Matematica e statistica 28/11/18

Ese exp/log

$\boxed{10}$ $$\log_2(x^2) + (1-\log_2(x))^2 = x^2 + (x+2)(2-x) + 6$$

$\underline{x > 0}$

$$\cancel{2\log_2(x)} + 1 \cancel{-2\log_2(x)} + (\log_2(x))^2 =$$

$$= \cancel{x^2} + 4 \cancel{-x^2} + 6$$

$$(\log_2(x))^2 = 9$$

$$\log_2(x) = \pm 3$$

Soluz: $x = 8$ o $x = \frac{1}{8}$.

$\boxed{12}$ Dire quante soluz. ha per $a \in \mathbb{R}$

$$a \cdot (\operatorname{Log}(x))^3 - (\operatorname{Log}(x))^2 - 4a \operatorname{Log}(x) + 4 = 0.$$

$t = \operatorname{Log}(x)$ $\qquad at^3 - t^2 - 4at + 4 = 0$

$$t^2(at-1) - 4(at-1) = 0$$

$$(t^2-4)(at-1)=0$$

$a=0$: $t=\pm 2$ $\quad \log(x)=\pm 2$ $\quad x=100,\ x=\frac{1}{100}$

due soluzioni.

$a\neq 0$ soluz $t=\pm 2$, $t=\frac{1}{a}$;

$a=\pm\frac{1}{2}$ due soluzioni (stesse di sopra).

$a\neq 0, \pm 1/2$ $\quad x=100,\ x=\frac{1}{100}$ $\quad x=10^{1/a}$

Tre soluzioni.

$\boxed{13}$

$$\frac{5\cdot 2^x - 2}{2^x - 1} > 6$$

$$\cancel{5\cdot 2^x - 2 > 6\cdot(2^x - 1)}$$

$$\frac{5\cdot 2^x - 2}{2^x - 1} - 6 > 0$$

$$\frac{5\cdot 2^x - 2 - 6\cdot 2^x + 6}{2^x - 1} > 0$$

$$\frac{-2^x + 4}{2^x - 1} > 0 \qquad \frac{2^x - 4}{2^x - 1} < 0$$

		0		2	
NUM	$-$		$-$		$+$
DEN	$-$		$+$		$+$

Solut: $0 < x < 2$.

$\boxed{15}$ $(3^x - 2)(3^x + 1 + 2^x) < 6^x - 2^{x+1}$

$3^{2x} + 3^x \cancel{+ 6^x} - 2 \cdot 3^x - 2 \cancel{- 2^{x+1}} < \cancel{6^x} \cancel{- 2^{x+1}}$

$(3^x)^2 - 3^x - 2 < 0$

$t = 3^x$ $\qquad t^2 - t - 2 < 0$

$(t-2)(t+1) < 0$

$$-1 < t < 2$$

$$-1 < 3^x < 2$$

sempre vera

<u>soluz</u>: $x < \log_3(2)$.

$\boxed{16}$ $\ln(x^2-1) + \ln(x+3) > \ln(x+1)$

C.E. $\begin{cases} x^2 - 1 > 0 \\ x + 3 > 0 \\ x + 1 > 0 \end{cases}$ $\begin{cases} x < -1 \quad o \quad x > 1 \\ x > -3 \\ x > -1 \end{cases}$

$x < -3$	$-3 < x < -1$	$-1 < x < 1$	$x > 1$
V	V	F	V
F	V	V	V
F	F	V	V

$$\boxed{x > 1}$$

$$\log(x^2-1) + \log(x+3) > \log(x+1)$$

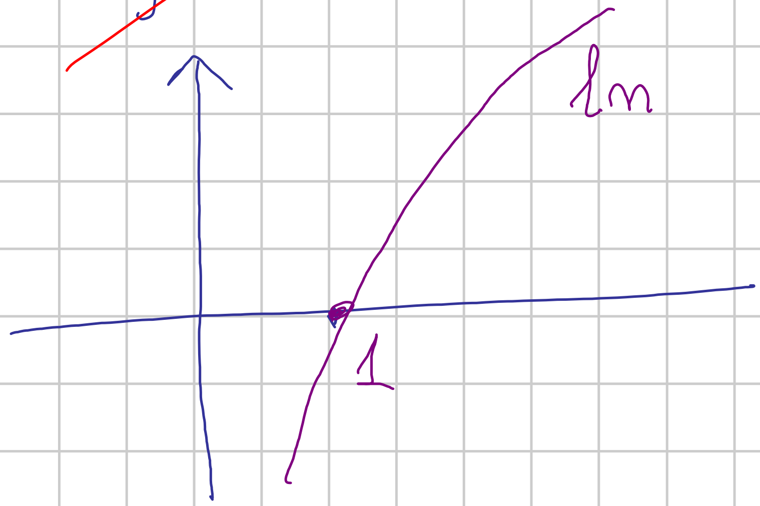
$$\cancel{\log(x+1)} + \log(x-1) + \log(x+3) > \cancel{\log(x+1)}$$

$$\log\big((x-1)(x+3)\big) > 0$$

$$(x-1)(x+3) > 1$$

$$x^2 + 2x - 4 > 0$$

$$x_{1,2} = -1 \pm \sqrt{5}$$



Senza tener conto condiz. esistenza:

$$x < -1-\sqrt{5} \quad o \quad x > -1+\sqrt{5}.$$

Considerando $x > 1$

		$-1-\sqrt{5}$		1		$-1+\sqrt{5}$	
CE	F		F		V		V
	V		F		F		V

Soluz: $x > -1+\sqrt{5}$

$\boxed{18}$ $$\left(\log_{\frac{1}{2}}(x)\right)^2 + 4\log_{\frac{1}{2}}(x) - 5 > 0$$

C.E. $x > 0$.

