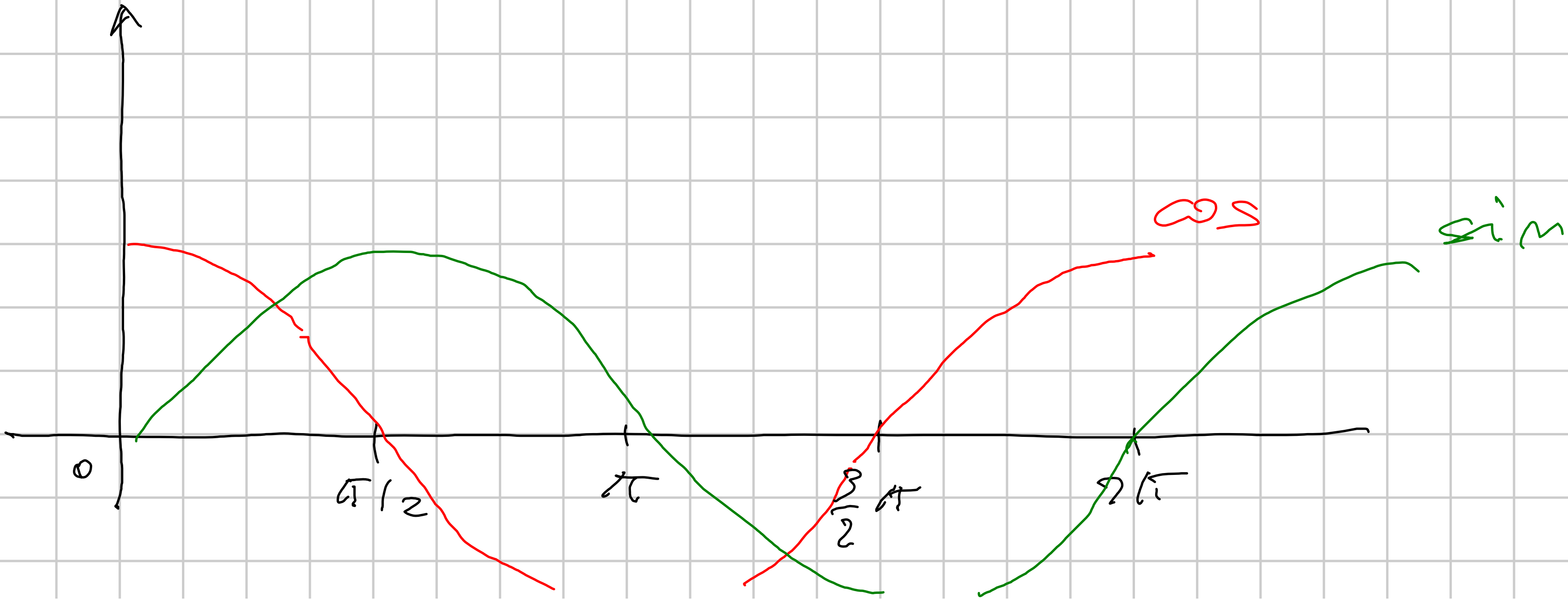
Matematica e statistica 27/11/18

Lez straord. lun 3/12 13:30 - 15 (qui).

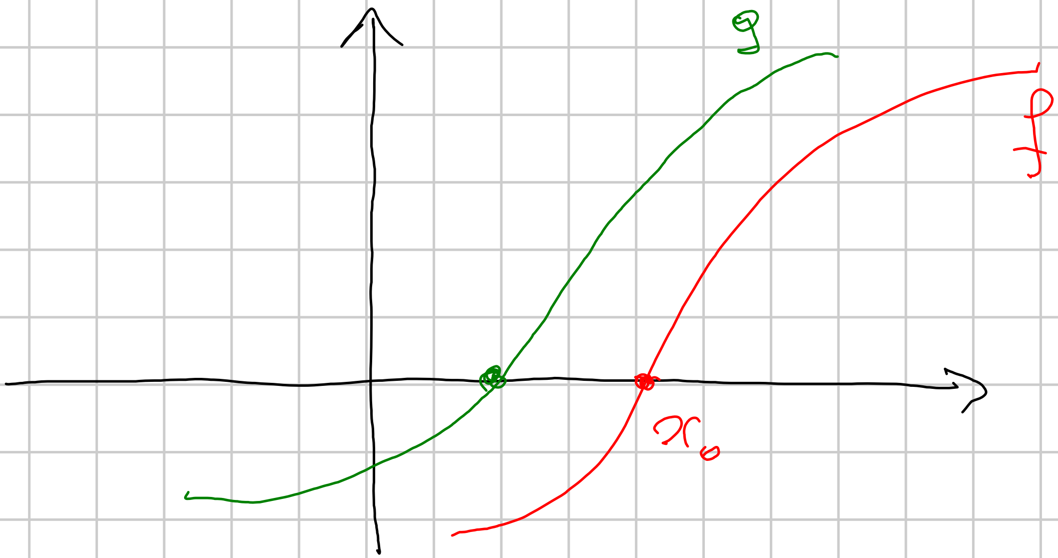


Oss: il grafico del sin è traslato a destra di $\frac{\pi}{2}$ rispetto al grafico del coseno. Dunque:

$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right).$$

In generale: operazioni sui grafici: data f

- $g(x) = f(x+k)$ $\Longrightarrow$ grafico di g traslato a sinistra di k rispetto a quello di f (a destra di $|k|$ se $k<0$)



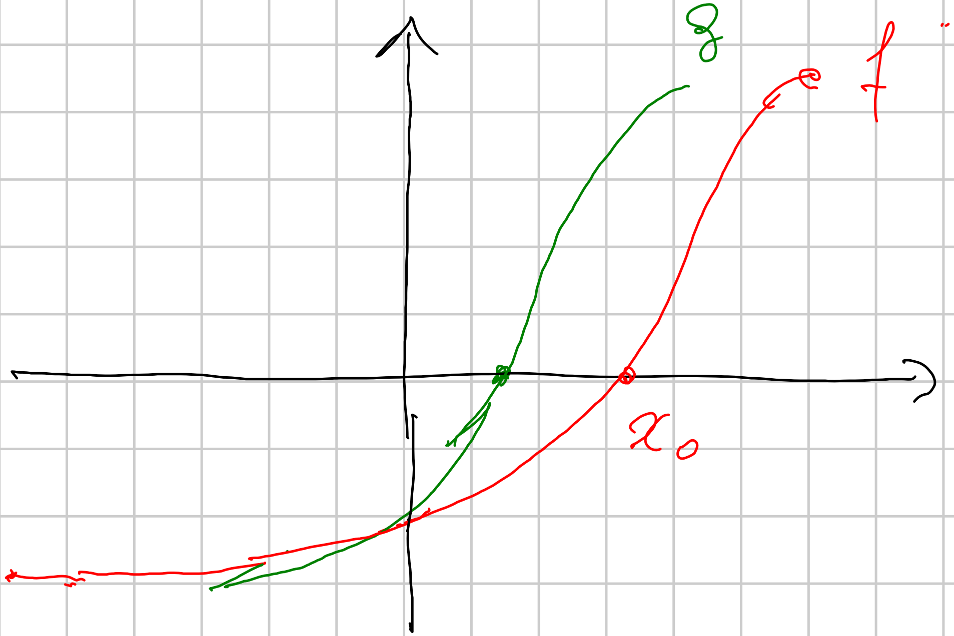
$g(x) = f(x+k) \quad (k>0)$

$g(x) = 0 \iff f(x+k) = 0$

$\iff x+k = x_0$

$\iff x = x_0 - k$

- $g(x) = f(k \cdot x) \implies$ grafico di g ottenuto da quello di f con una contrazione* di fattore $1/k$ nella direzione delle ascisse



$(k>1)$

$$g(x) = f(kx)$$

$$g(x) = 0 \iff f(kx) = 0$$

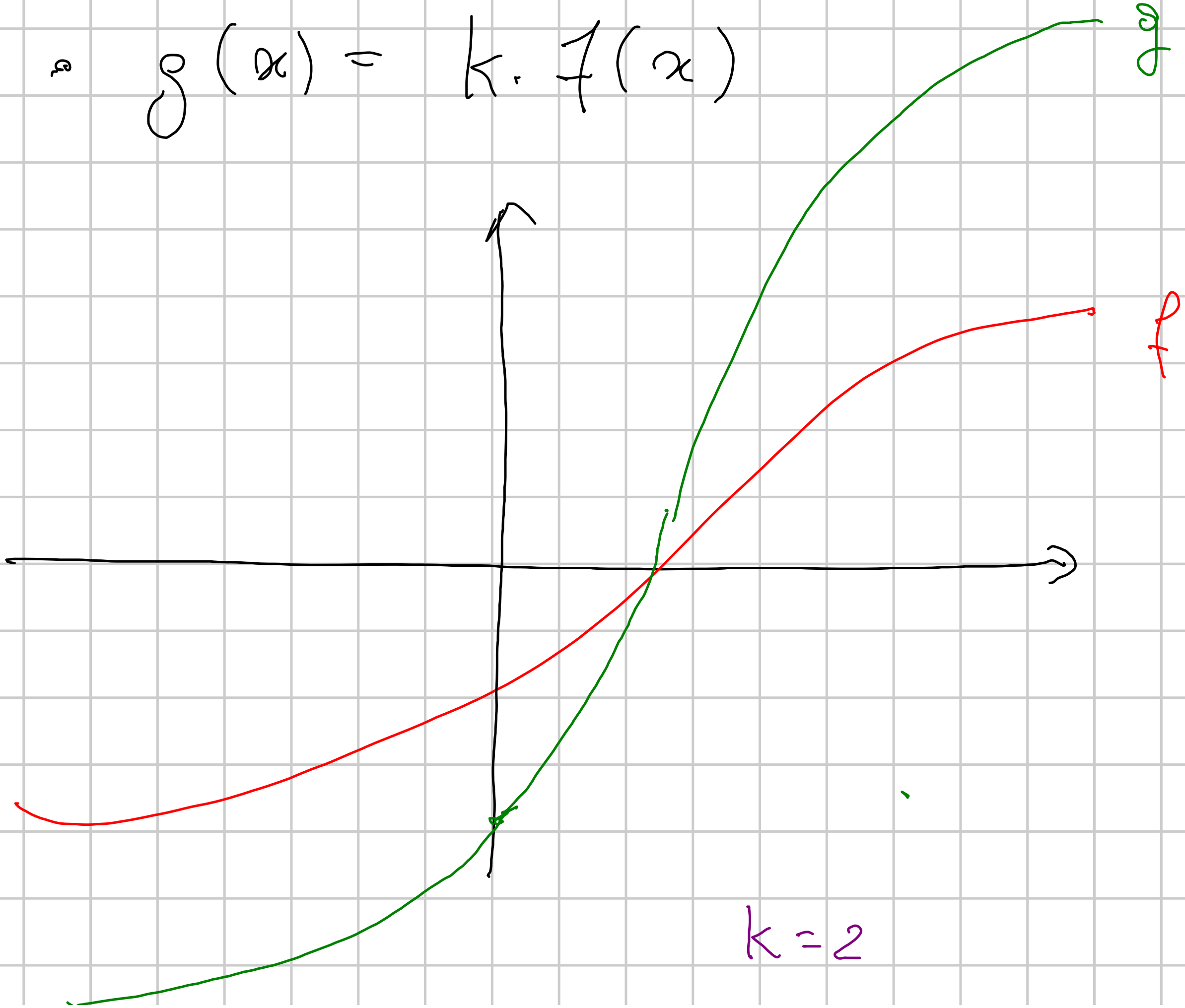
$$\iff kx = x_0$$

$$\iff x = \frac{1}{k} \cdot x_0$$

* se $0<k<1$ dilatazione di fattore $1/k$

se $k<0$ anche riflessione rispetto asse ordinate.

