Mat. Stat. 21/11/18

Equazioni exp/log.

- $a^{f(b)} = b$: equivale a $f(x) = \log_a(b)$
  (con $b > 0$)

<u>Es</u> : $3^{5x+7} = \frac{1}{27}$ $\left(\frac{1}{27} > 0\right)$

$\iff 3x + 7 = \log_3\left(\frac{1}{27}\right)$

$\iff 3x+7=-3 \iff x=-\frac{10}{3}$

- Se x compare solo nella forma $a^{f(x)}$ (con sempre lo stesso a e stesso f) risolvo sostituendo $t=a^{f(x)}$, risolvendo rispetto a t e poi a x.

Es: $4^{x+1}-5\cdot 2^{x}+1=0$

$\iff (2^2)^{x+1}-5\cdot 2^x+1=0$

$$\Longleftrightarrow \quad 2^{2x} \cdot 4 - 5 \cdot 2^{x} + 1 = 0$$

$$\Longleftrightarrow \quad 4 \cdot \left(2^{x}\right)^{2} - 5 \cdot 2^{x} + 1 = 0$$

$$\Longleftrightarrow \quad t = 2^{x} \qquad 4t^{2} - 5t + 1 = 0$$

$$\Longleftrightarrow \quad t = 2^{x} \qquad t = 1 \ \text{ oppure } \ t = \frac{1}{4}$$

$$\Longleftrightarrow \quad 2^{x} = 1 \quad \text{ oppure } \quad 2^{x} = \frac{1}{4}$$

$\iff \quad x = 0$ oppure $x = -2$.

* $\log_a(f(x)) = b$: equivale a $f(x) = a^b$

ha senso solo per $f(x) > 0$

per una soluz. di questa ho di certo $f(x) > 0$.

Es: $\log_5(9x-1) = 3$

$\iff 9x-1 = 5^3 \iff 9x-1 = 125$

$\iff x = \frac{126}{9} = 14.$

- Se x compare solo in espressioni $\log_a(f(x))$ (stesso a, stesse f) pongo $t = \log_a(f(x))$, risolvo in t, poi in x.

Es: $3\left(\log_2(3x+1)\right)^2 - \log_2(3x+1) - 10 = 0$

$\iff t = \log_2(3x+1)\ ,\quad 3t^2 - t - 10 = 0$

$\iff t = \log_2(3x+1) \qquad t_{1,2} = \dfrac{1 \pm \sqrt{1+120}}{6} = \begin{cases} 2 \\ -\dfrac{5}{3} \end{cases}$

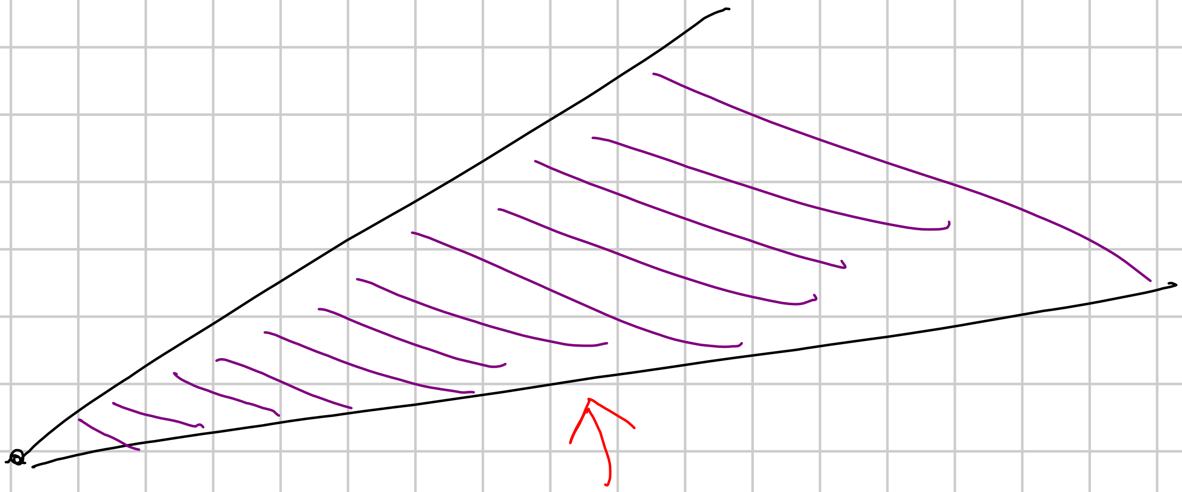
$\iff \log_2(3x+1) = 2$ oppure $\log_2(3x+1) = -\dfrac{5}{3}$

$\iff 3x+1 = 4$ oppure $3x+1 = 2^{-5/3} = \dfrac{1}{\sqrt[3]{32}}$

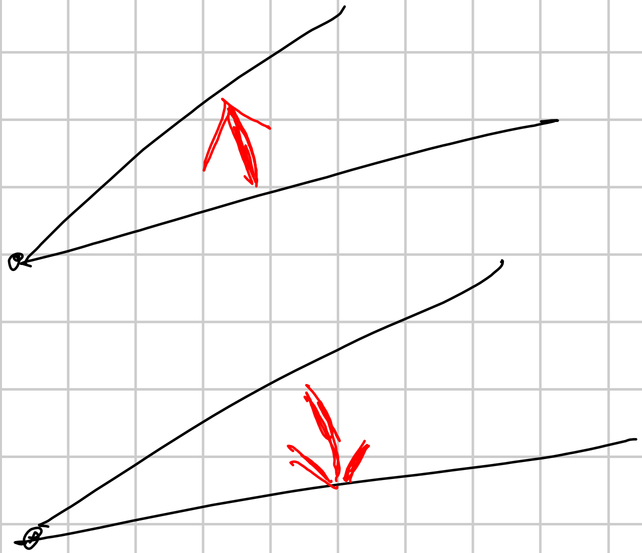
$\iff \quad x = 1 \quad$ oppure $\quad x = \frac{1}{3}\left(\frac{1}{\sqrt[3]{32}} - 1\right)$

