Algebra Lineare 17/12/19

$\boxed{31}$ $\boxed{1}$ $A = \begin{pmatrix} i & -2i & 1+i \\ 0 & 1 & 2-i \\ i-1 & 2 & 3+6i \end{pmatrix}$

calcolare $(A^{-1})_{(2,2)}$

3^a riga $\rightsquigarrow$ si somma $\underbrace{-\frac{i-1}{i}}$ $\cdot 1^a$

$i(i-1) = -1-i$

$\boxed{\begin{array}{l} 2+(1+i)(2i) \\ = 2+2-i-2 \end{array}}$

$(3+6i)-(1+i)^2$

$\det(A) = \begin{pmatrix} i & -2i & 1+i \\ 0 & 1 & 2-i \\ 0 & 2i & 3+4i \end{pmatrix}$

$= i\,(3+4i-4i-2) = i$

$(A^{-1})_{2,2} =$

$= \frac{+1}{i} \cdot (3i-6+2)$

$= -i\,(3i-4)$

$= 3+4i$

$\boxed{2}$ Date $\mathcal{B} = (3e_1-2e_2,\ e_1+e_2)$ $\quad \mathcal{C} = (2e_1+e_2,\ e_1-3e_2)$

trovare $[v]_{\mathcal{C}}$ sapendo che $[v]_{\mathcal{B}} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

$v = (+1)\cdot\begin{pmatrix} 3 \\ -2 \end{pmatrix} + (-1)\cdot\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \frac{3}{7}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \frac{8}{7}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$\frac{6+8}{7} = 2$

$\frac{1}{7}\begin{pmatrix} 3 \\ 8 \end{pmatrix}$ $\qquad \frac{3-24}{7} = -3$ ✓

$\boxed{3}$ $X = \{x \in \mathbb{C}^6 : ix_2+3x_5 = 0\}$, $f : \mathbb{C}^3 \to X$ lin.

$f(2e_1+ie_2) = f(e_2+2ie_3-e_5)$; $Y \oplus \mathrm{Im}(f) = X$;

$\dim(Y) = ?$

$f(2e_1+ie_2-e_2-2ie_3-e_5) = 0$

$$\overset{3}{} \qquad\qquad \overset{5}{}$$
$$f : \mathbb{C}^3 \longrightarrow X$$

dim Ker $f \geq 1$

Ker f	Im f	3
1	2	
2	1	
3	0	

Y	Im f	5
3	2	
4	1	
5	0	

Tra 3 e 5 compresi

$\boxed{4}$ Data $B = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ provare che

$$X = \{ A \in M_{2\times 2}(\mathbb{R}) : \det(A+B) = \det(A) + \det(B) \}$$

è sttosp. e calcolare dim.

$$A = \begin{pmatrix} x & y \\ z & w \end{pmatrix} : \det \begin{pmatrix} x+1 & y+1 \\ z+1 & w \end{pmatrix} = \det \begin{pmatrix} x & y \\ z & w \end{pmatrix} + \det \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$(x+1)\cdot w - (z+1)(y+1) = xw - zy - 1$$

$$\cancel{xw} + w - \cancel{zy} - y - z - \cancel{1} = \cancel{xw} - \cancel{zy} - \cancel{1}$$

$$w - y - z = 0.$$

Sì: è def. da equaz. lineare; $\dim = 4 - 1 = 3$.

5 Risolvere $\begin{cases} 2x-3y+5z=3 \\ 3x+5y-4z=-1 \\ 7x-y+6z=5 \end{cases}$

$\begin{cases} y=7x+6z-5 \\ 2x-21x-18z+15+5z=3 \\ 3x+35x+30z-25-4z=-1 \end{cases}$ $\begin{cases} +19x+13z=+12 \\ \cancel{38x+26z=24} \\ y=7x+6z-5 \end{cases}$

Soluz: $\begin{pmatrix} 12/19 \\ 7\cdot 12/19-5 \\ 0 \end{pmatrix} + \mathrm{Span}\begin{pmatrix} -13 \\ 23 \\ 19 \end{pmatrix}$