- $g(x) = k \cdot f(x)$



il grafico di g si ottiene da quello di f per dilatazione/contrazione/riflessione di fattore k nella direz. delle ordinate

$$g(x) = f(x) + k$$

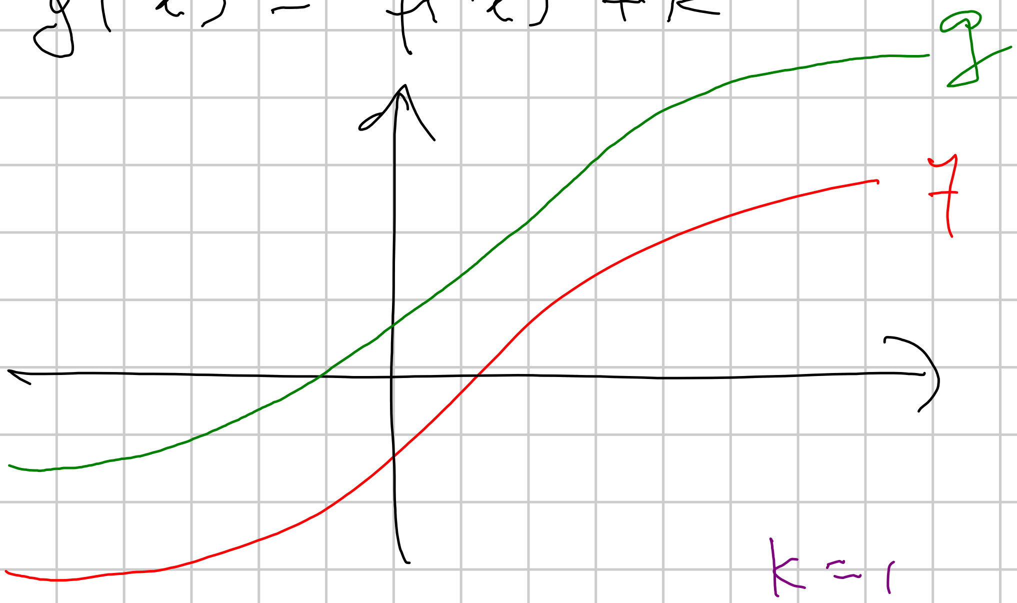
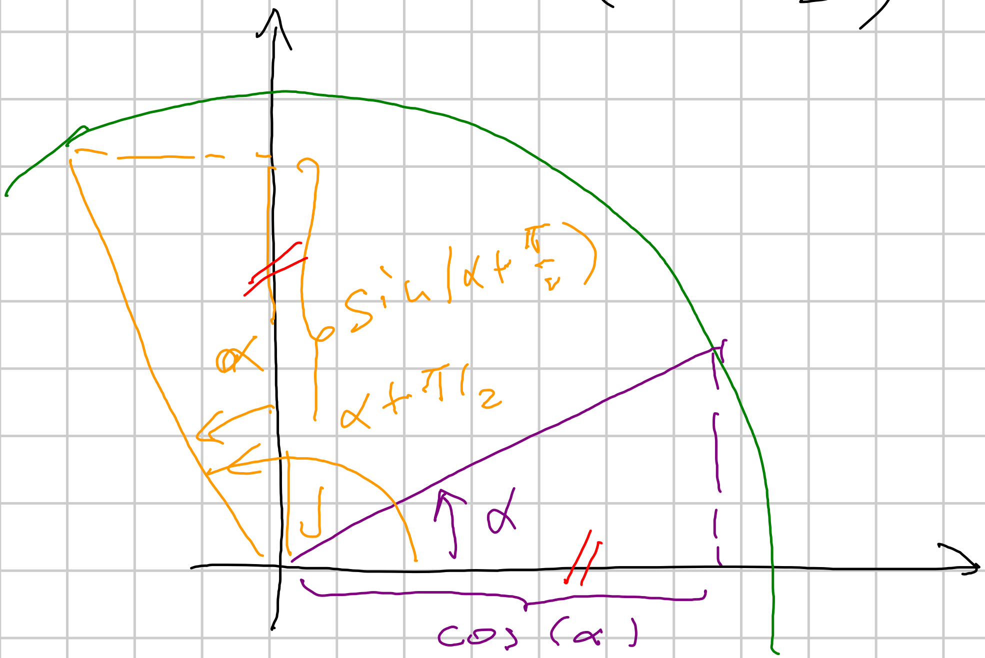


grafico di g ottenuto da quello di f traslando di k nella direzione delle ordinate.

$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right) \qquad \alpha = x - \frac{\pi}{2} \qquad x = \alpha + \frac{\pi}{2}$$

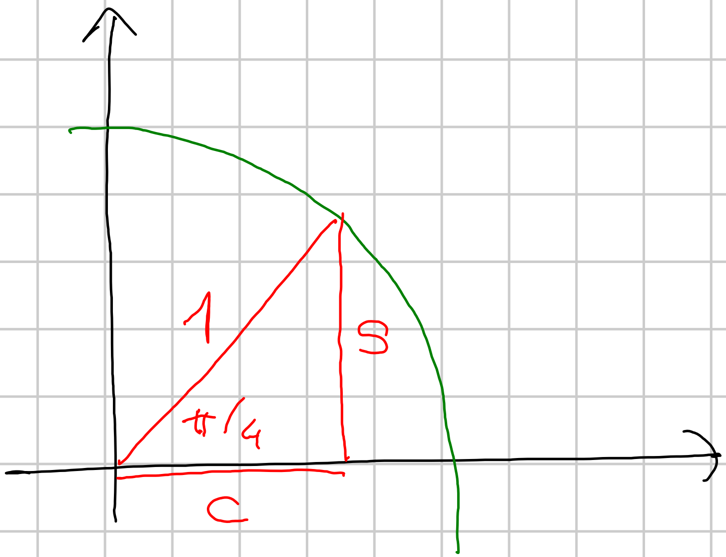
$$\cos(\alpha) = \sin\left(\alpha + \frac{\pi}{2}\right)$$

$\leftarrow$ grafico cos traslato a sinistra di $\pi/2$ da quello di sin.



cos/sin di alcuni angoli notevoli:

$\alpha = \frac{\pi}{4}$



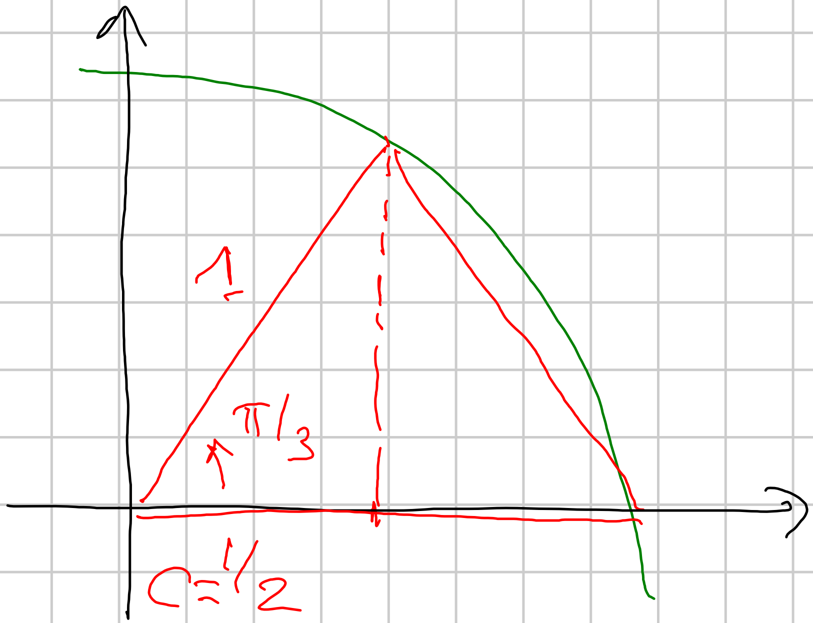
$c = s$

$c^2 + s^2 = 1$

$c^2 = \frac{1}{2}$

$c = s = \frac{1}{\sqrt{2}}$

$\alpha = \frac{\pi}{3}$



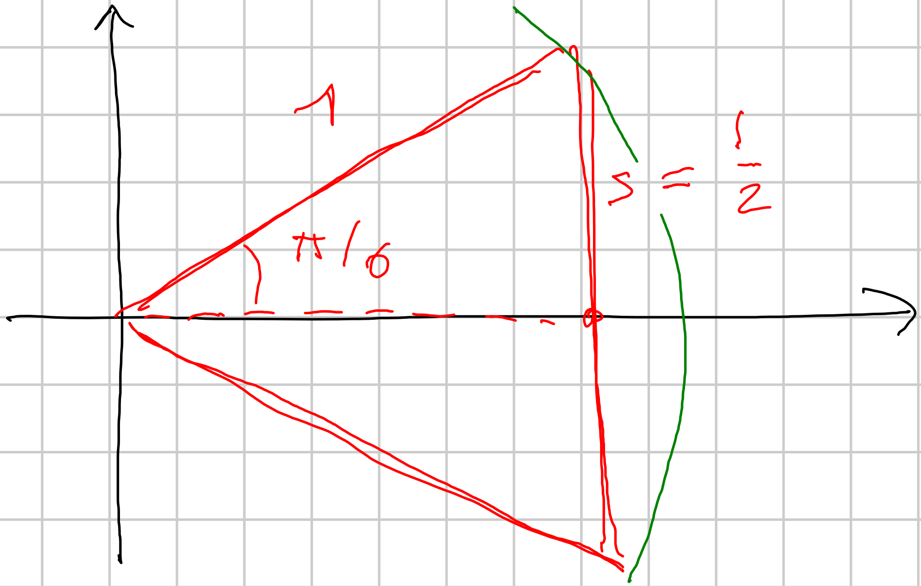
$$c^2 + s^2 = 1$$

$$\frac{1}{4} + s^2 = 1$$

$$s^2 = \frac{3}{4}$$

$$s = \frac{1}{2}\sqrt{3} \qquad c = \frac{1}{2}$$

$\alpha = \frac{\pi}{6}$



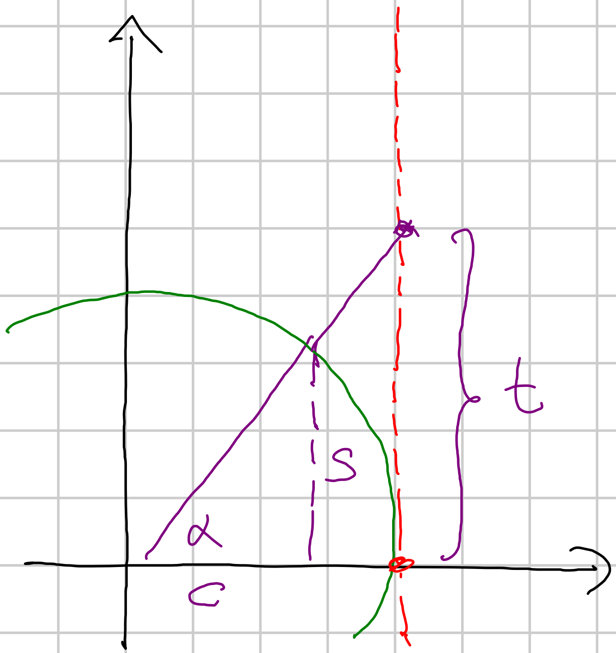
$c = \frac{1}{2}\sqrt{3} \qquad s = \frac{1}{2}$

Tangente:

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \qquad x \neq \frac{\pi}{2} + k\pi$$

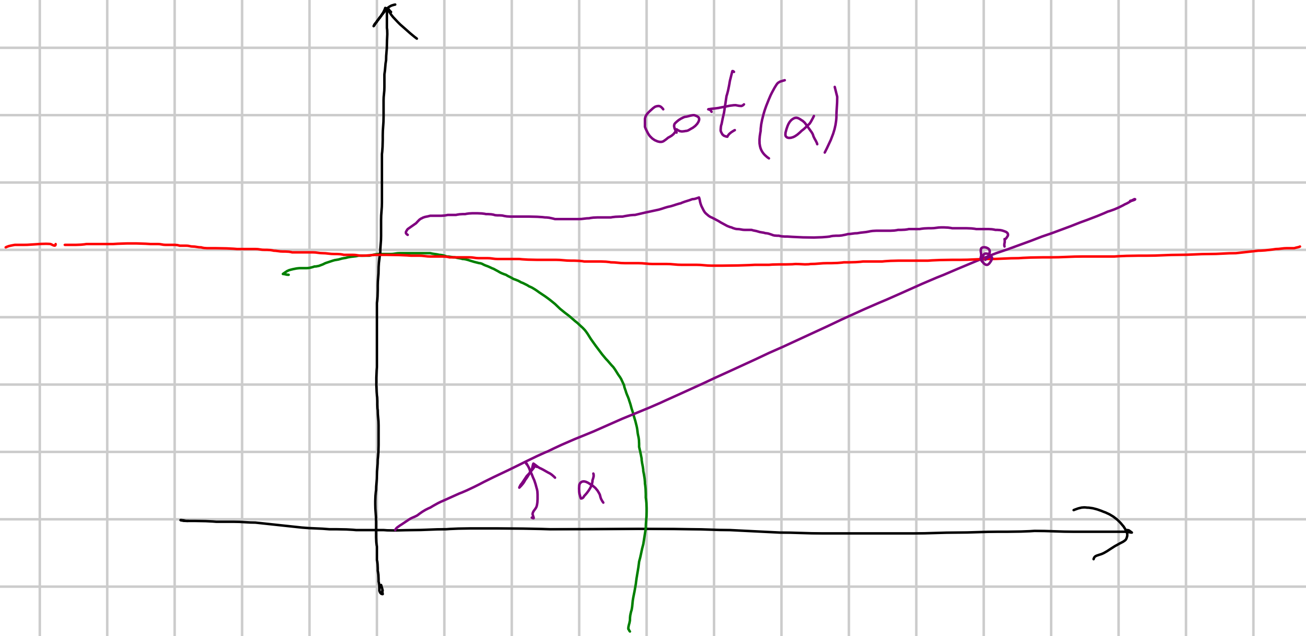
cotangente :

$$\cot(x) = \frac{\cos(x)}{\sin(x)} \qquad x \neq k\pi$$

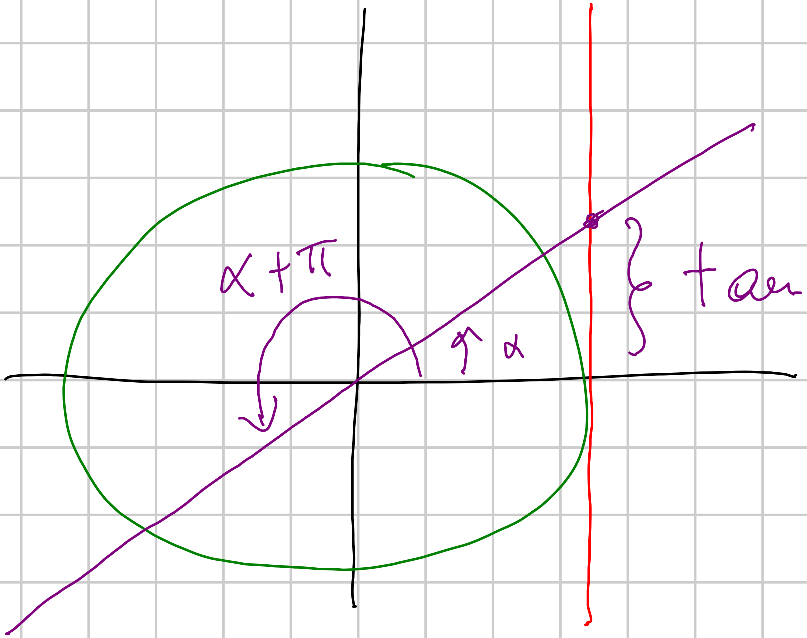


$$s : c = t : 1$$

$$t = \frac{s}{c} = \tan(\alpha)$$

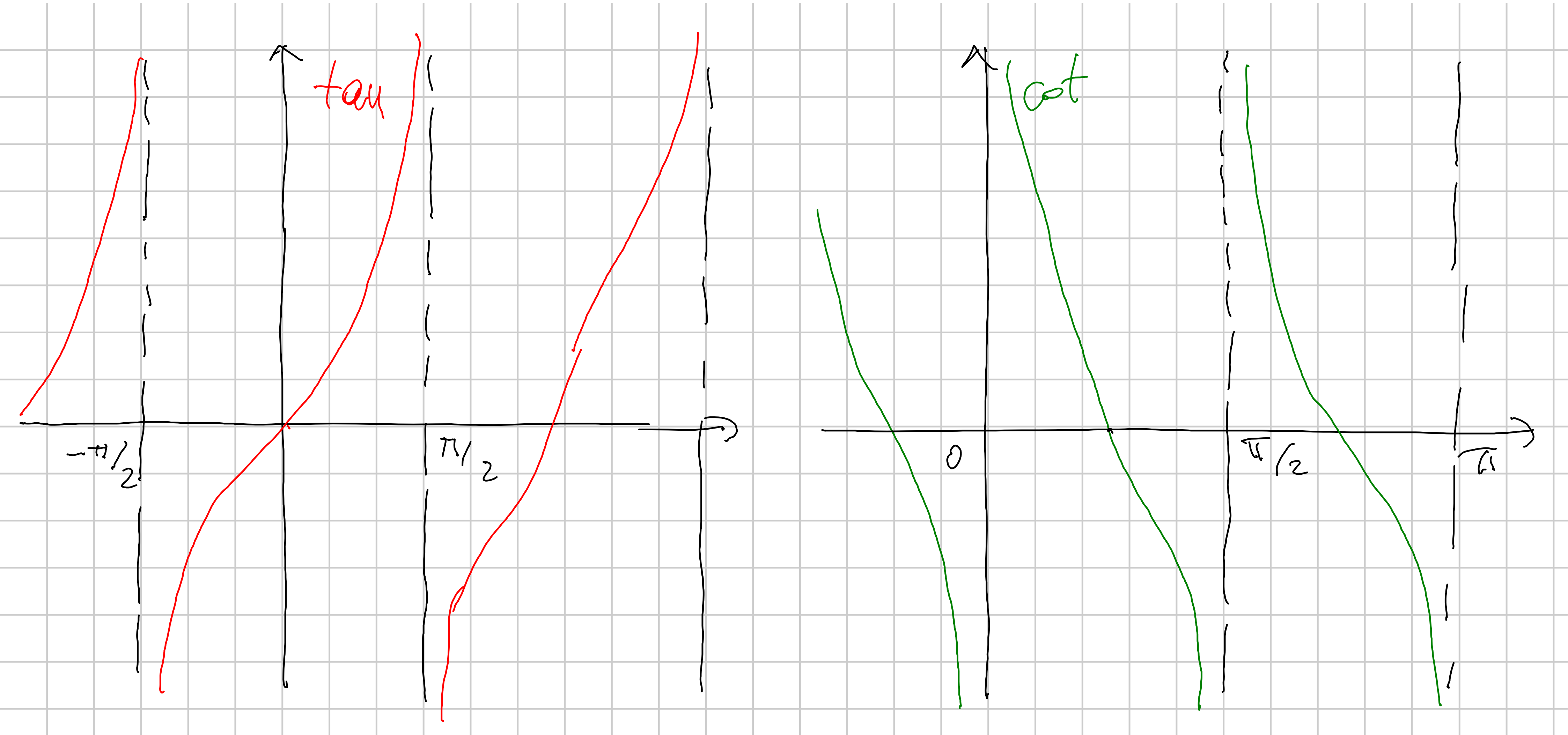


⟹ tan, cot sono periodiche di periodo π :