$$t = \log_{\frac{1}{2}}(x)$$

$$t^2 + 4t - 5 > 0$$

$$(t+5)(t-1) > 0$$

$$t < -5 \quad \circ \quad t > 1$$

$$\log_{\frac{1}{2}}(x) < -5 \quad \circ \quad \log_{\frac{1}{2}}(x) > 1$$

$$-\log_2(x) < -5 \quad \circ \quad -\log_2(x) > 1$$

$$\log_2(x) > 5 \quad \circ \quad \log_2(x) < -1$$

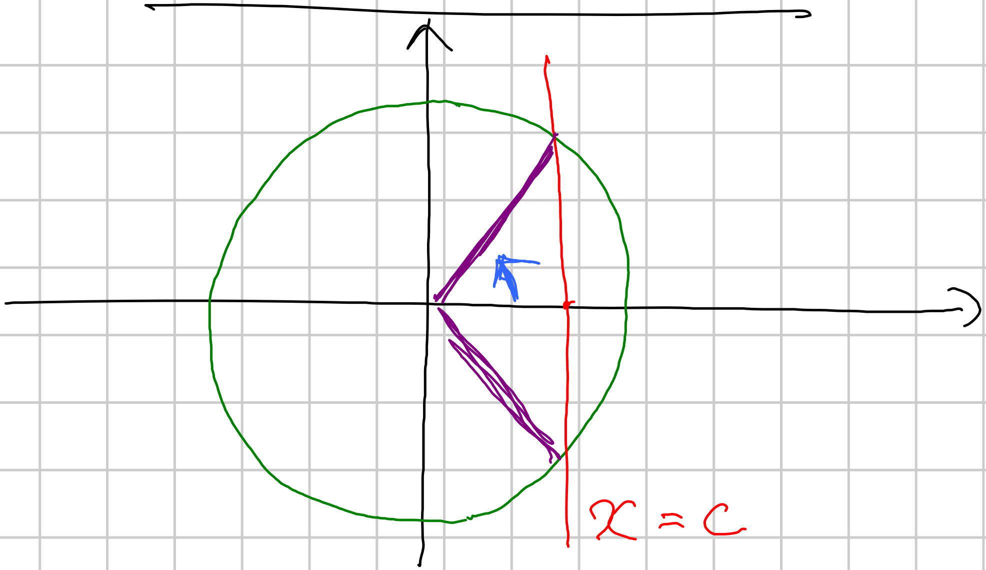
$$x > 2^5 = 32 \quad \circ \quad x < \frac{1}{2}$$

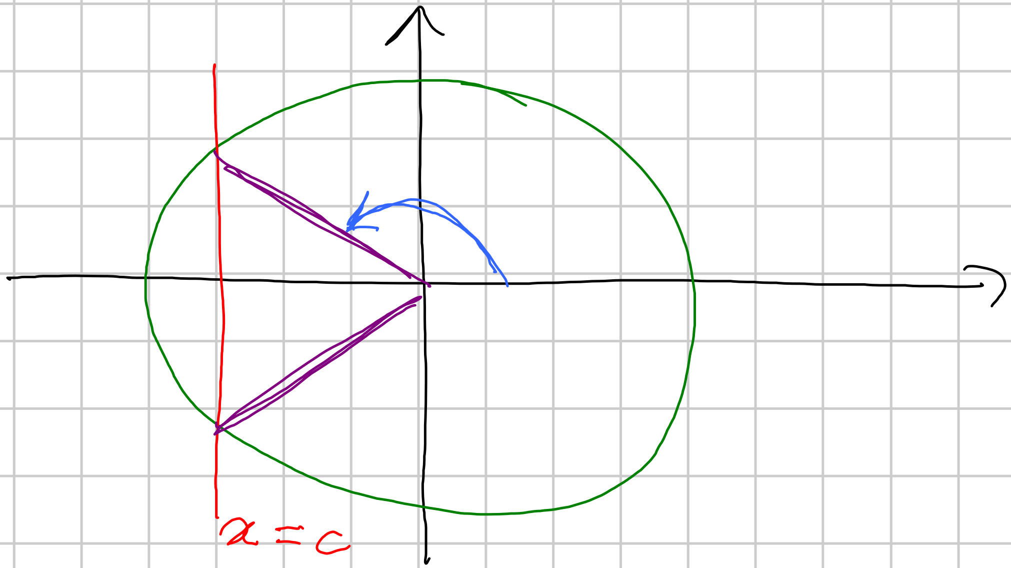
Soluz: $0 < x < 1/2$ oppure $x > 32$.

Equaz. e disequaz. trigonometriche.

$$\boxed{\cos(f(x)) = c}$$

Nessuna soluzione se $|c|>1$.



