Matematica e statistica

Limiti e derivate

Es 2 : studiare insieme di def, limiti agli estremi, intersezione con assi, crescenza e decrescenza per $f(x)$.

(a) $f(x) = \dfrac{x+2}{3-x}$

Definita per $x \neq 3$, cioè su $(-\infty, 3) \cup (3, +\infty)$.

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x}{-x} = -1$$

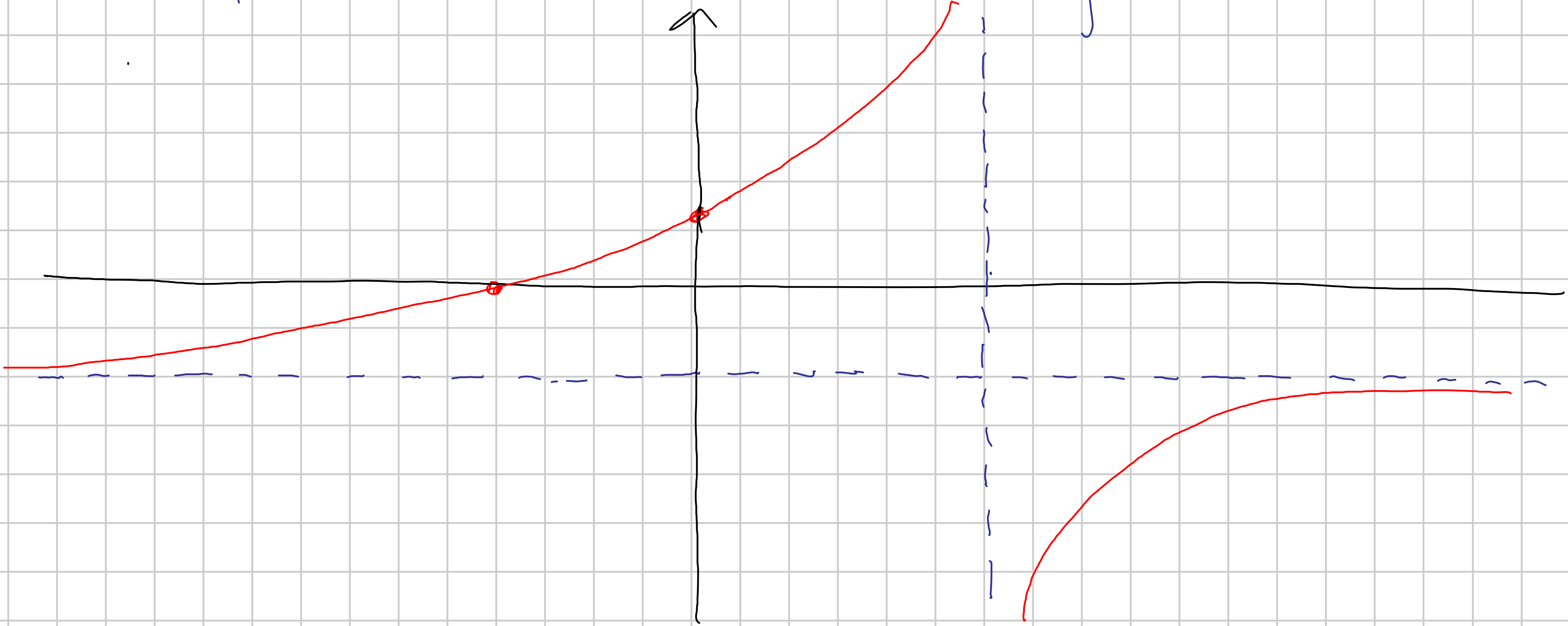
$$\lim_{x \to 3^-} f(x) = \frac{5}{0^+} = +\infty$$

$$\lim_{x \to 3^+} f(x) = \frac{5}{0^-} = -\infty$$

$f(0) = \frac{2}{3}$ $\qquad$ $f(x) = 0$ per $x = -2$

$$f'(x) = \frac{1 \cdot (3-x) - (-1)(x+2)}{(3-x)^2} = \frac{3 - x + x - 2}{(3-x)^2} = \frac{1}{(3-x)^2}$$

$\Rightarrow f'(x) > 0 \quad \forall x \Rightarrow f$ strictement croissante.