Trigonometria

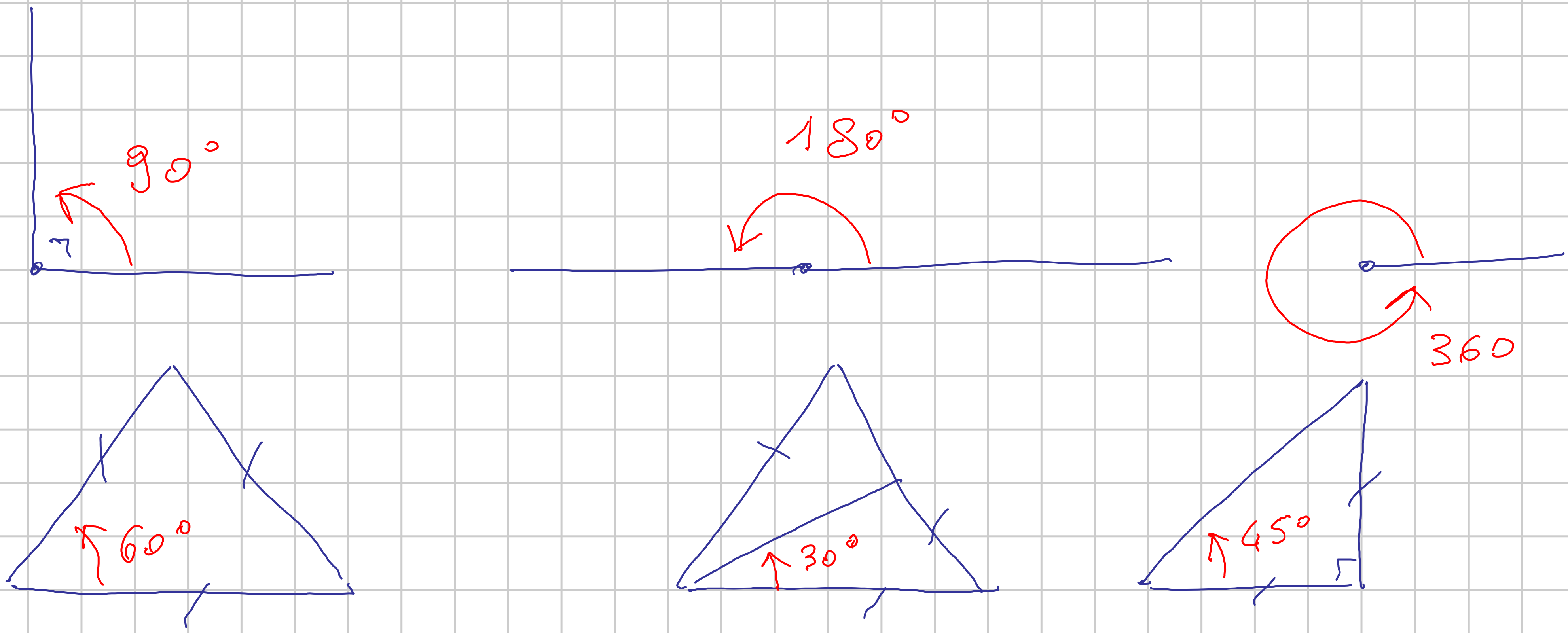
Angolo: regione di piano compresa tra due semirette con origine comune.
(con verso rotatorio orario o antiorario)



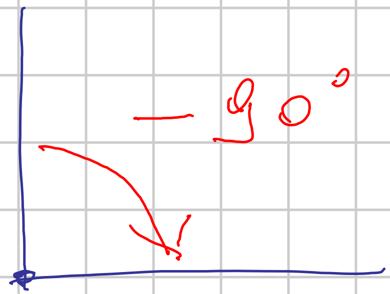
senza verso



Ampiezza angoli: misura in gradi


90°
180°
360
60°
30°
45°

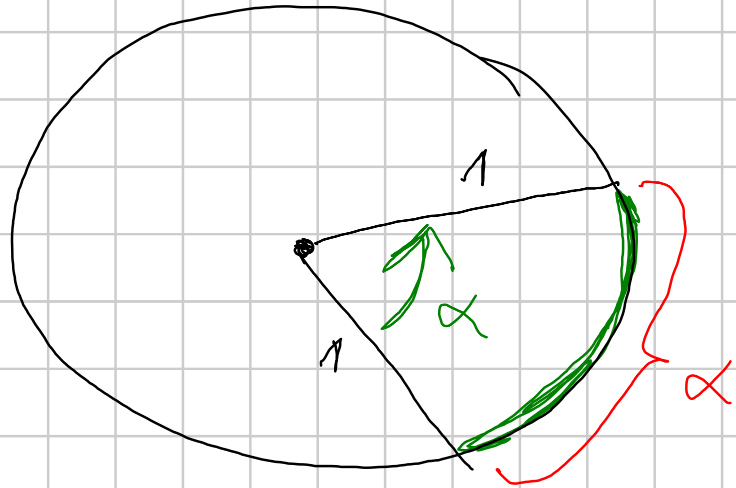
Inoltre: l'ampiezza di un angolo in verso antiorario è l'opposto dell'ampiezza dello stesso in verso orario