$-26-69+95=0$
$-39+115-76=0$
$-91 - - -$

6 Sapendo che $z=-\frac{i}{2}$ risolve $2z^3+(2+5i)z^2+3(3i-2)z-2(2+i)=0$ trovare altre soluz.

Il pol. dato è div. per $z+\frac{i}{2}$ dunque per $2z+i$

$$
\begin{array}{l|l}
2z^3+(2+5i)z^2+(-6+9i)z-4-2i & 2z+i \\
\hline
-2z^3 \quad -iz^2 & z^2+(1+2i)z-2+4i \\
\hline
(2+4i)z^2+(-6+9i)z-4-2i & \\
-(2+4i)z^2+(2-i)z & \\
\hline
(-4+8i)z-4-2i & \\
(-4+8i)z+4+2i & \\
\hline
0 &
\end{array}
$$

$$\Delta = 1+4i-4+8-16i = 5-12i$$

cerco $\sqrt{\Delta} = a+ib$:

$$\begin{cases} a^2-b^2=5 \\ 2ab=-6 \end{cases}$$

$a=3 \quad b=-2$

$$z_{2,3} = \frac{-1-2i \pm (3-2i)}{2} = \begin{cases} 1-2i \\ -2 \end{cases}$$

$\boxed{7}$ Trovare base $\mathcal{B}$ di $\mathbb{R}^2$ t.c. $[x]_{\mathcal{B}} = \begin{pmatrix} 4x_1+x_2 \\ -2x_2-7x_1 \end{pmatrix} \quad \forall x \in \mathbb{R}^2$.

(I) $\mathcal{B} = (v_1, v_2)$. Applico ipotesi a $x = e_1, e_2$:

$x = e_1$: $\quad [e_1]_{\mathcal{B}} = \begin{pmatrix} 4 \\ -7 \end{pmatrix} \qquad e_1 = 4v_1 - 7v_2$

$x = e_2$: $\quad [e_2]_{\mathcal{B}} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \qquad e_2 = v_1 - 2v_2$

$$\Rightarrow \quad v_1 = 2e_1 - 7e_2 = \begin{pmatrix} 2 \\ -7 \end{pmatrix}$$

$$v_2 = e_1 - 4e_2 = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

(II) $x = \mathcal{B} \cdot [x]_{\mathcal{B}}$ def coord.

$$[x]_{\mathcal{B}} = \begin{pmatrix} 4x_1+x_2 \\ -7x_1-2x_2 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ -7 & -2 \end{pmatrix} \cdot x$$

$$x = \underbrace{\mathcal{B} \cdot \begin{pmatrix} 4 & 1 \\ -7 & -2 \end{pmatrix}} \cdot x \qquad \forall x$$

$$\Rightarrow \mathcal{B} = \begin{pmatrix} 4 & 1 \\ -7 & -2 \end{pmatrix}^{-1} = \frac{1}{-8+7} \cdot \begin{pmatrix} -2 & -1 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -7 & -4 \end{pmatrix}$$

$\boxed{32}$ $\boxed{1}$ $V \oplus W = \{x \in \mathbb{R}^7 : x_1 = x_7\}$; $\dim V = 2$;

2 4 6

dat. 13 generat. di W quanti ne devo scartare per base?

$13 - 4 = 9$.