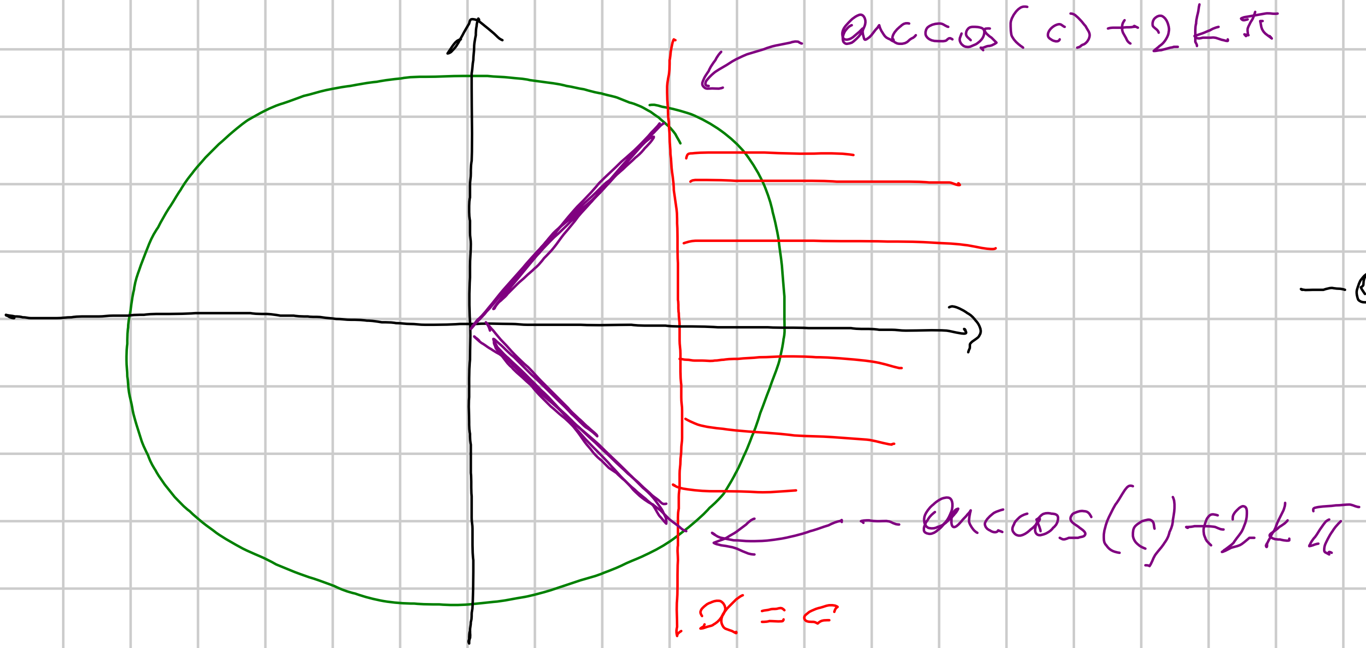


Ricordo: $\arccos : [-1, 1] \longrightarrow [0, \pi]$

L'equazione equivale a $f(x) = \pm \arccos(c) + 2k\pi$
$k \in \mathbb{Z}$

$$\boxed{\cos(f(x)) > c}$$

Sempre: cominciamo da equaz associata.



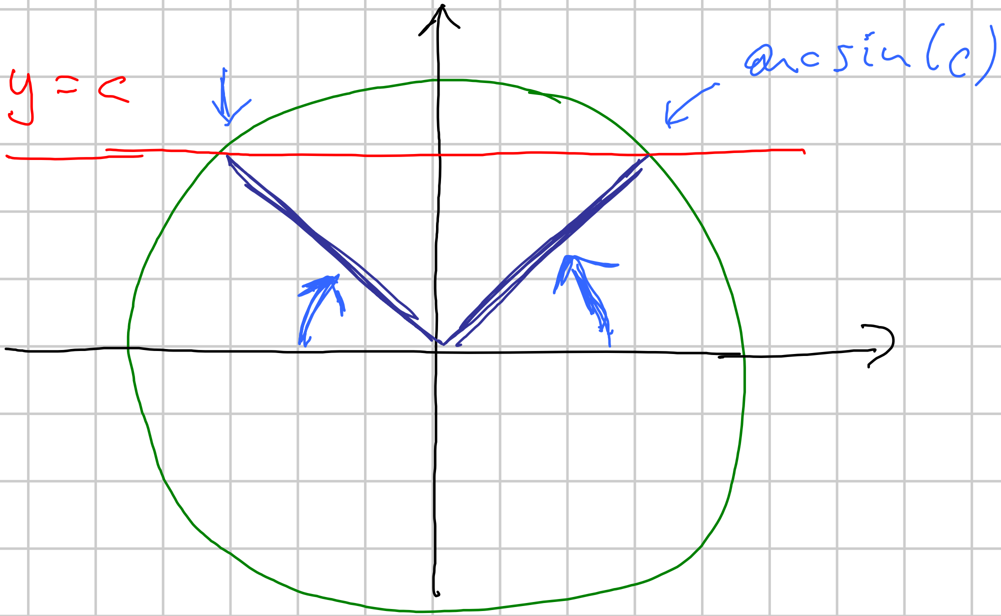
equivale a:

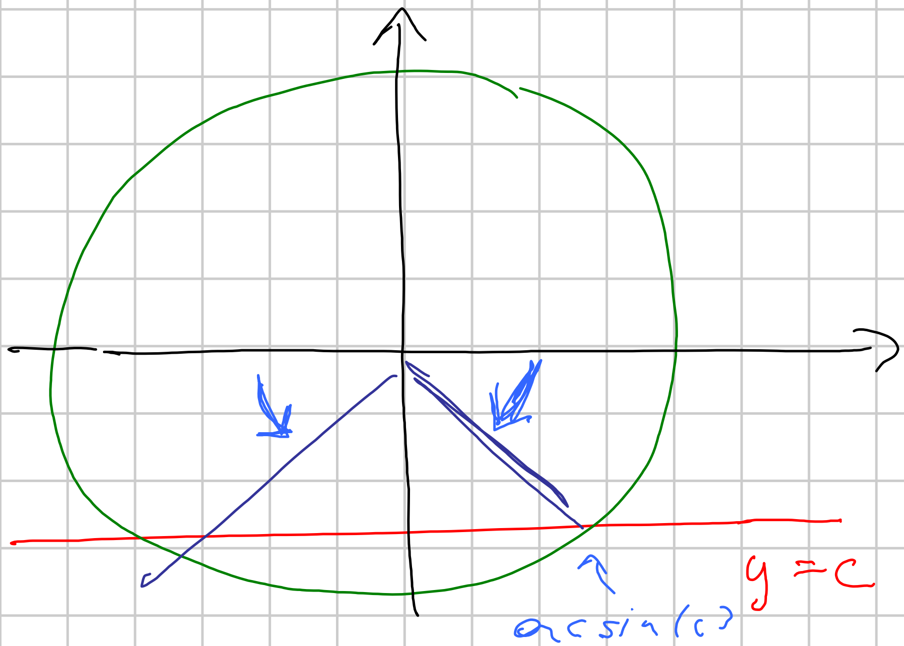
$$-\arccos(c) + 2k\pi < f(x) < \arccos(c) + 2k\pi$$

$$\boxed{\sin(f(x)) = c}$$

Nessuna soluz. per $|c| > 1$; se $|c| \le 1$

Ricordo: $\arcsin : [-1,1] \to [-\pi/2, \pi/2]$





equivale a $\quad f(x) = \arcsin(c) + 2k\pi$

oppure $\quad f(x) = \pi - \arcsin(c) + 2k\pi$

$k \in \mathbb{Z}$

ovvero
$$\begin{cases} f(x) = \arcsin(c) + 2k\pi \\ \quad \text{oppure} \\ f(x) = -\arcsin(c) + (2k+1)\pi \end{cases} \qquad k \in \mathbb{Z}$$