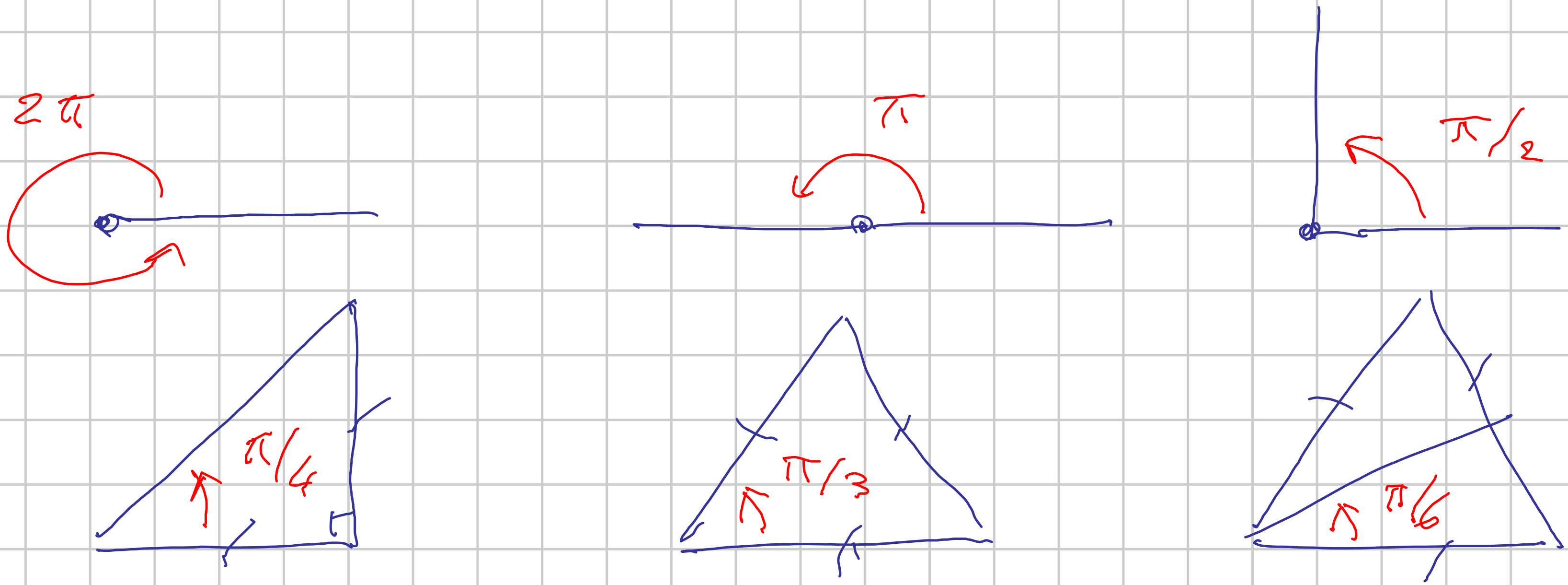
Misure in radianti:

ricordo che la circonf. di raggio 1 ha

lunghezza 2π; se considero un angolo al centro di tale circonferenza di ampiezza α la sua ampiezza in radianti la lunghezza dell'arco sotteso:

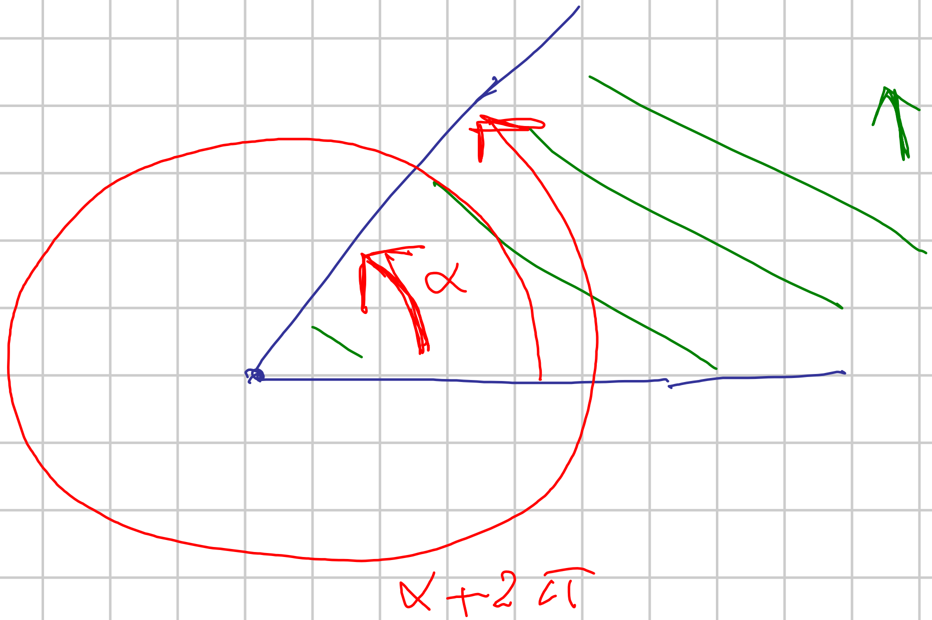


<u>Oss</u>: se un angolo misura x gradi allora misura $\frac{2\pi}{360} \cdot x$ radianti.