(b) $f(x) = \dfrac{12x - 17}{1 + 2x^2}$

definita su $\mathbb{R}$; $\lim\limits_{x \to \pm\infty} f(x) = \lim\limits_{x \to \pm\infty} \dfrac{12x}{2x^2} = 0^{\pm}$

$f(0) = -7$ $\quad f(x) = 0$ per $x = 17/12$

$$f'(x) = \frac{12(1+2x^2) - 4x(12x-17)}{(1+2x^2)^2} = \frac{12+24x^2-48x^2+68x}{(\quad)^2}$$

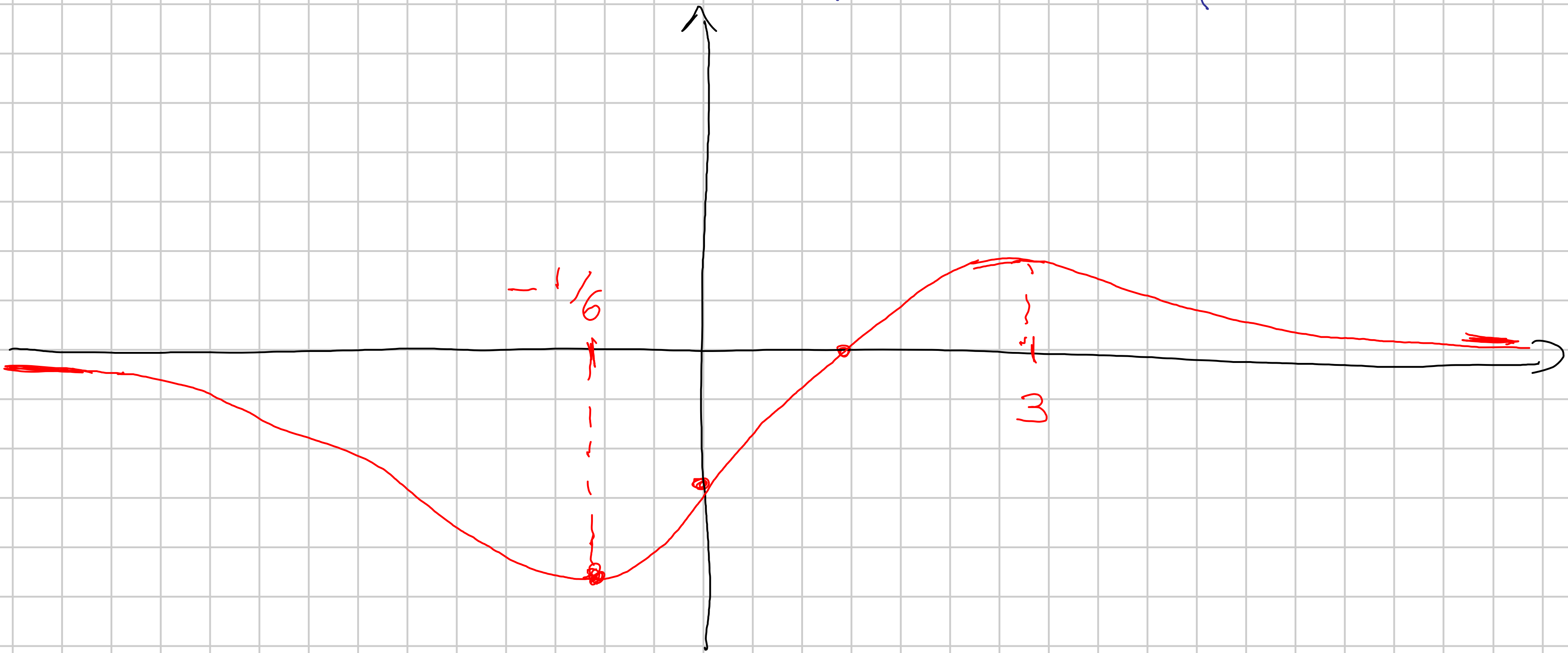
$$= -4 \cdot \frac{6x^2-17x-3}{(\quad)^2}$$