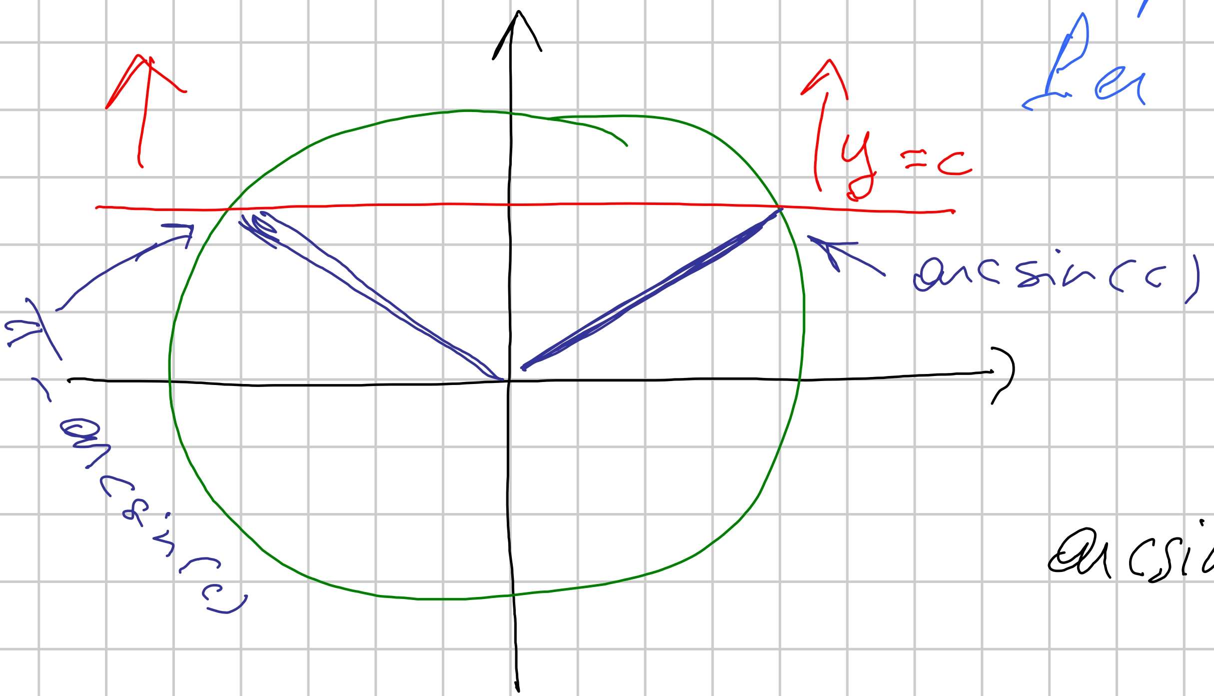
ovvero $\quad f(x) = (-1)^h \cdot \arcsin(c) + h\pi \qquad h \in \mathbb{Z}$

$$\boxed{\sin(f(x)) \geq c}$$

Impossibile se $c > 1$

Sempre vero se $c < -1$

Per $|c| \leq 1$



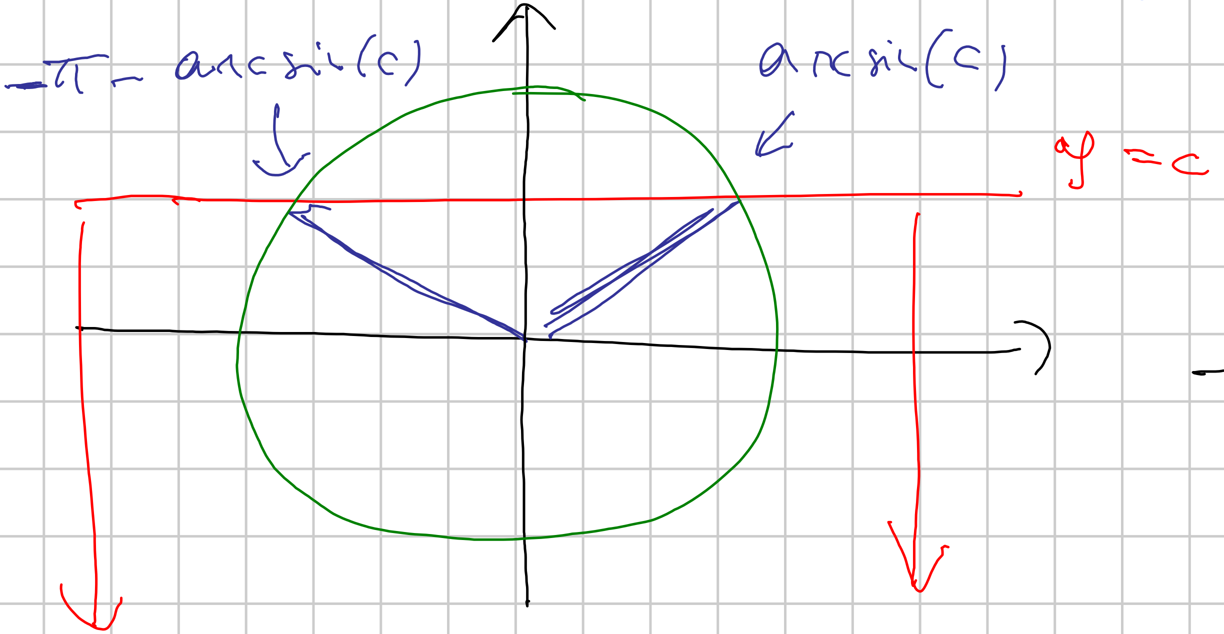
equivale a

$$\arcsin(c) + 2k\pi \leq f(x) \leq \pi - \arcsin(c) + 2k\pi$$

$$\boxed{\sin(f(x)) \le c}$$

Se $|c| > 1$ ...