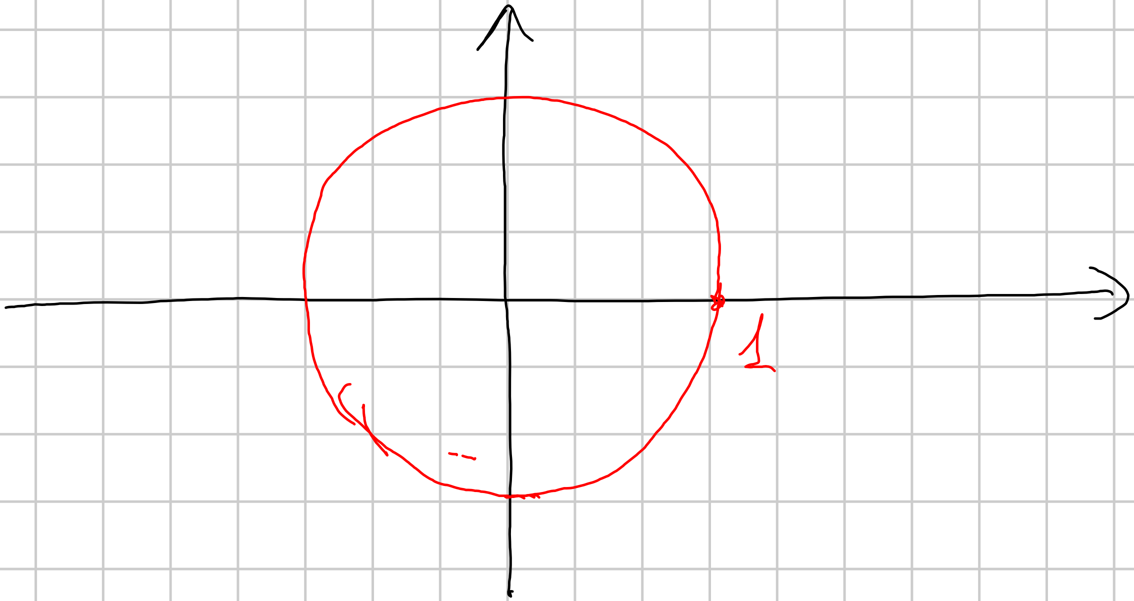


Come prima: $-\pi/4$ = "$\pi/4$ con verso orario"

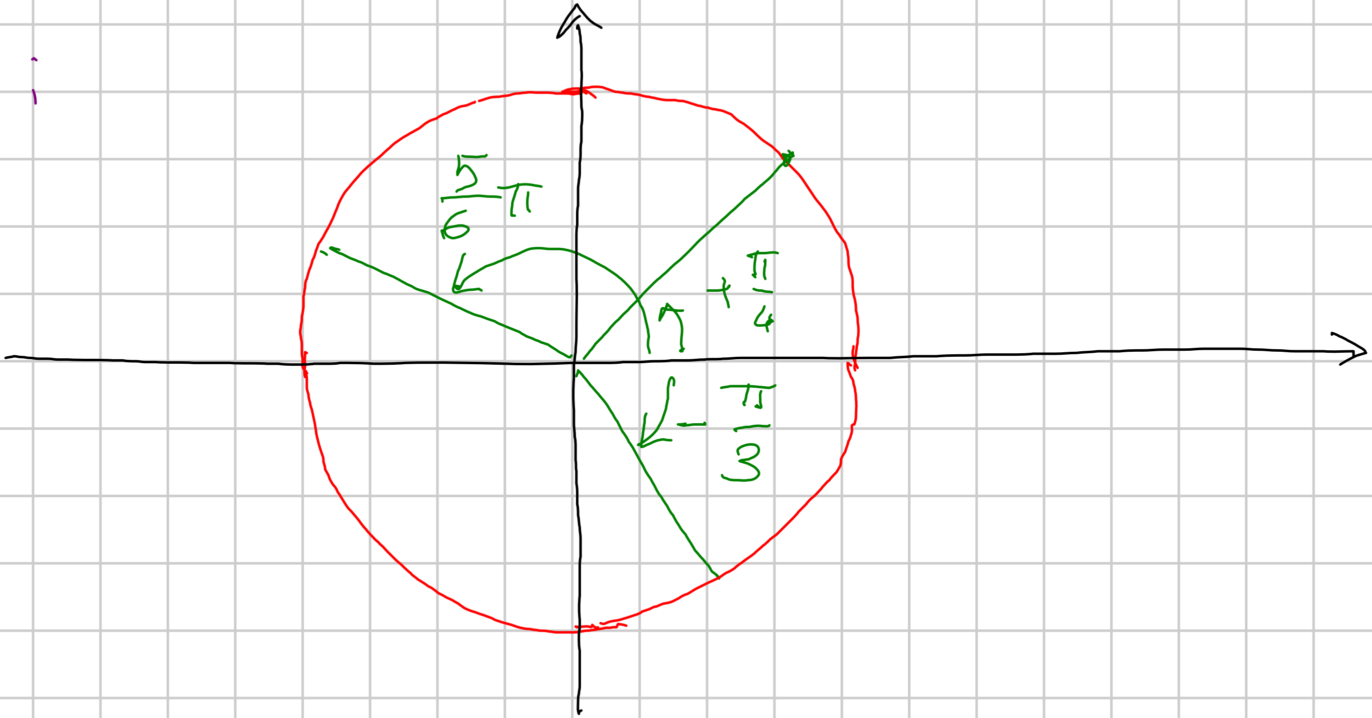
Oss: α e $\alpha + 2\pi$ sono lo stesso angolo