$f'(x) = 0$ per $x_{1,2} = \dfrac{17 \pm \sqrt{289 + 72}}{12} =$

$$= \frac{17 \pm \sqrt{361}}{12} = \frac{17 \pm 19}{12} = \begin{cases} 3 \\ -\frac{1}{6} \end{cases}$$

Dunque in $(-\infty, -1/6)$	$f'(x) < 0$	f decrescente
$x = -1/6$		pto di min rel
$-1/6 < x < 3$	$f'(x) > 0$	f crescente
$x = 3$		pto di max rel.

$(3, +\infty)$ $\quad f'(x) < 0 \quad$ f decrescente



(c) $f(x) = \dfrac{1-3x-x^2}{x-2}$

Definita su $(-\infty, 2) \cup (2, +\infty)$.

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{-x^2}{x} = \lim_{x \to -\infty} (-x) = +\infty$$

$$\lim_{x \to 2^-} f(x) = \frac{1-6-4}{0^-} = +\infty$$

$$\lim_{x \to 2^+} f(x) = -\infty \qquad \lim_{x \to +\infty} f(x) = -\infty$$

$$f(0) = -\frac{1}{2} \qquad f(x) = 0 \quad \text{per} \quad x^2 + 3x - 1 = 0 \quad \sim -3.2$$

$$x_{1,2} = \frac{-3 \pm \sqrt{13}}{2} = \begin{cases} \sim -3.2 \\ \sim 0.1 \end{cases}$$