$\boxed{2}$ $f : \{y \in \mathbb{R}^5 : 2y_1 + 3y_2 = 0\} \to \mathbb{R}^8$; $\underbrace{f(e_3) = 4f(e_5)}$; $\dim(\mathrm{Im} f) = ?$

4

$f(e_3 - 4e_5) = 0$

$\dim(\mathrm{Ker} f) \geq 1$

Ker	Im	= 4
1	3	
2	2	
3	1	
4	0	

$\boxed{0 \leq \cdot \leq 3}$

$\boxed{3}$ Trovare $[f]_{\mathcal{B}}^{\mathcal{B}}$ dove $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x - y \\ y - 2x \end{pmatrix}$ $\mathcal{B} = \left(\begin{pmatrix} 2 \\ -1 \end{pmatrix}\begin{pmatrix} 5 \\ -2 \end{pmatrix}\right)$

$f\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 7 \\ -5 \end{pmatrix} = 11\begin{pmatrix} 2 \\ -1 \end{pmatrix} - 3\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$f\begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 17 \\ -12 \end{pmatrix} = 26\begin{pmatrix} 2 \\ -1 \end{pmatrix} - 7\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

$22 - 15 = 7$
$-11 + 6 = -5$

$52 - 35 = 17$
$-26 + 14 = -12$

$$\begin{pmatrix} 11 & 26 \\ -3 & -7 \end{pmatrix}$$

$\boxed{4}$ $A = \begin{pmatrix} 2 & 3 & 0 \\ -1 & 0 & 4 \\ 2 & 2 & -1 \end{pmatrix}$ $\quad (A^{-1})_{2,1} = ?$

$$\det(A) = 24 - 3 - 16 = 5$$

$$(A^{-1})_{2,1} = \frac{-1}{5} \cdot (-7) = \frac{7}{5}.$$

$\boxed{5}$ calcolare det orlati di $\begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$ in $\begin{pmatrix} 2 & 1 & 0 & -3 \\ 1 & 2 & -1 & 0 \\ 4 & 0 & 2 & 1 \end{pmatrix}$

$$\det\begin{pmatrix} 2 & 1 & -3 \\ 1 & 2 & 0 \\ 4 & 0 & 1 \end{pmatrix} = \dots \qquad \det\begin{pmatrix} 2 & 0 & -3 \\ 1 & -1 & 0 \\ 4 & 2 & 1 \end{pmatrix} = \dots$$

$\boxed{6}$ Calcolare modulo e tangente dell'argomento di $\frac{3-2i}{4-3i}$.

$$z = \frac{3-2i}{4-3i} = \frac{(3-2i)(4+3i)}{25} = \frac{18+i}{25}$$

$$|z| = \frac{1}{25}\sqrt{18^2+1}\,; \qquad z = |z| \cdot (\cos\vartheta + i \sin\vartheta)$$

$$\operatorname{tg}(\vartheta) = \frac{\sin\vartheta}{\cos\vartheta} = \frac{|z| \cdot \sin\vartheta}{|z| \cdot \cos\vartheta} = \frac{\operatorname{Im}(z)}{\operatorname{Re}(z)} = \frac{1}{18}$$

$$z = \frac{1}{7}(a+ib) \qquad |z| = \frac{1}{7}\sqrt{a^2+b^2}$$

$\boxed{7}$ Data $\mathbb{R}^2 = \operatorname{Span}\begin{pmatrix} 3 \\ -1 \end{pmatrix} \oplus \operatorname{Span}\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ trovare proiez. di $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ sul I addendo.

$$\begin{pmatrix} -2 \\ 1 \end{pmatrix} = \underbrace{-\frac{9}{11}\begin{pmatrix} 3 \\ -1 \end{pmatrix}} + \underbrace{\frac{1}{11}\begin{pmatrix} 5 \\ 2 \end{pmatrix}}$$

proiez. nel I addendo

proiez. nel II addendo

[34] [3] Per quali $k \in \mathbb{R}$ si ha $\det\begin{pmatrix} 3 & 1 & k \\ 2 & k & 1 \\ -1 & 2 & 1 \end{pmatrix}$ fa 9?

$$\det\begin{pmatrix} 3 & 7 & k+3 \\ 2 & k+4 & 3 \\ -1 & 0 & 0 \end{pmatrix} = 9$$

$$-(21 - k^2 - 7k - 12) = 9$$

$$k^2 + 7k - 18 = 0$$

$$(k+9)(k-2) = 0 \qquad k = 2, \ k = -9.$$

[4] $f : \{z \in \mathbb{C}^5 : z_1 + z_5 = 0\} \to \mathbb{C}^7$ $\quad f(e_2) = f(e_4)$