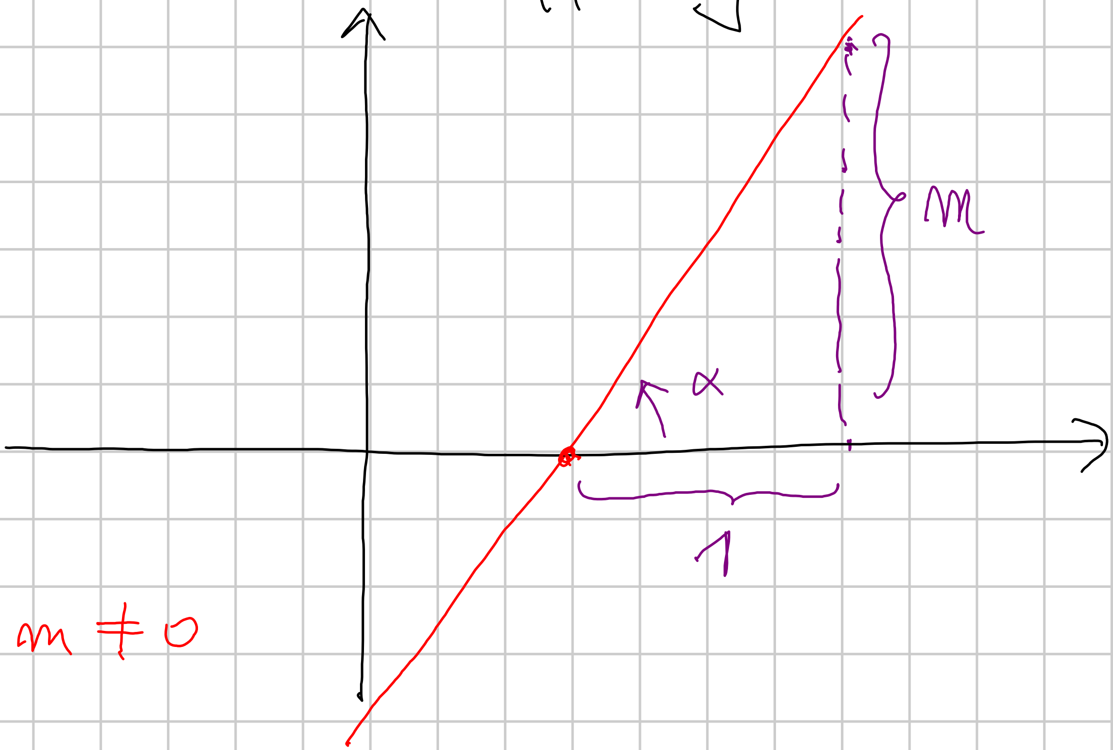
$$\tan(\alpha) = \tan(\alpha + \pi)$$

Gyakci:


tan
$-\pi/2$
$\pi/2$
cot
0
$\pi/2$
π

Oss: per la retta $r: \; y = mx + q$

m = coeff. angolare è



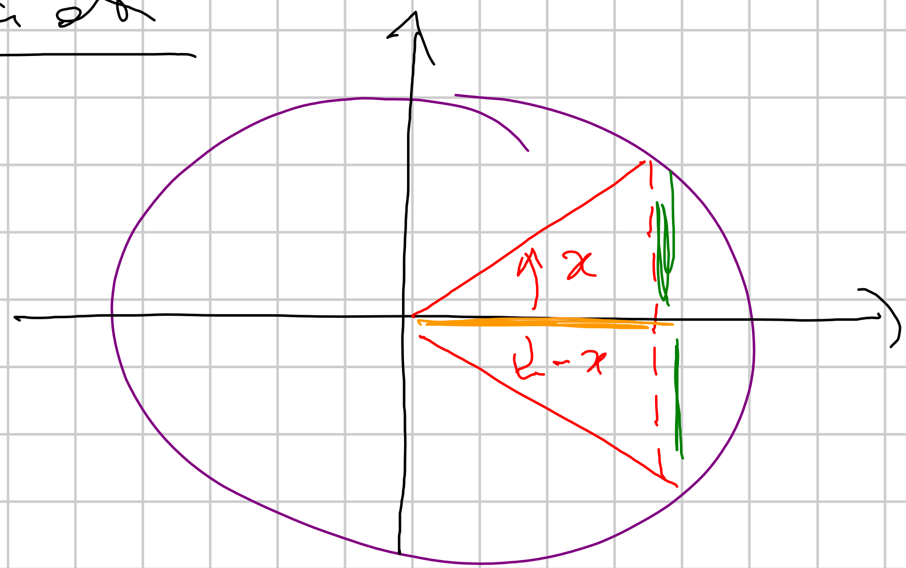
$\Longrightarrow m = \mathrm{Tg}(\alpha)$

m = tangente angolo formato da r e semiasse positivo ascisse.

Sin/cos di angoli associati

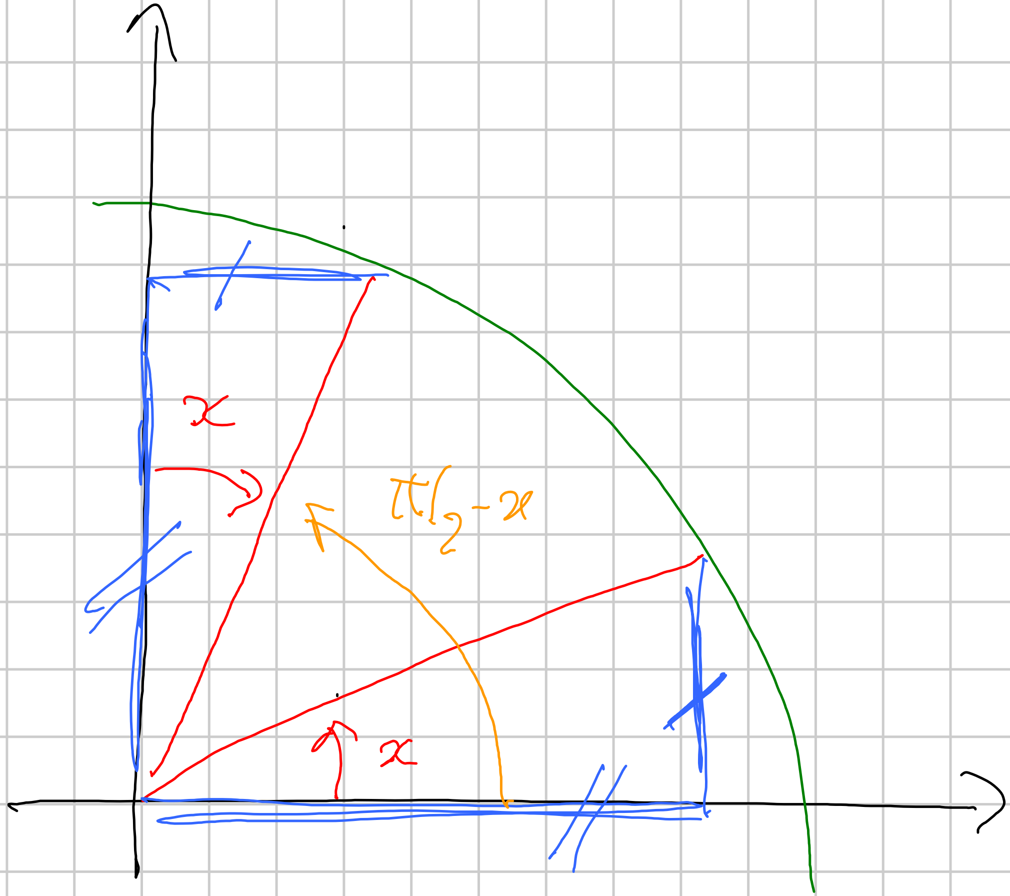
$$\sin(-x) = -\sin(x)$$

$$\cos(-x) = \cos(x)$$



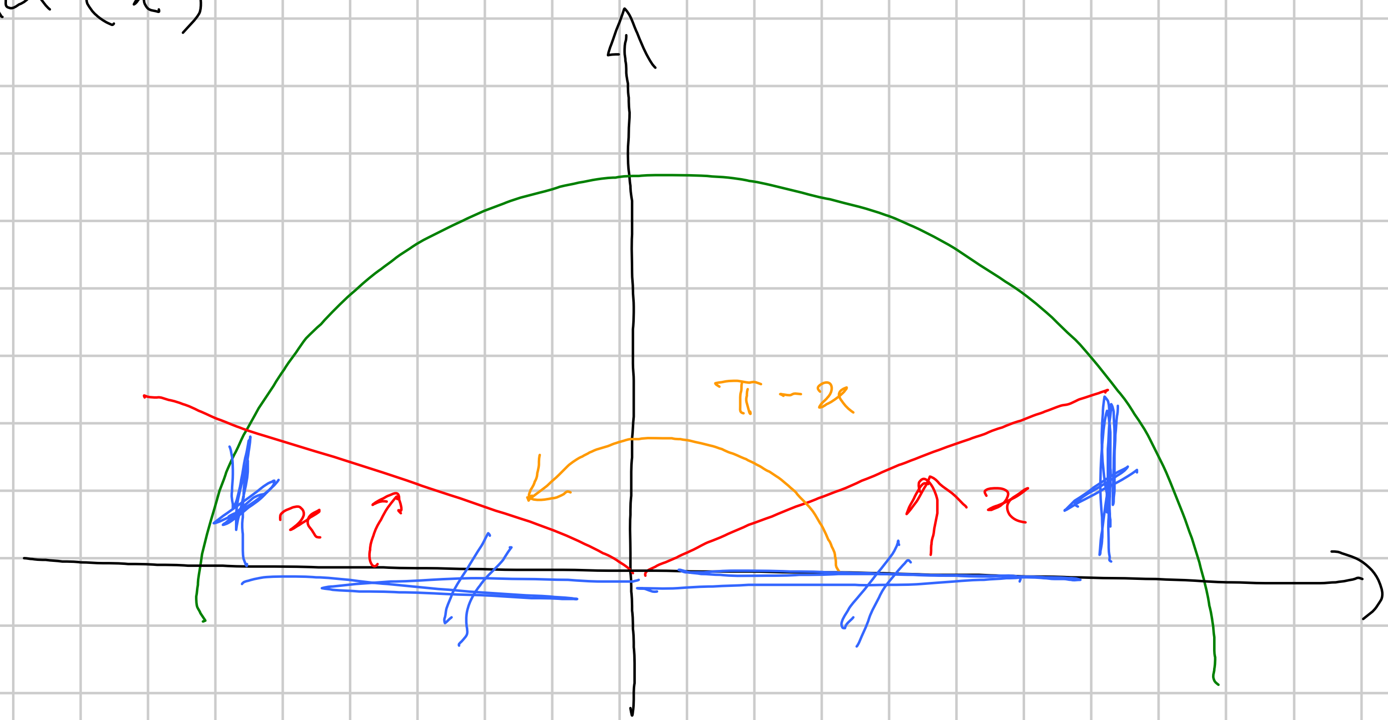
$$\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$$