$$f(x) = \frac{1 - 3x - x^2}{x - 2}$$

$$f'(x) = \frac{(-3 - 2x)(x-2) - 1 \cdot (1 - 3x - x^2)}{(x-2)^2} =$$

$$= \frac{-3x+6-2x^2+4x-1+3x+x^2}{(x-2)^2}$$

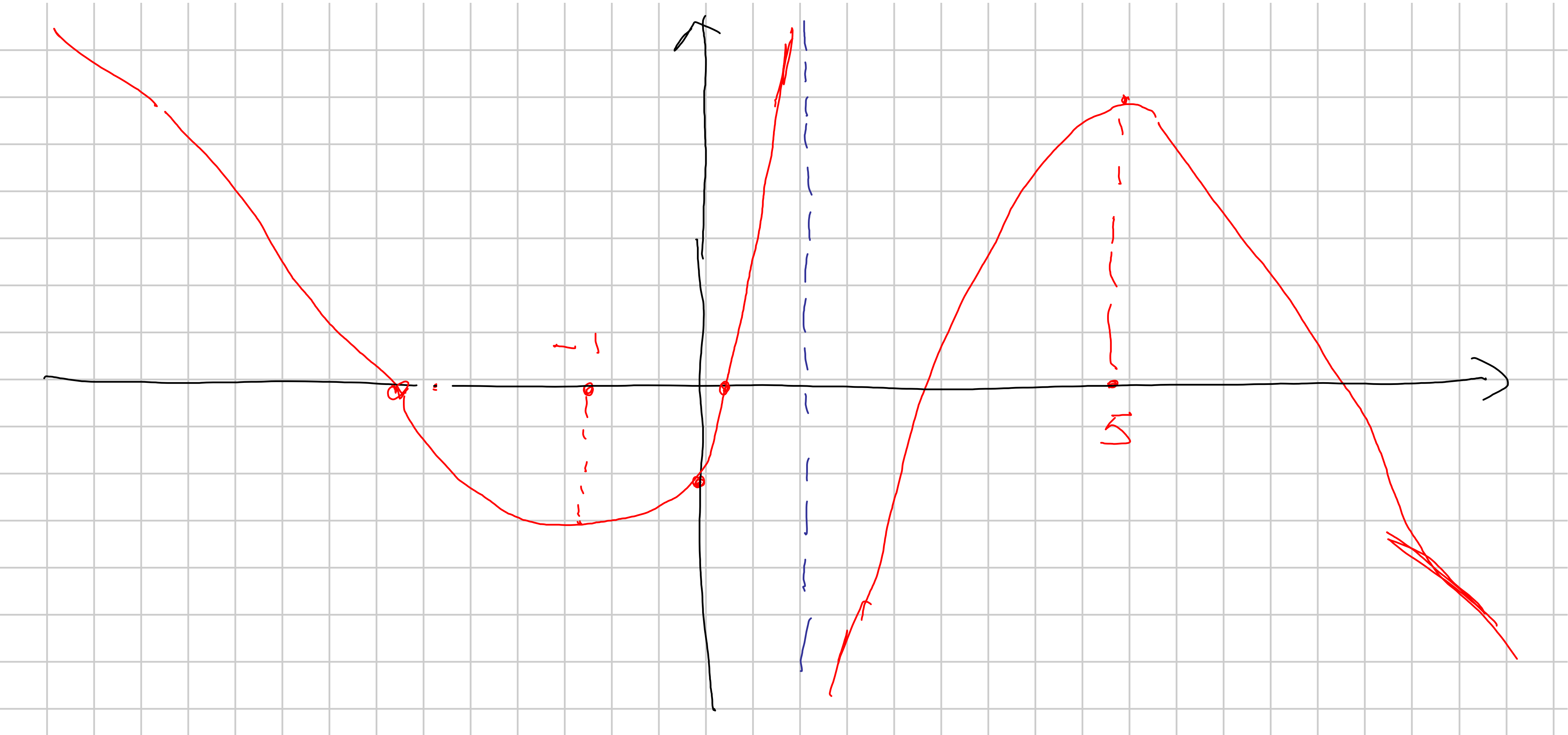
$$= -\frac{x^2-4x-5}{(x-2)^2} = -\frac{(x-5)(x+1)}{(x-2)}$$

$(-\infty, -1)$ $\quad f' < 0 \quad f$ decr

$x = -1$ $\quad$ min rel $\quad f(-1) = -1$

$(-1, 2)$ $\quad f' > 0 \quad f$ cresc

$(2, 5)$	$f' > 0$	f cresc
$x = 5$	max rel	$f(5) = -13$
$(5, +\infty)$	$f' < 0$	f decr



-1
5

(d) $f(x) = \dfrac{5x - 13}{2x^2 - 3x - 5}$

Zeri denominatore: $\dfrac{3 \pm \sqrt{9+40}}{4} = \dfrac{3 \pm 7}{4} = \begin{cases} 5/2 \\ -1 \end{cases}$

f def su $(-\infty, -1) \cup (-1, 5/2) \cup (5/2, +\infty)$

$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \dfrac{5x}{2x^2} = 0^-$

$$\lim_{x \to +\infty} f(x) = 0^+$$

$$\lim_{x \to (-1)^-} f(x) = \lim_{x \to (-1)^-} \frac{5x-13}{(x+1)(2x-5)} = \frac{-5-13}{0^- \cdot (-2-7)}$$

$$= -\infty$$

$$\lim_{x \to (-1)^+} f(x) = +\infty$$

$$\lim_{x \to (5/2)^-} f(x) = \frac{25/2 - 13}{(5/2+1) \cdot 0^-} = +\infty$$

$$\lim_{x \to (5/2)^+} f(x) = -\infty$$

$f(0) = 13/5$ $\quad$ $f(x) = 0$ pour $x = 13/5$

$$f(x) = \frac{5x - 13}{2x^2 - 3x - 5}$$

$$f'(x) = \frac{5 \cdot (2x^2 - 3x - 5) - (4x - 3)(5x - 13)}{(2x^2 - 3x - 5)^2}$$