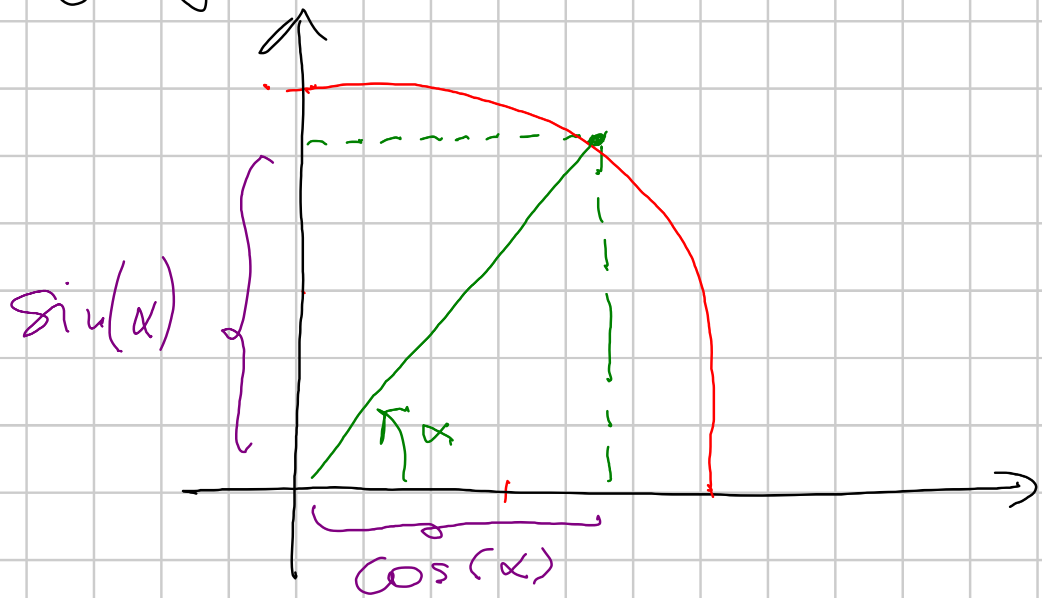
Chiamo circonferenza goniometrica la circonf. di centro O e raggio 1 nel piano cartesiano $\mathbb{R}^2$:



Su essa riportiamo le ampiezze degli angoli in radianti a partire dal semiasse positivo delle ascisse;



Def: chiamo coseno e seno dell'angolo α l'ascissa e l'ordinata del punto sulla circonf. gonio che corrisponde ad α:



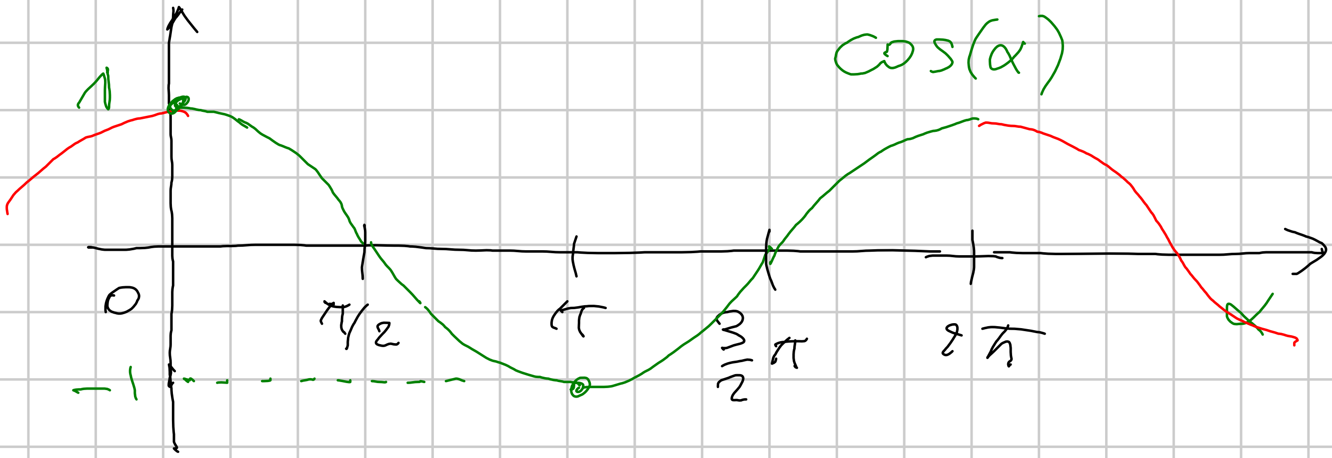
Oss: cos, sin sono periodiche di periodo 2π:

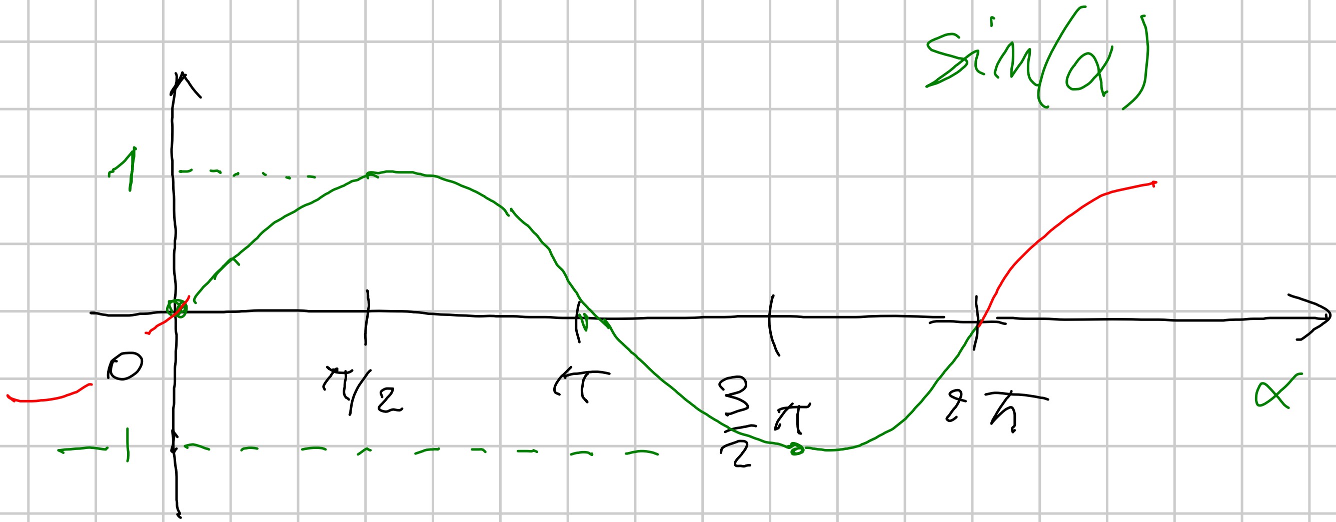
$$\cos(\alpha+2\pi) = \cos(\alpha) \qquad \cos(\alpha+2k\pi)=\cos(\alpha)$$