$W + \mathrm{Im}(f) = \mathbb{C}^7 \quad ; \quad \dim(W) = ?$

Ker	Im = 4		W	chiedendo ⊕
1	3	←	$4 \le \cdot \le 7$	4
2	2	←	$5 \le \cdot \le 7$	5
3	1	←	$6 \le \cdot \le 7$	6
4	0	←	$= 7$	7

$\to \boxed{4 \le \cdot \le 7}$

$4 \le \cdot \le 7$

[6] Per quali $z \in \mathbb{C}$ i vett. $\begin{pmatrix} 1-4i \\ z-i \end{pmatrix} \begin{pmatrix} 2+i \\ 3-2i \end{pmatrix}$ sono lin. dip.?

$$-\det\begin{pmatrix} 1-4i & 2+i \\ z-i & 3-2i \end{pmatrix} = 0$$

$$z = \frac{1}{2+i}\left(3 - 2i - 12i - 8 + 2i - 1 \right)$$

$$= \frac{-6-12i}{2+i} = -6\,\frac{(1+2i)(2-i)}{5} = -\frac{6}{5}(4+3i)$$

$\boxed{7}$ Sapendo che $\mathbb{R}^3 = W \oplus Z$ e che la proiez. su W è associata alla matr.

$$P = \begin{pmatrix} 8 & -3 & -5 \\ 7 & -2 & -5 \\ 7 & -3 & -4 \end{pmatrix}$$

trovare base di Z.

$V = W \oplus Z$ $\qquad$ p proiez. su W

$\Longrightarrow$ $W = \mathrm{Im}(p)$, $Z = \mathrm{Ker}(p)$

$$\mathrm{Ker}(P): \begin{cases} 8x - 3y - 5z = 0 \\ 7x - 2y - 5z = 0 \\ 7x - 3y - 4z = 0 \end{cases} \qquad \begin{cases} x - y = 0 \\ y - z = 0 \\ \dots \end{cases} \qquad \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

$$Z = \mathrm{Span}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Dubbio: il testo è giusto? Sì se $P \cdot P = P$

$$\begin{pmatrix} 8 & -3 & -5 \\ 7 & -2 & -5 \\ 7 & -3 & -4 \end{pmatrix} \cdot \begin{pmatrix} 8 & -3 & -5 \\ 7 & -2 & -5 \\ 7 & -3 & -4 \end{pmatrix} =$$

$$= \begin{pmatrix} 64-21-35 & -24+6+15 & \cdot \\ \cdot & -21+4+15 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} 8 & -3 & \cdot \\ \cdot & -2 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

Proiez. su Z : $I - P$.

Base di W: $\begin{pmatrix} 8 \\ 7 \\ 7 \end{pmatrix} \begin{pmatrix} -3 \\ -2 \\ -3 \end{pmatrix}$

$\boxed{36}$ $\boxed{1}$ Il numero $\sqrt{2}+\sqrt{7}$ è razionale?

$$(3-5\sqrt{7})^2 + 30\sqrt{7} = 9 - \cancel{30\sqrt{7}} + 25 \cdot 7 + \cancel{30\sqrt{7}}$$ Sì $\in \mathbb{Q}$

$$2\sqrt{2} - \sqrt{8} = 0$$

Poniamo $x = \sqrt{2}+\sqrt{7}$. Se x fosse razionale lo sarebbe anche $x^2 = 2 + 2\sqrt{14} + 7$; ma allora lo sarebbe anche $\frac{1}{2}(x^2-9) = \sqrt{14}$. Invece non lo è:

se fosse $\sqrt{14} = \frac{p}{q}$ p, q coprimi

$p^2 = 14 q^2 \Rightarrow p$ pari

$4m^2 = 14 q^2$

$7q^2 = 2m^2 \Rightarrow q$ pari. Assurdo.