$$
\begin{array}{rrr}
10x^2 & -15x & -25 \\
-20x^2 & +52x & -39 \\
 & +15x &
\end{array}
$$

$$= -\left(10x^2 - 52x + 64\right)$$

$$= -2\left(5x^2 - 26x + 32\right)$$

$$x_{1,2} = \frac{13 \pm \sqrt{169 - 160}}{5} = \frac{13 \pm 3}{5} = \begin{cases} \frac{16}{5} \\ 2 \end{cases}$$

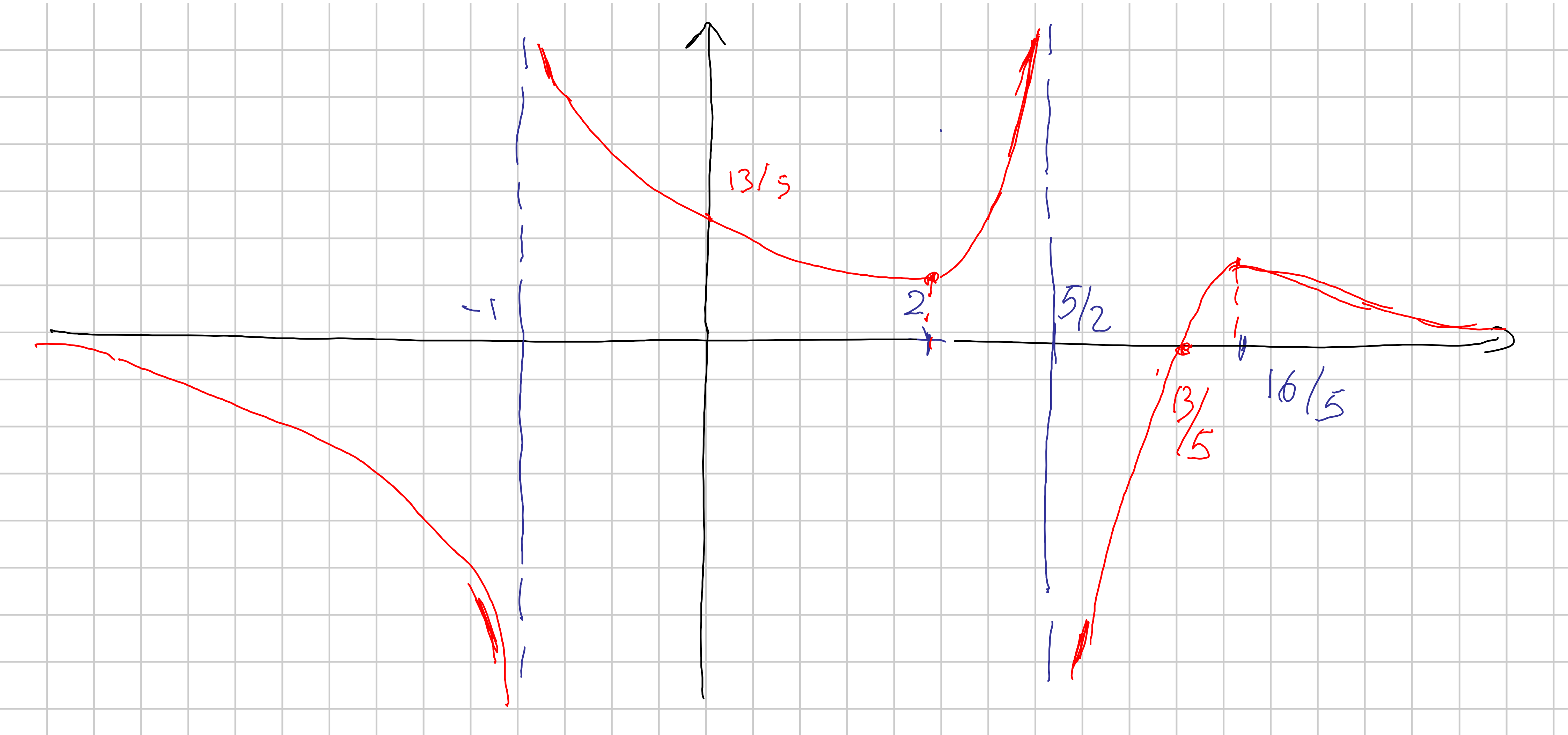
$$f'(x) = (-2) \cdot \frac{(x-2)(5x-16)}{(2x^2-3x-5)^2}$$