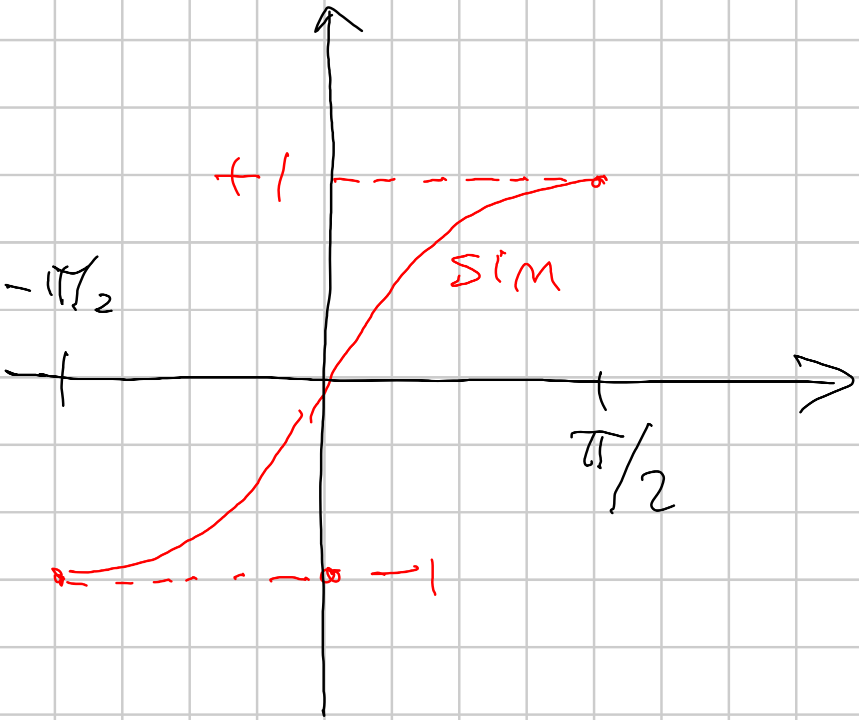


$$\cos(\pi - x) = -\cos(x)$$

$$\sin(\pi - x) = \sin(x)$$



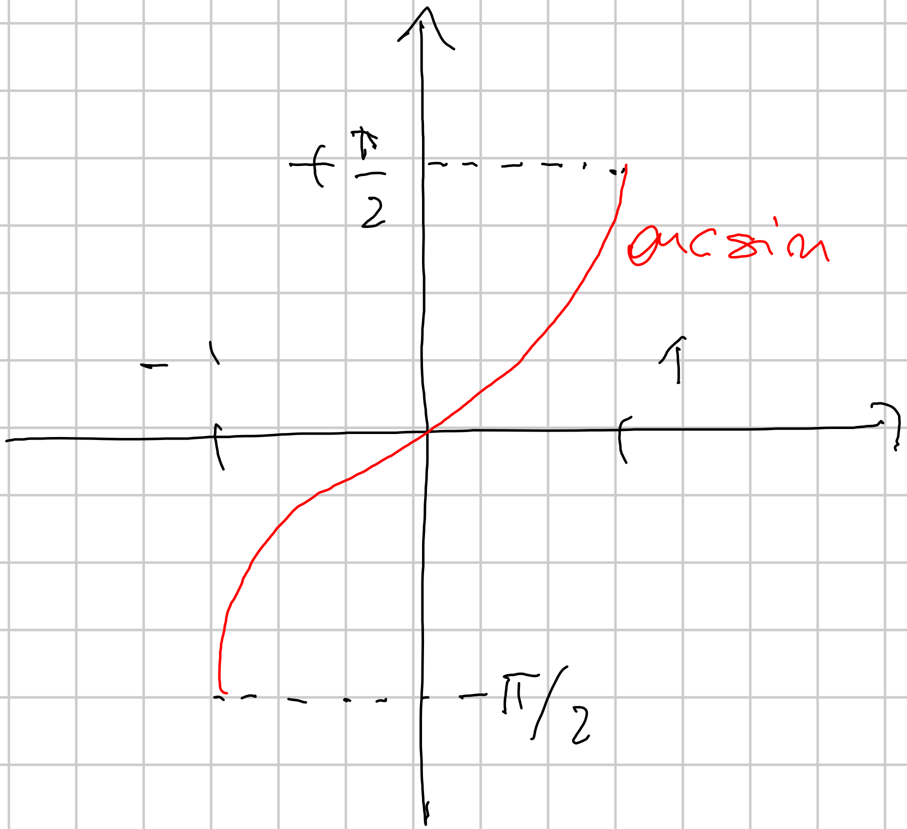
Funzioni trigonometriche inverse.



sin è biiezione tra $[-\pi/2, \pi/2]$ e $[-1,1]$.

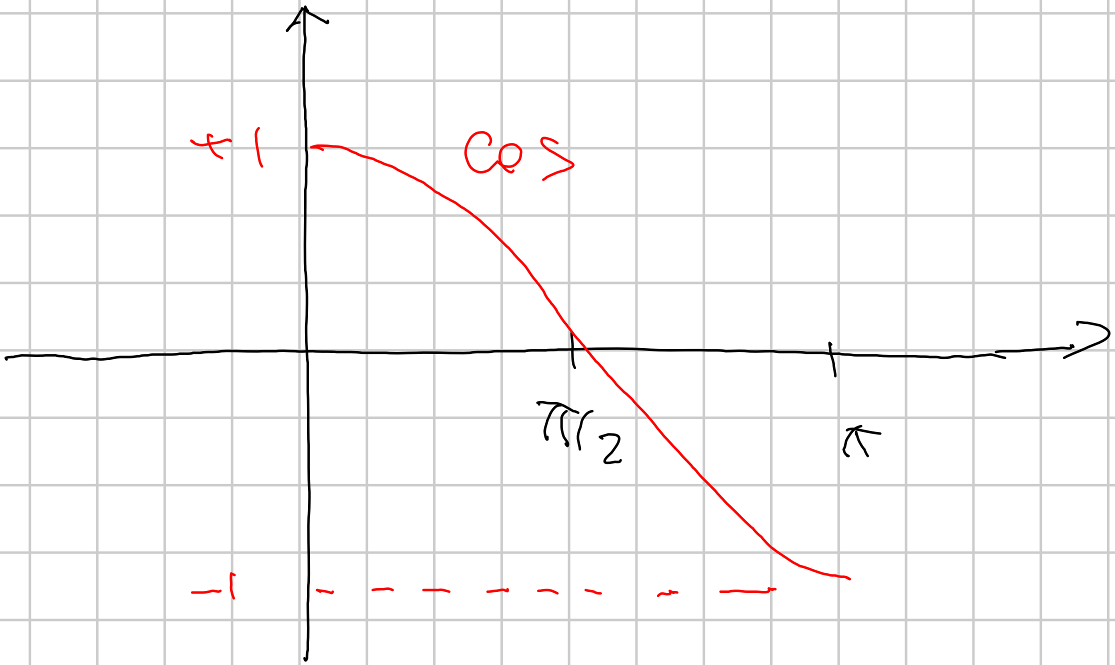
Chiamo arcsin l'inversa:

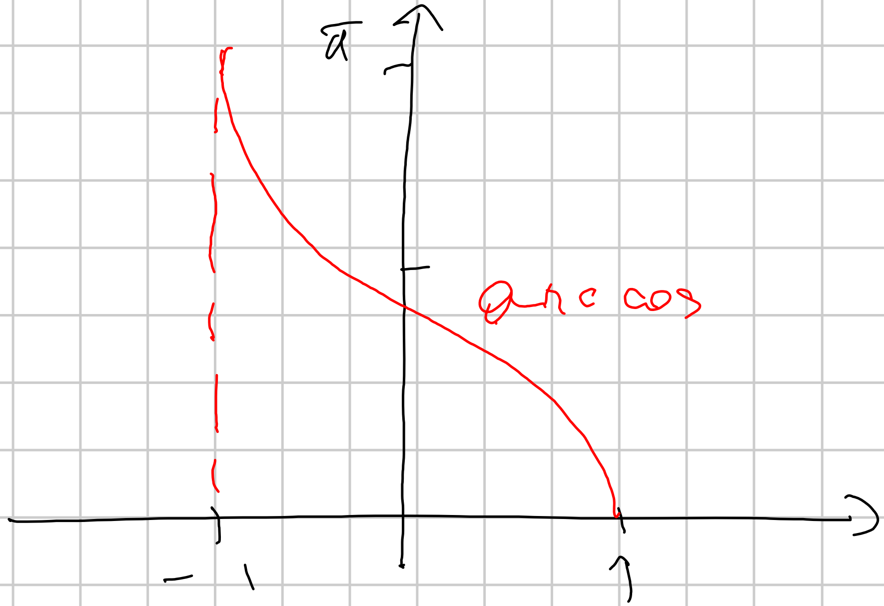
$$\arcsin : [-1,1] \to [-\pi/2, \pi/2]$$