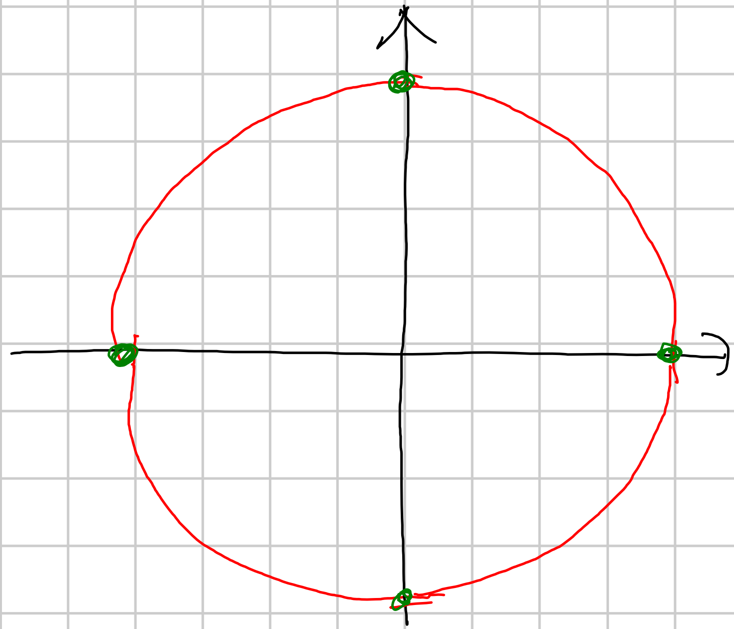
$$\sin(\alpha+2\pi) = \sin(\alpha) \qquad \sin(\alpha+2k\pi)=\sin(\alpha)$$

$$\forall k \in \mathbb{Z}$$

Conseguenze: se so il grafico su $[0, 2\pi]$ lo posso replicare $[2\pi, 4\pi]$ ... $[-2\pi, 0]$ ...


cos(α)
1
0
-1
π/2
π
3/2 π
2π
α


sin(α)
1
0
-1
π/2
π
3/2 π
2π
α



Esercizi

$f: \mathbb{R} \to \mathbb{R} \quad f(x) = \dfrac{x-3}{x^4+1}$ iniettiva?

NO: $f(0) = -3$; risolviamo $f(x) = -3$:

$\dfrac{x-3}{x^4+1} = -3 \qquad x - 3 = -3x^4 - 3$

$3x^4 + x = 0 \qquad x \cdot (3x^3 + 1) = 0$

Soluz: $x = 0$ $\quad x = -\dfrac{1}{\sqrt[3]{3}}$.

$\boxed{\text{Polinomi}}$ $\boxed{16}$ $x^4 - 2x^3 - 6x^2 + 8x + 8 \geq 0$.

Cerco radici dell'equaz. associata; se sono raz

sono $\pm 1, \pm 2, \pm 4, \pm 8$; provo:

$\pm 1 \quad 1 \mp 2 - 6 \pm 8 + 8 \quad$ No

$\pm 2 \quad 16 \mp 16 - 24 \pm 16 + 8 \quad$ Sì per ± 2

$$
\begin{array}{c|cccc|c}
 & 1 & -2 & -6 & 8 & 8 \\
2 & & 2 & 0 & -12 & -8 \\
\hline
 & 1 & 0 & -6 & -4 & / \\
-2 & & -2 & 4 & 4 & \\
\hline
 & 1 & -2 & -2 & / &
\end{array}
$$

Soluz:

$x_1 = -2$ $\quad x_2 = 2$ $\quad$ e quelle di $x^2 - 2x - 2 = 0$

$x_{3,4} = 1 \pm \sqrt{3}$

Disp:

$$(x-2)(x+2)(x-(1+\sqrt{3}))(x-(1-\sqrt{3}))\geq 0$$

$-2 \qquad 1-\sqrt{3} \qquad 2 \qquad 1+\sqrt{3}$

$-$	$+$	$+$	$+$	$+$
$-$	$-$	$+$	$+$	$+$
$-$	$-$	$-$	$+$	$+$
$-$	$-$	$-$	$-$	$+$
$+$	$-$	$+$	$-$	$+$

Soluz $x\leq -2$ oppure $1-\sqrt{3}\leq x\leq 2$ oppure $x\geq 1+\sqrt{3}$.

$\boxed{19}$ $$\frac{1-x}{x^2+2x}+\frac{2}{x}\geq\frac{3}{x+3}$$

$$\frac{(1-x)(x+3)+2(x+2)(x+3)-3(x^2+2x)}{x(x+2)(x+3)}\geq 0$$

$-x^2+x+3$
$-3x$
$+2x^2+10x+12$
$-3x^2-6x$

$$\frac{-2x^2-2x+15}{x(x+2)(x+3)}\geq 0$$

$$\frac{2x^2+2x-15}{x(x+2)(x+3)} \le 0$$

$$x_{1,2} = \frac{-1 \pm \sqrt{31}}{2} \begin{cases} -2.28 \\ 1.39 \end{cases}$$

	-3		$\frac{-1-\sqrt{31}}{2}$		-2		0		$\frac{-1+\sqrt{31}}{2}$	
NUM	+		+		−		−		−	+
DEN	−		+		+		−		+	+
	−		+		−		+		−	+

