdot(-i)\det(v_1,v_3,v_2)\ +$

$+\, i\det(v_2,v_1,v_3) = 2\cdot i + i\cdot i = 2i-1.$

Oppure:

$$B = (v_1, v_2, v_3) \cdot \begin{pmatrix} 1 & 1 & 0 \\ i & 0 & 2i \\ 0 & -i & 1 \end{pmatrix}$$

$$\Rightarrow \det(B) = (-i)\cdot(-i-2) = 2i-1.$$

$\boxed{43}$ $\boxed{1}$ Completare $e_1+e_2-e_4$ a base di $\{x \in \mathbb{R}^4 : 3x_1+2x_2-4x_3+5x_4=0\}$.

[Non nec: ha senso? $3+2-5=0$ ✓.]

Quanti servono: $(4-1)-1 = 2$.

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ -1 \end{pmatrix}\checkmark \quad \underbrace{\begin{pmatrix} 4 \\ 0 \\ 3 \\ 0 \end{pmatrix}\checkmark \begin{pmatrix} 0 \\ 2 \\ 1 \\ 0 \end{pmatrix}\checkmark \cancel{\begin{pmatrix} 0 \\ 0 \\ 5 \\ 4 \end{pmatrix}}}$$

$\boxed{5}$ Tra le orlate di $(1+i)$ in $\begin{pmatrix} 0 & i & -2 \\ 2-i & 3 & 1+i \\ 1 & 0 & 3-2i \end{pmatrix}$ trovare quella con det di modulo max.

$$\begin{pmatrix} 0 & -2 \\ 2-i & 1+i \end{pmatrix} \longrightarrow |\quad|^2 = 4\cdot 5 = 20$$

$$\begin{pmatrix} i & -2 \\ 3 & 1+i \end{pmatrix} \longrightarrow |i-1+6|^2 = 25+1 = 26$$

$$\begin{pmatrix} 2-i & 1+i \\ 1 & 3-2i \end{pmatrix} \longrightarrow |6-4i-3i-2-1-i|^2 = 9+64 = 73$$

$$* \begin{pmatrix} 3 & 1+i \\ 0 & 3-2i \end{pmatrix} \longrightarrow |\quad|^2 = 9\cdot 13 = 111$$

[6] Posto $W = \{x \in \mathbb{R}^3 : 3x_1 + 6x_2 - 2x_3 = 0\}$

provare che $g(x) = \begin{pmatrix} 2x_2 - 2x_1 \\ 2x_1 - x_3 \\ -3x_2 - x_3 \end{pmatrix}$ definisce $g : W \to W$ lin.

e trovare $[g]_{\mathcal{B}}^{\mathcal{B}}$ con $\mathcal{B} = \left(\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \right)$.

[W è def. da $w \cdot x = 0$ $\quad w = (3, 6, -2)$;
g se fosse $\mathbb{R}^3 \to \mathbb{R}^3$ sarebbe associata alla matrice
$A = \begin{pmatrix} -2 & 2 & 0 \\ 2 & 0 & -1 \\ 0 & -3 & -1 \end{pmatrix}$; per provare che la formula $g(x) = A \cdot x$ definisce $g : W \to W$ devo verificare che $A(W) \subset W$ cioè che $w \cdot A \cdot x = 0$ $\forall x$ t.c. $w \cdot x = 0$; basta calcolare

$$w \cdot A = (3, 6, -2) \cdot \begin{pmatrix} -2 & 2 & 0 \\ 2 & 0 & -1 \\ 0 & -3 & -1 \end{pmatrix} = (6, 12, -4) = 2 \cdot w$$

$\Rightarrow$ se $w \cdot x = 0$ ho $w \cdot A \cdot x = 2 w \cdot x = 0$. OK]

In realtà posso rispondere alla prima domanda insieme alla seconda: se verifico che posto $\mathcal{B} = (v_1, v_2)$

$A \cdot v_1 = c_{11} v_1 + c_{21} v_2 \qquad A \cdot v_2 = c_{12} v_1 + c_{22} v_2$

ho verificato in part. che $A(v_1) \subset W$, $A(v_2) \subset W$

ho che $A(W) \subset W$

e $[g]_{\mathcal{B}}^{\mathcal{B}} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$.

$$\begin{pmatrix} -2 & 2 & 0 \\ 2 & 0 & -1 \\ 0 & -3 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \\ -3 \end{pmatrix} = -4 \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + (-1) \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 2 & 0 \\ 2 & 0 & -1 \\ 0 & -3 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix} = 1\cdot\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} + (-1)\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$$

OK

$$[g]^{\mathcal{B}}_{\mathcal{B}} = \begin{pmatrix} -4 & 1 \\ -1 & -1 \end{pmatrix}.$$

$\boxed{4.5}$ $\boxed{6}$ Dire quanti par. posso usare per pres. param. di

$$X: \begin{cases} 3x_1 - 4x_2 + 5x_3 - 7x_4 = -2 & \leftarrow \neq 0 \\ 2x_1 - 4x_2 + 4x_3 - 5x_4 = 1 & \leftarrow \text{non mult. I} \\ 5x_1 - 4x_2 + 7x_3 - 12x_4 = -8 \end{cases}$$

$\mathrm{III} = \alpha\cdot\mathrm{I} + \beta\cdot\mathrm{II}$ omogenea?

- No: $\dim(X) = 4-3 = 1 \Rightarrow$ posso usare ≥ 1 param.
- Sì; termini noti: $\mathrm{III} = \alpha\mathrm{I} + \beta\mathrm{II}$?
  - No: $X = \emptyset \rightarrow 0$ param.
  - Sì: $\dim(X) = 4-2 = 2$ $\rightarrow$ posso usare ≥ 2 param.