Se $|c| \le 1$

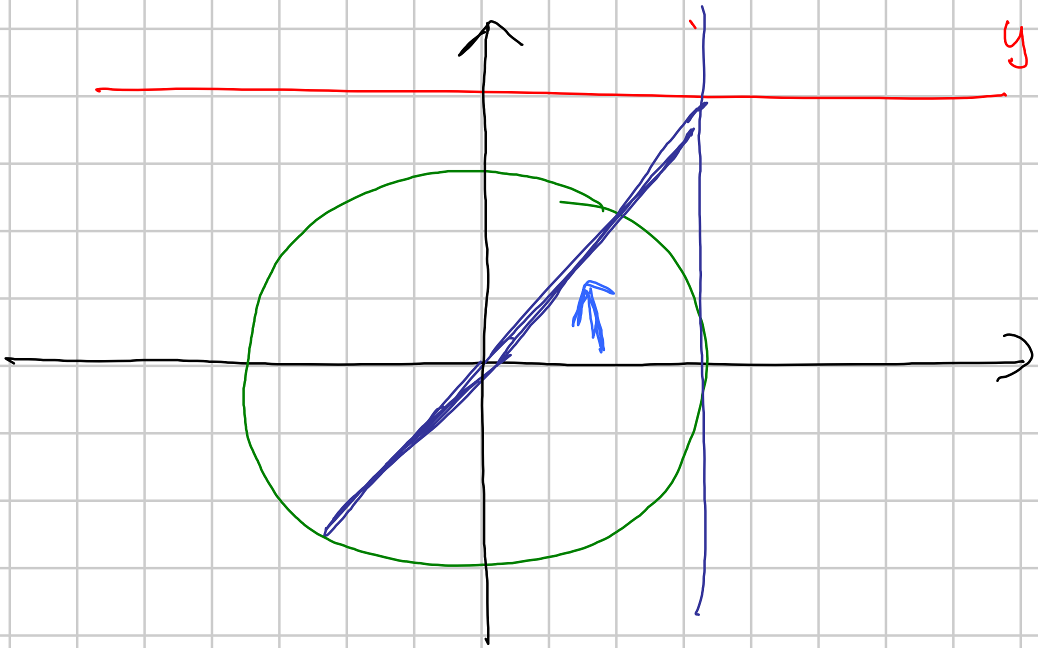


equivale a

$$-\pi - \arcsin(c) + 2k\pi \le$$
$$\le f(x) \le \arcsin(c) + 2k\pi$$

$$\boxed{\tan(f(x)) = c}$$

Ricordo: $\arctan : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

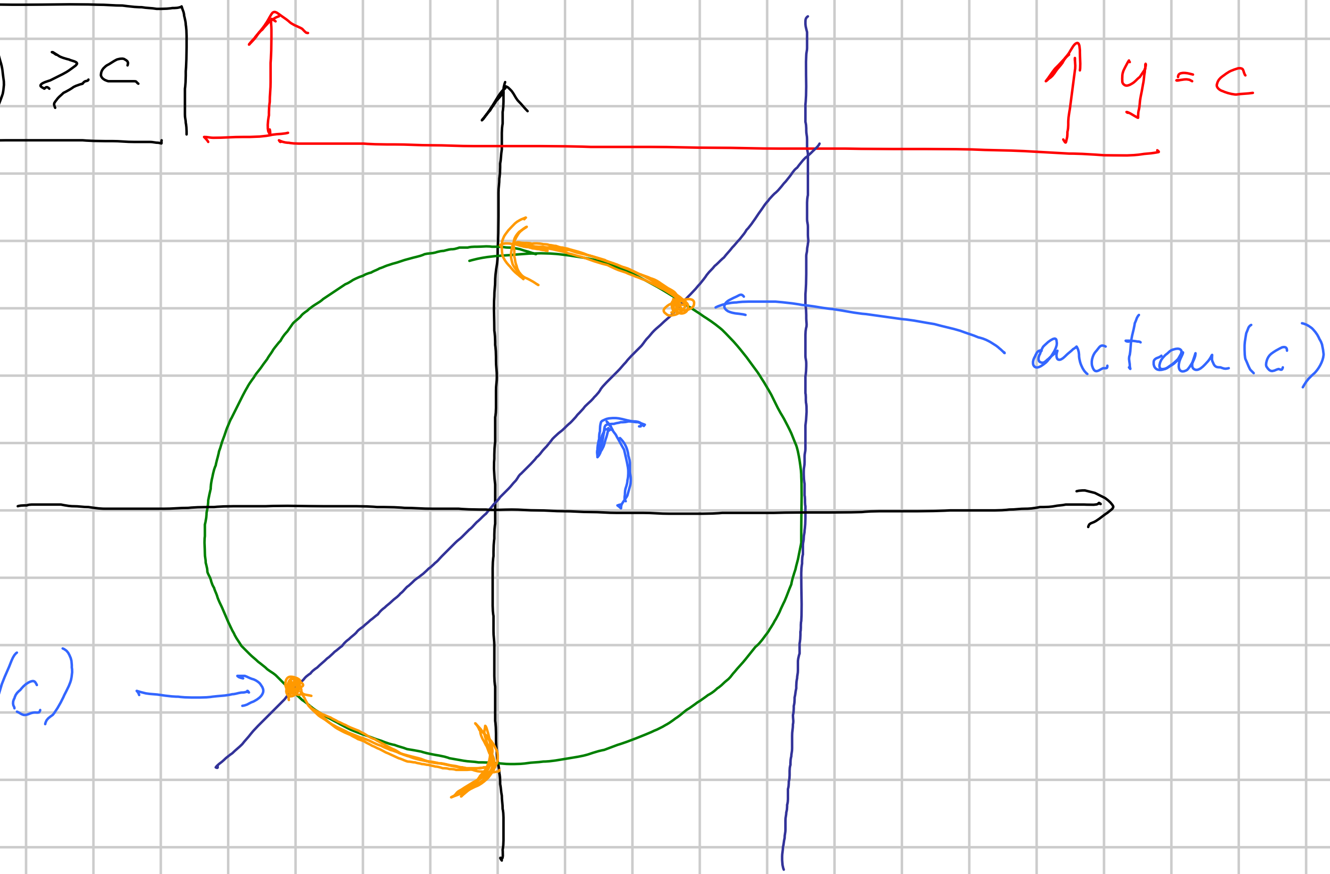


equivale a