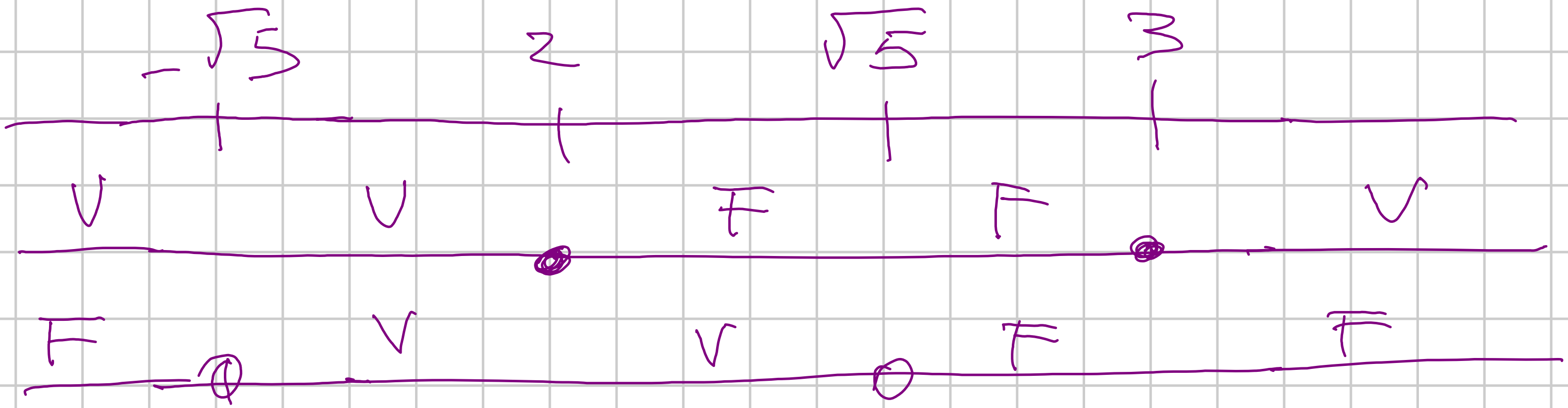
Soluz: $x < -3$ o $\frac{-1-\sqrt{31}}{2} \le x < -2$ o $0 < x \le \frac{-1+\sqrt{31}}{2}$

$\boxed{21}$

$$\begin{cases} x^2 - 2x + 8 \geq 3x + 2 \\ 3x^2 - 4x + 7 < 2x(x-1) - 2(x-6) \end{cases}$$

$$\begin{cases} x^2 - 5x + 6 \geq 0 \\ x^2 < 5 \end{cases} \qquad \begin{cases} (x-3)(x-2) \geq 0 \\ x^2 < 5 \end{cases}$$

$$\begin{cases} x \leq 2 \quad \text{oppure} \quad x \geq 3 \\ -\sqrt{5} < x < \sqrt{5} \end{cases}$$



Soluz: $-\sqrt{5} < x \leq 2$

IRRAZIONALI

$\boxed{1}$ $\sqrt{x+8} = 2x+1$

$$\begin{cases} (x+8 \geq 0) \\ 2x+1 \geq 0 \\ x+8 = (2x+1)^2 \end{cases} \qquad \begin{cases} x \geq -\frac{1}{2} \\ 4x^2+3x-7=0 \end{cases}$$

$$x_{1,2} = \frac{-3 \pm \sqrt{9+112}}{8} = \begin{cases} 1 & \text{SI} \\ -\frac{7}{4} & \text{NO} \end{cases}$$

Soluz: $x = 1$

$\boxed{2}$ $\sqrt{-x^2+4x-3} = x-2$

$$\begin{cases} x \geq 2 \\ -x^2+4x-3 = x^2-4x+4 \end{cases} \qquad \begin{cases} x \geq 2 \\ 2x^2-8x+7=0 \end{cases}$$

$$x_{1,2} = \frac{4 \pm \sqrt{16-14}}{2} = 2 \pm \frac{1}{\sqrt{2}}$$

Soluz: $x = 2 + \frac{1}{\sqrt{2}}$

$\boxed{14}$ $$\sqrt[3]{2\sqrt{2}\,x^2+7x+3\sqrt{2}} = x+\sqrt{2}$$

$$2\sqrt{2}\,x^2+7x+3\sqrt{2} = x^3+3\sqrt{2}\,x^2+6x+2\sqrt{2}$$

$$x^3+\sqrt{2}\,x^2-x-\sqrt{2}=0$$

$$(x+\sqrt{2})(x^2-1)=0$$

soluz $x=-\sqrt{2}$ o $x=\pm 1.$

$\boxed{6}$ $\sqrt{x+1} = kx - 1$

<u>soluz. algebrico</u>

$k=0$: impossibile

$k>0$: $\begin{cases} kx-1 \geq 0 \\ x+1 = k^2x^2 - 2kx + 1 \end{cases}$

$\begin{cases} x \geq \frac{1}{k} \\ k^2x^2 - (2k+1)x = 0 \end{cases}$

$$\begin{cases} x \geq 1/k \\ \underbrace{x = 0}_{\text{non accettabile}} \quad \text{oppure} \quad x = \dfrac{2k+1}{k^2} \end{cases}$$

accettabile se

$$\frac{2k+1}{k^2} \geq \frac{1}{k}$$

$$2k+1 \geq k$$

$$k \geq -1$$

si (caso $k > 0$).