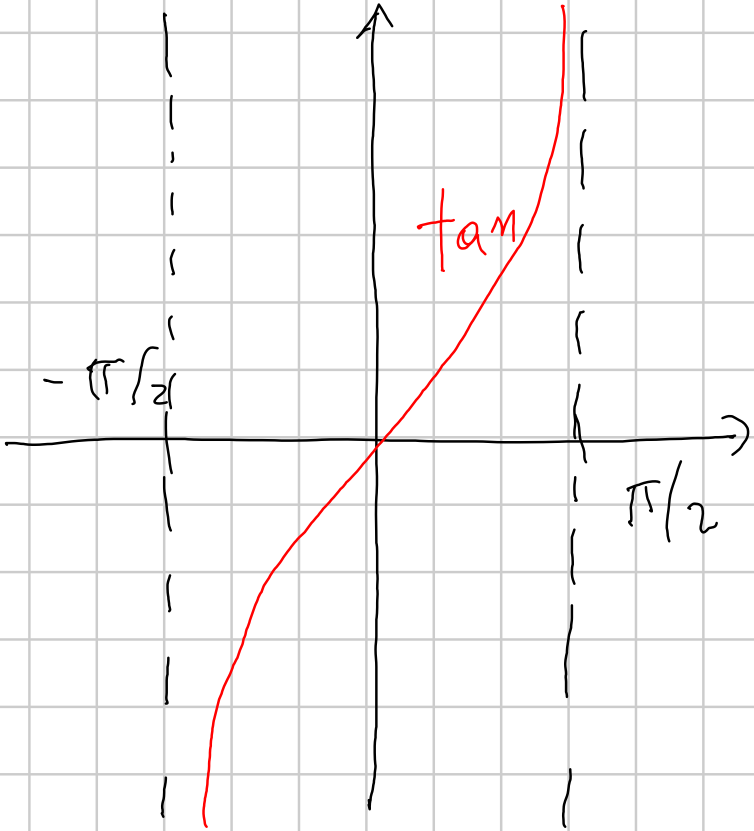
$+\frac{\pi}{2}$
arcsin
-1
1
$-\pi/2$

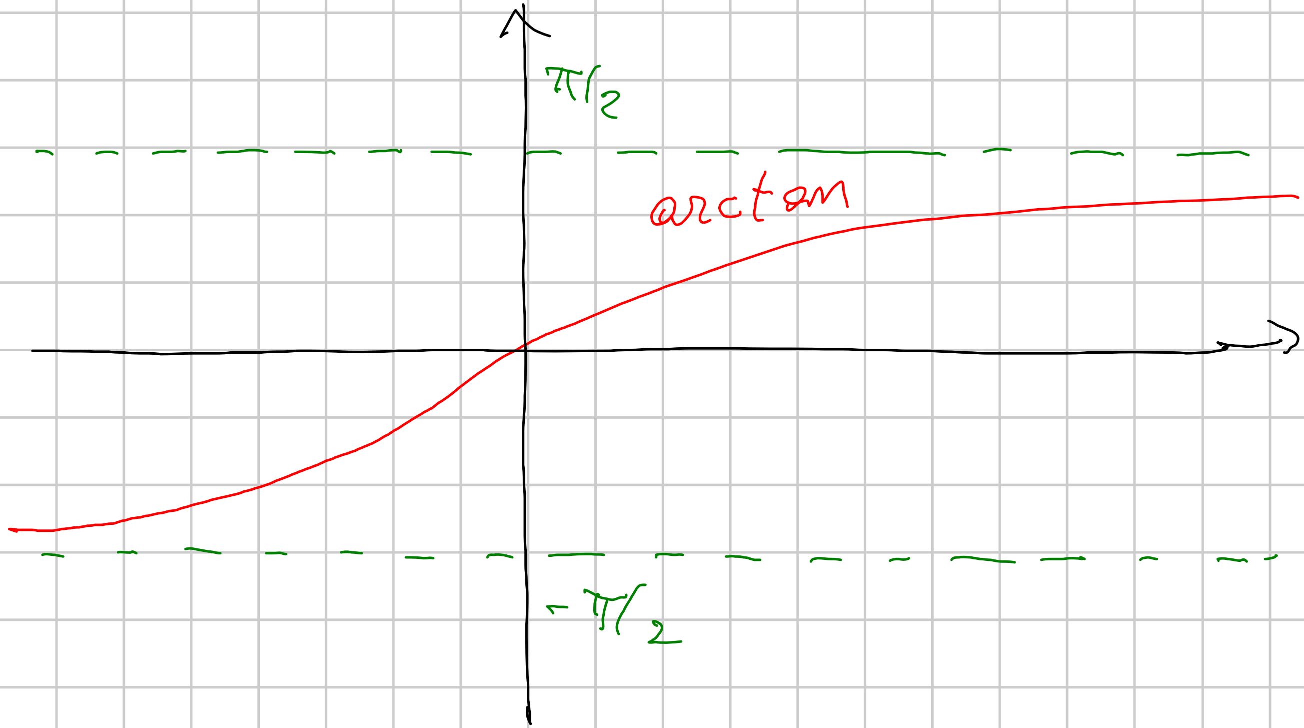
arccos : [-1,1] $\rightarrow$ [0, π]





$\arctan : \mathbb{R} \to (-\pi/2, \pi/2)$





Formule di addizione e collegate

$$\begin{cases} \sin(\alpha+\beta) = \sin(\alpha)\cdot\cos(\beta)+\cos(\alpha)\cdot\sin(\beta) \\ \cos(\alpha+\beta) = \cos(\alpha)\cdot\cos(\beta)-\sin(\alpha)\cdot\sin(\beta) \end{cases}$$

duplicazione:

$$\begin{cases} \sin(2\alpha) = 2\sin(\alpha)\cos(\alpha) \\ \cos(2\alpha) = \cos^2(\alpha)-\sin^2(\alpha) \end{cases}$$

$$= 2\cos^2(x) - 1 = 1 - 2\sin^2(\alpha)$$

Formule parametriche:

Se $\alpha \neq k \cdot \pi$ con $k \in \mathbb{Z}$ e $t = \tan(\alpha/2)$

$$\sin(\alpha) = \frac{2t}{1+t^2} \qquad \cos(\alpha) = \frac{1-t^2}{1+t^2}.$$

"Funzioni irrazionali"

$\boxed{7}$ $\sqrt{1-4x} \leq 2x+1$.

algebricamente:

(sviluppo sistema equiv. a equaz. iniz.)

$$\begin{cases} 1-4x \geq 0 \\ 2x+1 \geq 0 \\ 1-4x \leq (2x+1)^2 \end{cases}$$

$$\begin{cases} x \leq 1/4 \\ x \geq -1/2 \\ 4x^2+8x \geq 0 \end{cases}$$

$$\begin{cases} -1/2 \leq x \leq 1/4 \\ x(x+2) \geq 0 \end{cases}$$

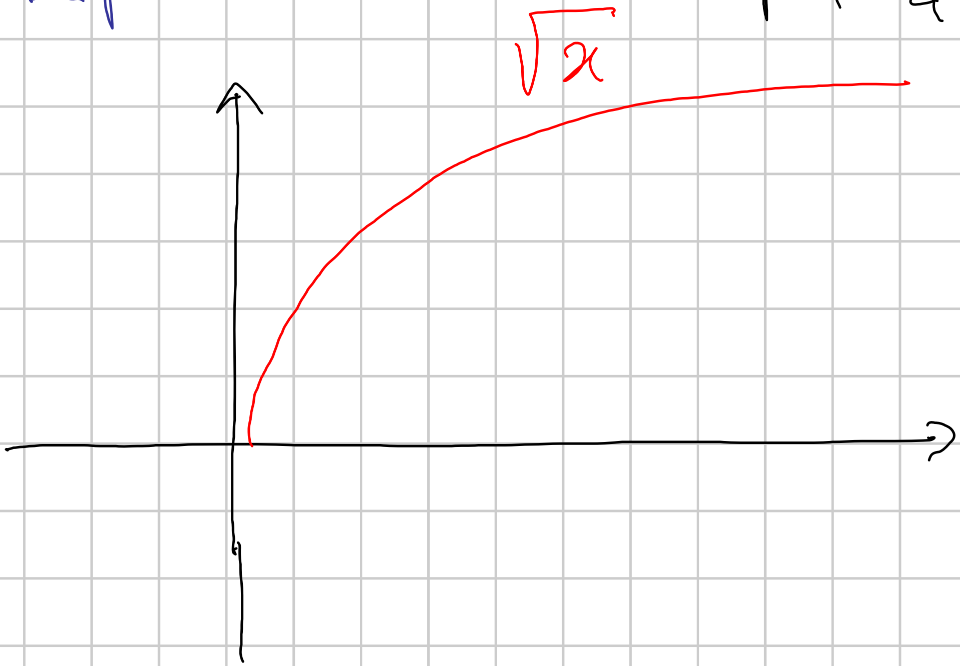
$$\begin{cases} -\frac{1}{2} \le x \le \frac{1}{4} \\ x \le -2 \quad \text{o} \quad x \ge 0 \end{cases}$$

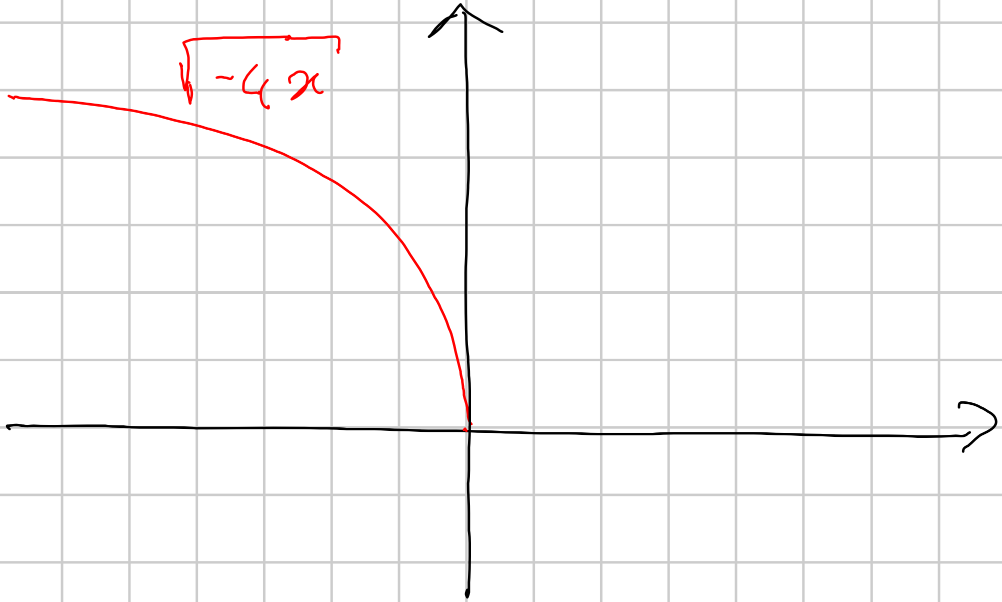
	-2		$-\frac{1}{2}$		0		$\frac{1}{4}$	
F		F		V		V		F
V		F		F		V		V

