15.5.3 (a) $\mathcal{B} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ $\mathcal{B}' = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$\mathcal{B}' = \mathcal{B} \cdot A$ $\mathcal{B} = \mathcal{B}' \cdot X$ $A \cdot X = I_2$.

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \frac{25}{29}\begin{pmatrix} 2 \\ -5 \end{pmatrix} + \frac{22}{29}\begin{pmatrix} 3 \\ 7 \end{pmatrix}$$
$$\begin{pmatrix} 2 \\ -3 \end{pmatrix} = \frac{23}{29}\begin{pmatrix} 2 \\ -5 \end{pmatrix} + \frac{4}{29}\begin{pmatrix} 3 \\ 7 \end{pmatrix}$$
$$A = \frac{1}{29}\begin{pmatrix} 25 & 23 \\ 22 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -5 \end{pmatrix} = -\frac{4}{14}\begin{pmatrix} 4 \\ 1 \end{pmatrix} + \frac{22}{14}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$$
$$\begin{pmatrix} 3 \\ 7 \end{pmatrix} = \frac{23}{14}\begin{pmatrix} 4 \\ 1 \end{pmatrix} - \frac{25}{14}\begin{pmatrix} 2 \\ -3 \end{pmatrix}$$
$$X = \frac{1}{14}\begin{pmatrix} -4 & 23 \\ 22 & -25 \end{pmatrix}$$

$$A \cdot X = \frac{1}{406}\begin{pmatrix} -100 + 506 & 0 \\ 0 & 506 - 100 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2.$$