$$-1 < 2 < \frac{5}{2} < \frac{16}{5}$$

$x < -1$ $\quad f' < 0 \quad$ dec

$-1 < x < 2$ $\quad f' < 0 \quad$ dec

$x = 2$	$f' = 0$	min rel $f(2) = 3/7$
$2 < x < 5/2$	$f' > 0$	cresce
$5/2 < x < 16/5$	$f' > 0$	cresce
$x = 16/5$	$f' = 0$	max rel $f(16/5) \sim \frac{3}{5}$
$x > 16/5$	$f' < 0$	decresce.


-1
13/5
2
5/2
13/5
16/5

(e) $f(x) = \dfrac{x^2 + x - 15}{2x^2 + x - 15}$

Zeri denominatore: $\dfrac{-1 \pm \sqrt{1+120}}{4} = \dfrac{-1 \pm 11}{4} = \begin{cases} 5/2 \\ -3 \end{cases}$

$$f(x) = \frac{x^2 + x - 15}{(x+3)(2x-5)}$$

f def su $(-\infty, -3) \cup (-3, 5/2) \cup (5/2, +\infty)$.

$$f(x) = \frac{x^2 + x - 15}{(x+3)(2x-5)}$$

$$\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{x^2}{2x^2} = \frac{1}{2}$$

$$\lim_{x \to (-3)^-} f(x) = \frac{9 - 3 - 15}{0^- \cdot (-6-5)} = -\infty$$

$$\lim_{x \to (-3)^+} f(x) = +\infty$$

$$\lim_{x \to (5/2)^-} f(x) = \frac{25/4 + 5/2 - 15}{(5/2 + 3) \cdot 0^-} = \frac{\frac{25 + 10 - 60}{4}}{(5/2 + 3) \cdot 0^-} =$$

$$\lim_{x \to (5/2)^+} f(x) = -\infty \qquad = +\infty$$

$$f(x) = \frac{x^2 + x - 15}{2x^2 + x - 15} \qquad f(0) = 1$$

$$f(x) = 0 \quad \text{pn} \quad x = \frac{-1 \pm \sqrt{61}}{2} = \begin{cases} \sim +3.5 \\ \sim -4.5 \end{cases}$$

$$f'(x) = \frac{(2x+1)(2x^2+x-15) - (4x+1)(x^2+x-15)}{(\qquad)^2}$$

$$\cancel{4x^3} + \cancel{2x^2} - 30x$$
$$+ \cancel{7x^2} \cancel{+x} \cancel{-15}$$
$$\cancel{-4x^3} \cancel{-4x^2} + 60x$$
$$-x^2 \cancel{-x} \cancel{+15} = -x^2 + 30x$$
$$= x(30 - x)$$