$$f(x) = \arctan(c) + k\pi$$

$$k \in \mathbb{Z}$$

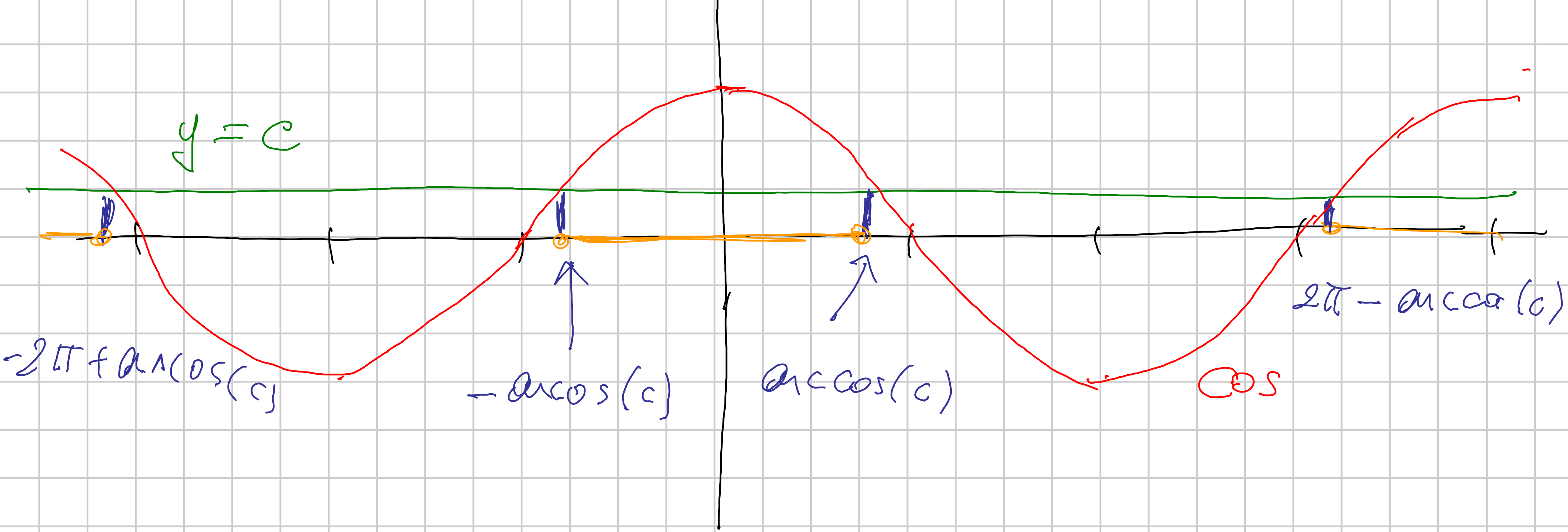
$$\boxed{\tan(f(x)) \geq c}$$



equivale a $\quad \arctan(c) + k\pi \leq f(x) < \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}.$

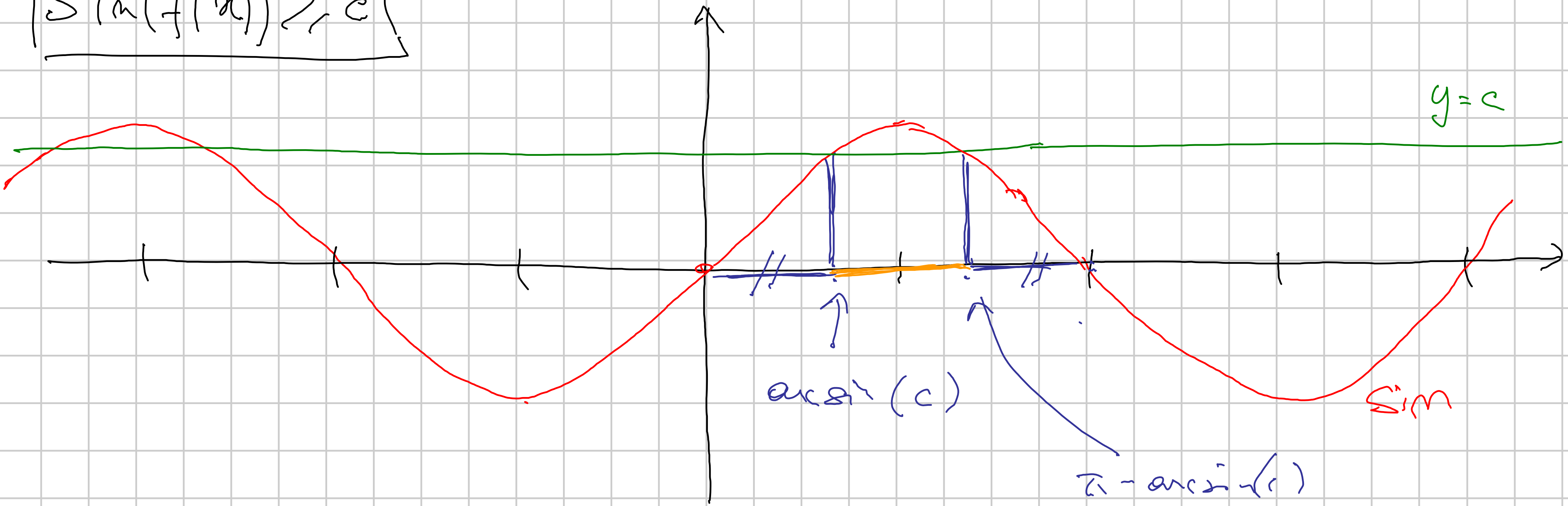
STUDIO GRAFICO ALTERNATIVO PER DISEQ.