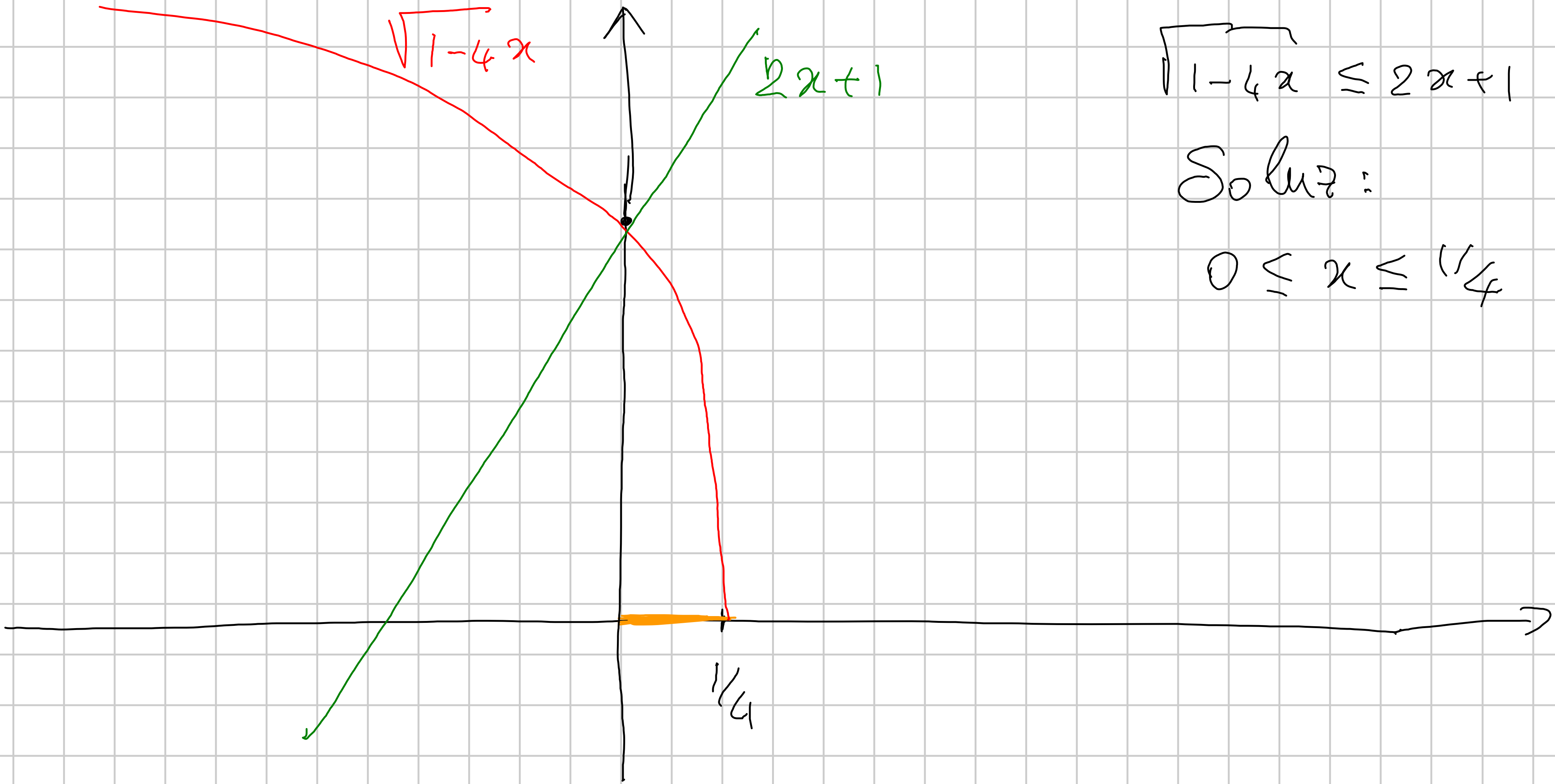
Solut: $0 \le x \le \frac{1}{4}$

graficamente :

$$\sqrt{1-4x} \leq 2x+1$$







$\sqrt{1-4x} \leq 2x+1$

Soluz:

$0 \leq x \leq 1/4$

$\boxed{8}$ $\sqrt{6x - x^2} > \frac{4}{3}x - 1$

algebricamente:

$$\begin{cases} 0 \leq x \leq 6 \\ \frac{4}{3}x - 1 < 0 \end{cases} \quad \text{oppure} \quad \begin{cases} 0 \leq x \leq 6 \\ \frac{4}{3}x - 1 \geq 0 \\ 6x - x^2 > \frac{16}{9}x^2 - \frac{8}{3}x + 1 \end{cases}$$

$$\begin{cases} 0 \leq x \leq 6 \\ x < 3/4 \end{cases} \quad \text{oppure} \quad \begin{cases} 0 \leq x \leq 6 \\ x \geq 3/4 \\ 25x^2 - 78x + 9 < 0 \end{cases}$$

$x_{1,2} = \begin{cases} 3 \\ 3/25 \end{cases}$

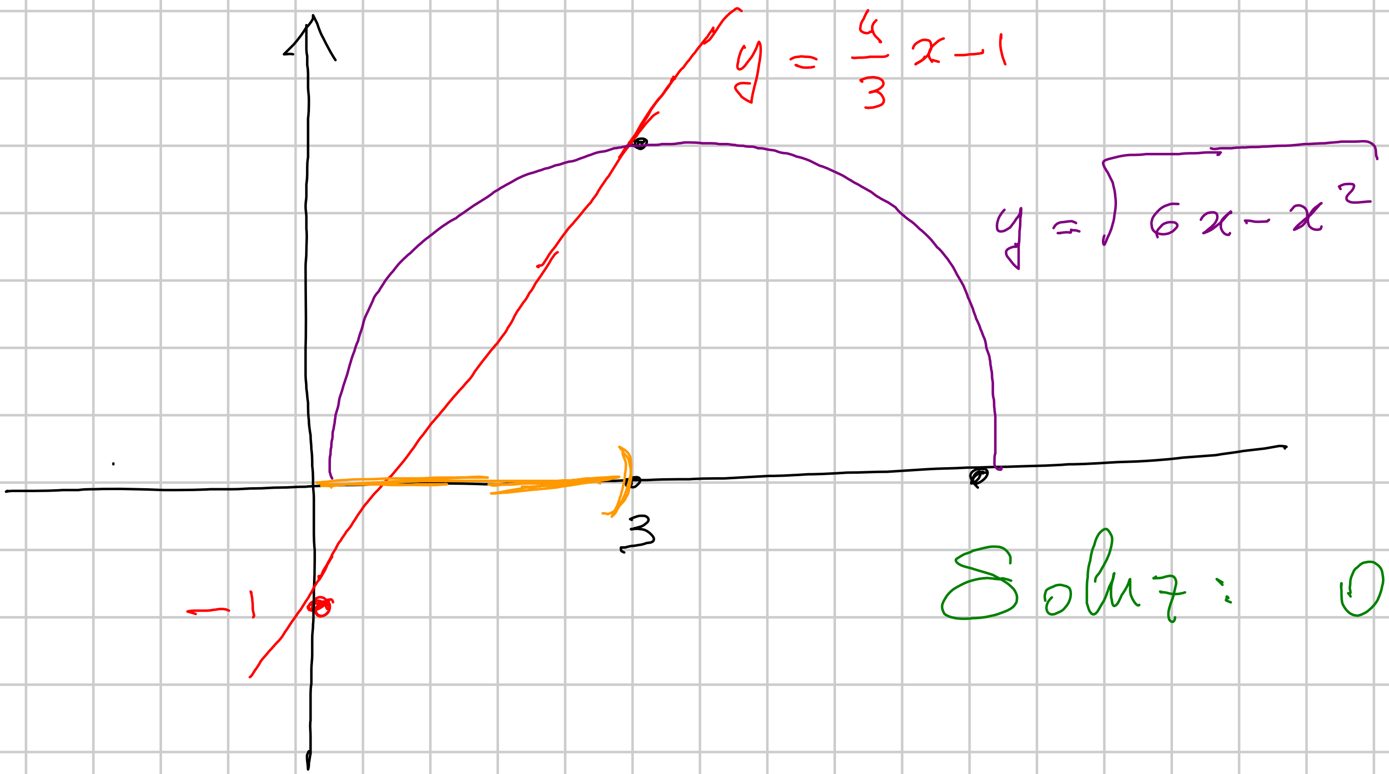
$0 \leq x < 3/4$ oppure $\begin{cases} 3/4 \leq x \leq 6 \\ 3/25 < x < 3 \end{cases}$

$0 \leq x < 3/4$ oppure $3/4 \leq x < 3$

Soluz: $0 \leq x < 3$,

Verificamento: $\sqrt{6x - x^2} > \frac{4}{3}x - 1$

$$y = \sqrt{6x - x^2} \iff \begin{cases} y \geq 0 \\ y^2 = 6x - x^2 \end{cases} \iff \begin{cases} y \geq 0 \\ x^2 + y^2 - 6x = 0 \end{cases}$$