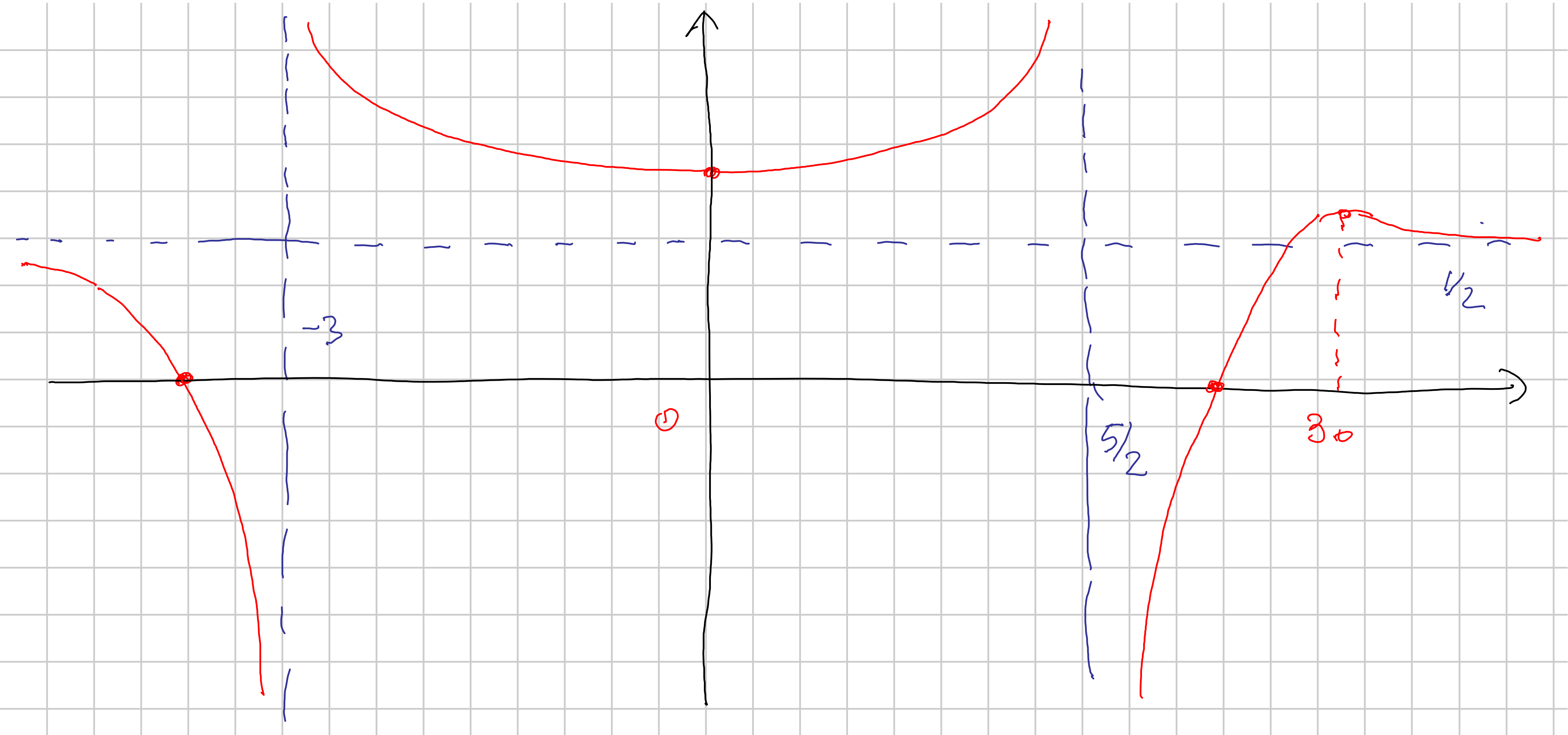
esclusi -3 e $5/2$ ho

$f' < 0$ per $x < 0$ e $x > 30 \Rightarrow f$ decr

$f' > 0$ per $0 < x < 30 \Rightarrow f$ cresc

$x = 0$ min rel $f(0) = 1$

$x = 30$ max rel $f(30) = \frac{915}{1815} = \frac{1}{2} + \text{poco}$


-3
0
5/2
3
1/2