$\cos(f(x)) \geq c$



equivale a $-\arccos(c)+2k\pi \leq f(x) \leq \arccos(c)+2k\pi$

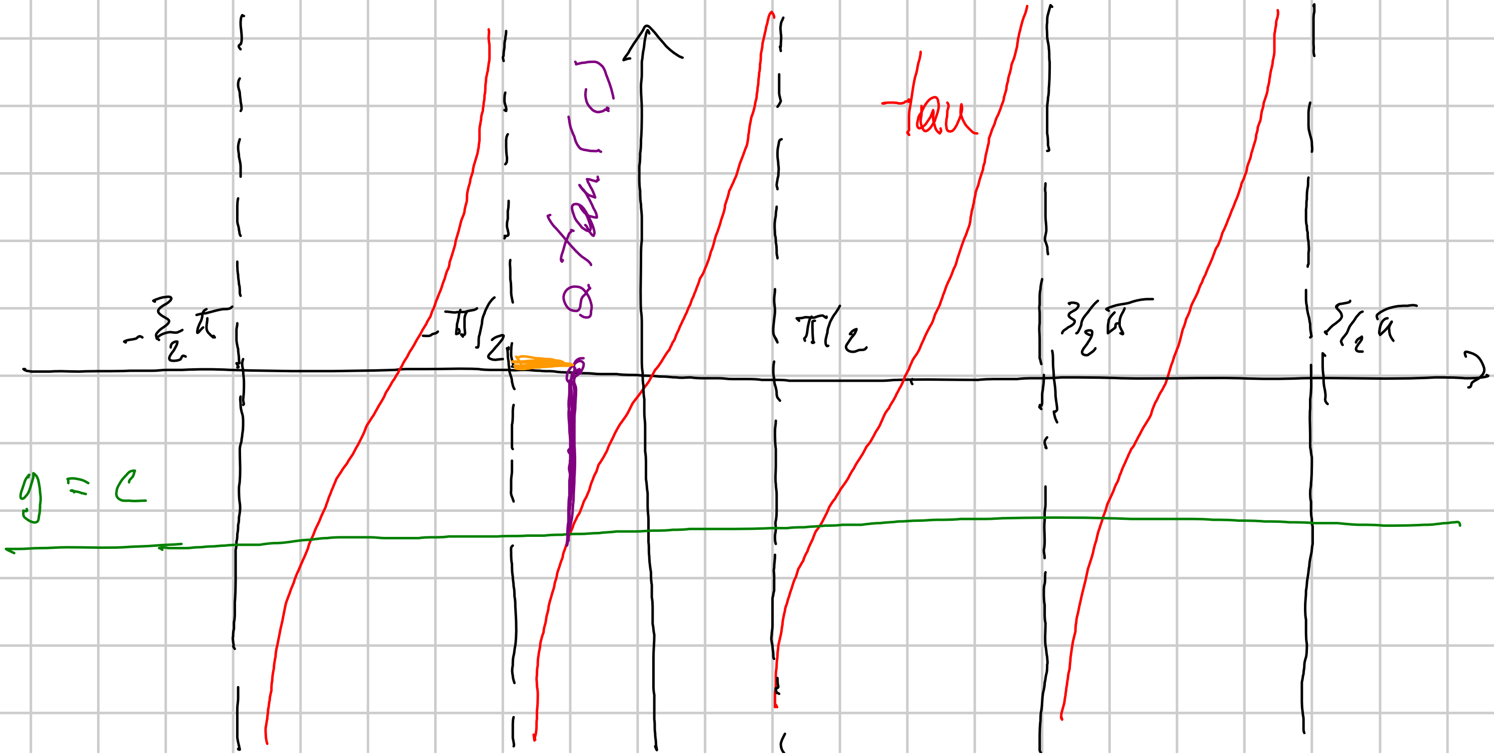
$$\boxed{\sin(f(x)) \geq c}$$



$$\boxed{tg(f(x)) \leq c}$$

equivale a

$$\arcsin(c) + 2k\pi \leq f(x) \leq \pi - \arcsin(c) + 2k\pi$$



equivale a $\quad -\frac{\pi}{2} + k\pi < f(x) \leq \operatorname{atan}(c) + k\pi$