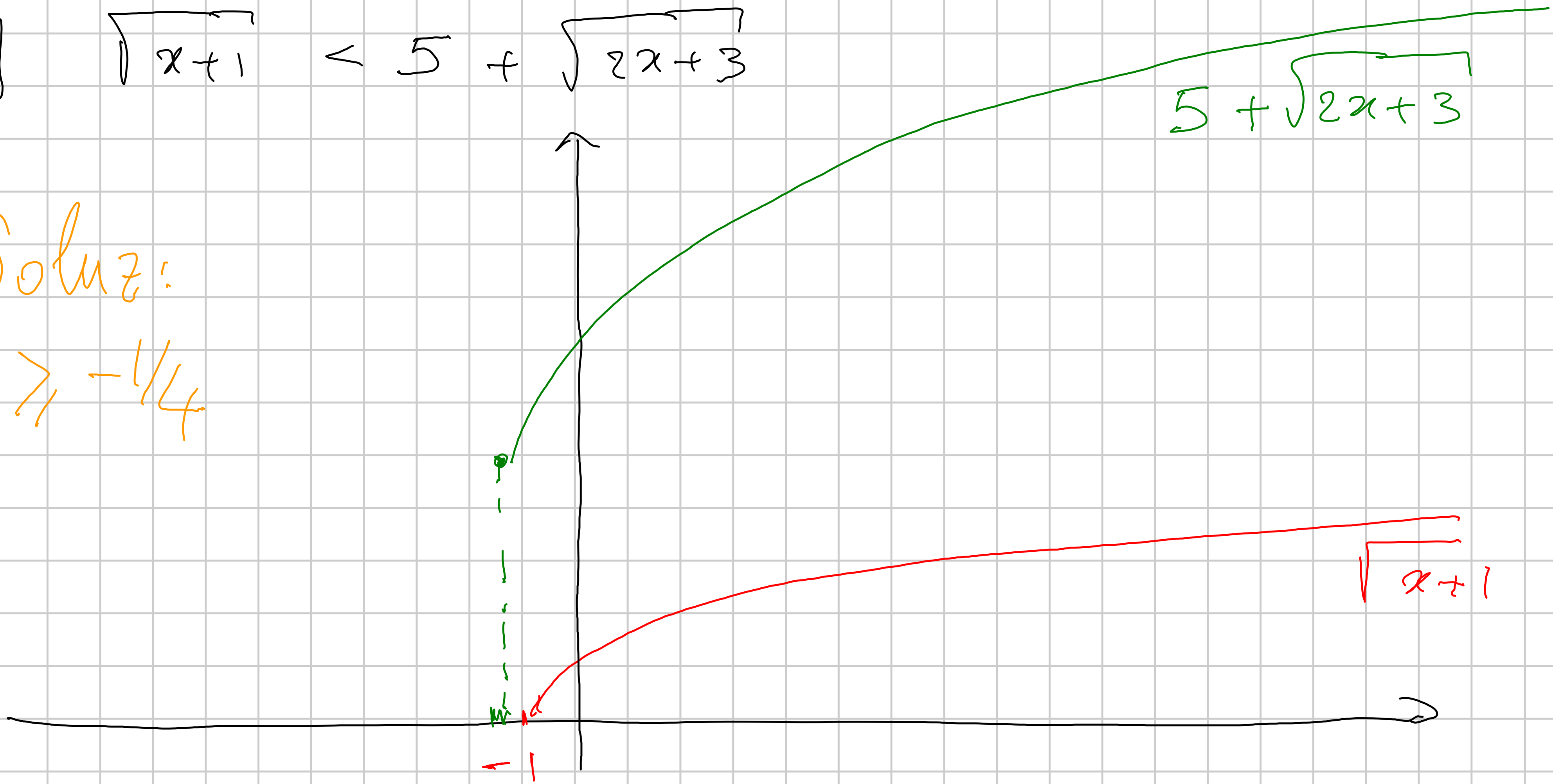


Soluz: $0 \leq x < 3$

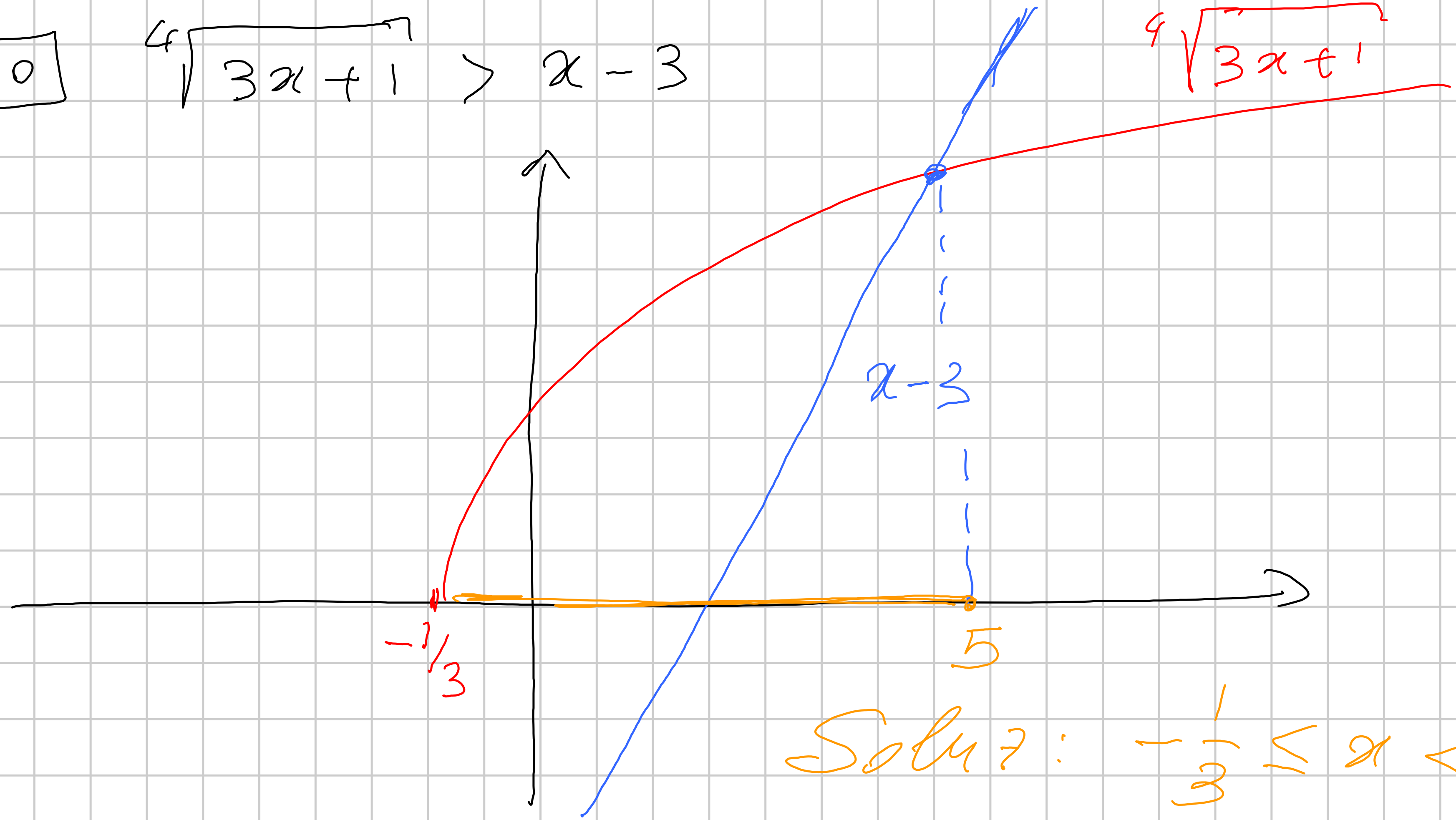
$\boxed{8}$ $\sqrt{x+1} < 5 + \sqrt{2x+3}$

Soluz:

$x \geq -1/4$



$\boxed{10}$ $\sqrt[4]{3x+1} > x-3$



Soluz: $-\frac{1}{3} \le x < 5$

Foglio exp/log

$\boxed{1}$ $\left(e^{x-1}\right)^3 \cdot e^{3-2x} = 7$

$e^{3x-3+3-2x} = 7$

$e^x = 7$

$x = \ln(7)$

$\boxed{2}$ $\left(3^{x-2}\right)^{2x-1} \cdot \left(3^{2x}\right)^{4-x} = 5$

$3^{2x^2 - x - 4x + 2 + 8x - 2x^2} = 5$

$3^{3x+2} = 5$

$3x + 2 = \log_3(5)$

$x = \frac{1}{3}\left(\log_3(5) - 2\right)$

$\boxed{3}$

$$\underbrace{100^{2x}}_{(100^x)^2} + 100^x \cdot (1-\sqrt{10}) - \sqrt{10} = 0$$

$t = 100^x$ $\qquad$ $t^2 + (1-\sqrt{10})\,t - \sqrt{10} = 0$

$$(t+1)(t-\sqrt{10}) = 0$$

$t = 100^x$ $\qquad$ $t = -1$ oppure $t = \sqrt{10}$

non accettabile

$$100^x = \sqrt{10}$$

$$x = \frac{1}{4}$$

$$\left(x = \log_{100} \sqrt{10} \right.$$

$$x = \log_{10^2} \left(10^{1/2} \right)$$

$$\left. x = \frac{1}{2} \cdot \frac{1}{2} \cdot \log_{10} (10) = \frac{1}{4} \right)$$

$\boxed{6}$ Dire quante sono le soluz. di

$$\left(a^2+1 \right)^{x^2 - x(a+\sqrt{3}) + a\sqrt{3}} = 1$$

al variare di $a \in \mathbb{R}$.

Se $a = 0$ valida $\forall x \in \mathbb{R}$ (infinite soluz)

Se $a \neq 0$ equivale a

$$x^2 - x(a+\sqrt{3}) + a\sqrt{3} = 0$$

$$(x - a)(x - \sqrt{3}) = 0$$

Se $a \neq \sqrt{3}$ 2 soluz distinte $a, \sqrt{3}$

Ze $a = \sqrt{3}$ solut. unice $\sqrt{3}$.

$\boxed{7}$ $\log_3\left(2x^2 - 5x + 2\right) = 3$

$$2x^2 - 5x + 2 = 27$$

$$2x^2 - 5x - 25 = 0$$

$$x_{1,2} = \frac{5 \pm \sqrt{25 + 200}}{4} = \frac{5 \pm 15}{4} \begin{cases} 5 \\ -5/2 \end{cases}$$

$\boxed{9}$ $$\log_2(x-1) - \underbrace{1}_{\log_2 2} = \log_2(x+5) + \underbrace{\log_{\frac{1}{2}}(x)}_{-\log_2(x)}$$

Condizione necessaria di esistenza: $x \geq 1$.

$$\log_2 \frac{(x-1)x}{2(x+5)} = 0$$

$$\frac{x^2 - x}{2(x+5)} = 1$$

calcoli: $x^2 - 3x + 10 = 0$

$$x_{1,2} = \begin{cases} 5 & \underline{\underline{\text{sì}}} \\ -2 & \underline{\underline{\text{no}}} \end{cases}$$

Soluz. unica $x = 5$