$\underline{k<0}$:

$$\begin{cases} x \leq \frac{1}{k} \\ \underbrace{x=0}_{\text{non accettabile}} \quad o \quad x = \frac{2k+1}{k^2} \end{cases}$$

accettabile se

$$\frac{2k+1}{k^2} \leq \frac{1}{k}$$

$$2k+1 \leq k$$

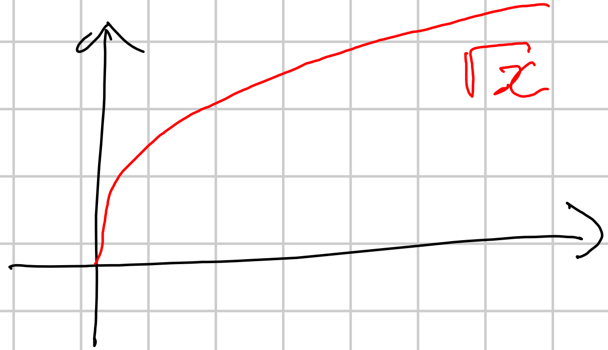
$$k \leq -1$$

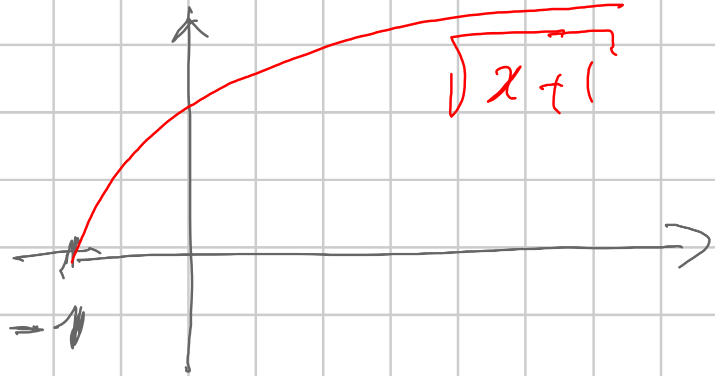
Conclusione: soluz. unica $x = \frac{2k+1}{k^2}$ per $k \leq -1$ e $k > 0$

nessuna soluz. altrimenti.

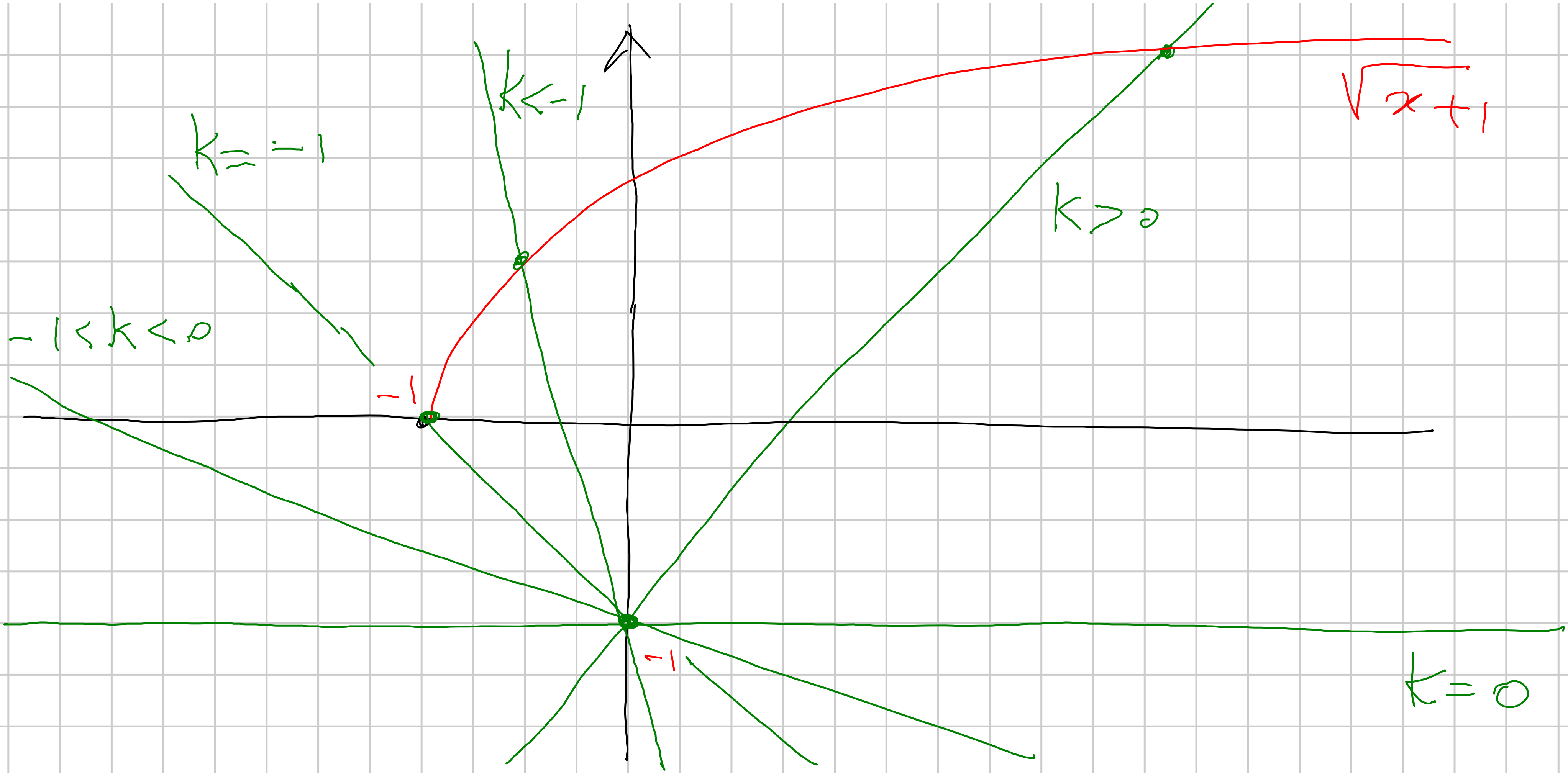
Soluz. grafica: $\sqrt{x+1} = kx - 1$

Devo trovare i pti di intersez. tra i grafici di $y = \sqrt{x+1}$ e $y = kx - 1$.





$y = kx - 1$ è la retta di coeff. ang. k e ordinata all'origine -1.



$k = -1$
$k < -1$
$\sqrt{x+1}$
$k > 0$
$-1 < k < 0$
-1
-1
$k = 0$