$k \in \mathbb{Z}$

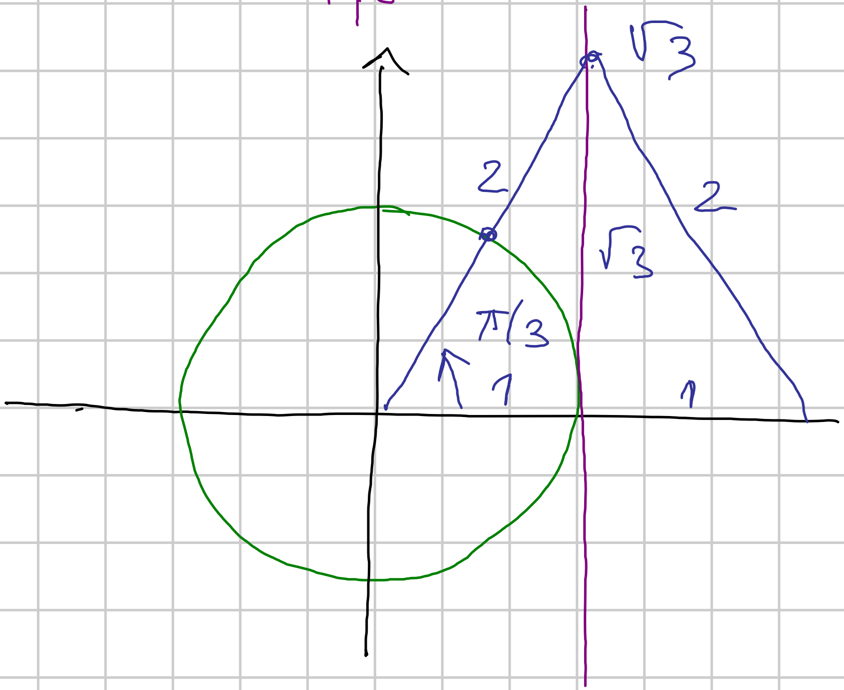
Risoluzione per sostituzione

Ese $\boxed{5}$: $\quad 3\tan^2(x) - 4\sqrt{3}\tan(x) + 3 = 0$

Sostituisco $\quad t = \tan(x)$:

$$3t^2 - 4\sqrt{3}\,t + 3 = 0$$

$$t_{1,2} = \frac{2\sqrt{3} \pm \sqrt{12 - 9}}{3} = \frac{2\sqrt{3} \pm \sqrt{3}}{3} \begin{cases} \sqrt{3} \\ \dfrac{1}{\sqrt{3}} \end{cases}$$



$$\tan\left(\pi/3\right) = \frac{\sin(\pi/3)}{\cos(\pi/3)} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$$

Analog.: $\operatorname{atan}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$: $\tan(\pi/6) = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

Soluz.: $\frac{\pi}{6} + k\pi$ o $\frac{\pi}{3} + k\pi$ $k \in \mathbb{Z}$.

$\underline{\qquad\qquad}$ o $\underline{\qquad\qquad}$

Equazioni del tipo

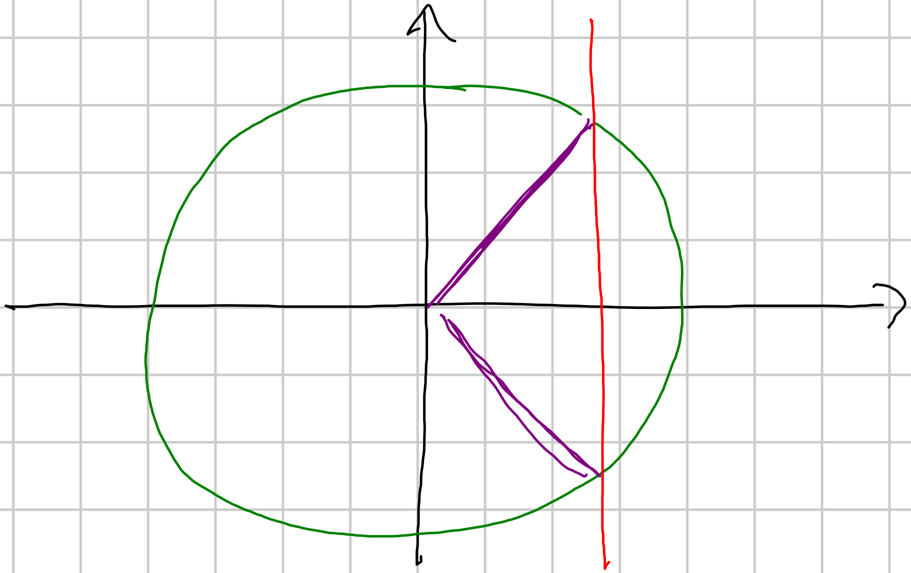
$f(g(x)) = f(h(x))$ con f funzione trigonometrica

Es: $\cos(2x+1) = \cos(1-7x)$

NON equivale a $g(x) = h(x)$

Per $\cos(2x+1) = \cos(1-7x)$:

quali sono gli angoli che hanno coseno uguale a quello di $2x+1$?



equivale a

$1-7x = 2x+1+2k\pi$

oppure

$1-7x = -(2x+1)+2k\pi$

$9x = -2k\pi$

oppure

$5x-2 = -2k\pi$

$$x = \frac{2}{5} k\pi \qquad k \in \mathbb{Z}$$

oppure

$$x = \frac{2}{5} + \frac{2}{5} k\pi \qquad k \in \mathbb{Z}$$

equazioni lineari in sin/cos.

$$a \cdot \sin(x) + b \cdot \cos(x) = c$$

Due metodi:

I. → verificare se $x = k\pi$ è soluz.

→ altrimenti pongo $t = \tan\left(\frac{x}{2}\right)$ e uso

$$\sin(x) = \frac{2t}{1+t^2} \qquad \cos(x) = \frac{1-t^2}{1+t^2}$$

viene eq. II grado in t

II riscrivo l'equaz come

$$\frac{a}{\sqrt{a^2+b^2}} \sin(x) + \frac{b}{\sqrt{a^2+b^2}} \cdot \cos(x) = \frac{c}{\sqrt{a^2+b^2}}$$

e trovo l'angolo α t.c.

$$\sin(\alpha) = b / \sqrt{a^2+b^2}$$

$$\cos(\alpha) = a / \sqrt{a^2+b^2}$$

l'equazione equivale a

$$\sin(x+\alpha) = \frac{c}{\sqrt{a^2+b^2}}$$