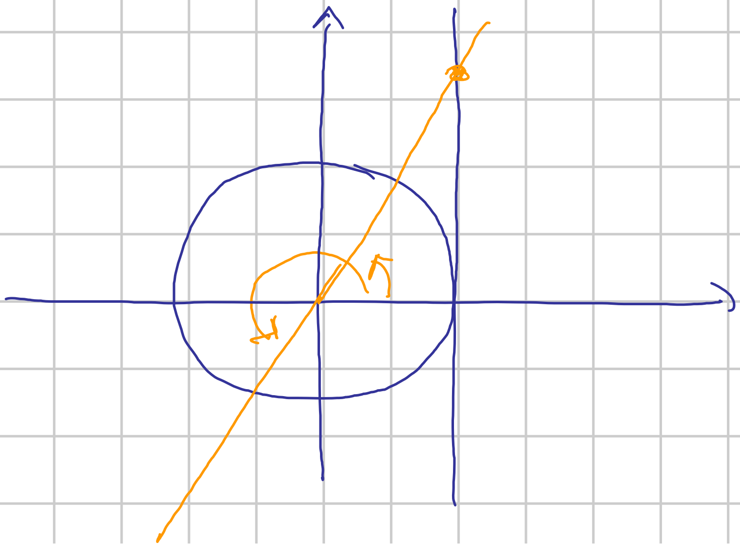
Matematica e statistica

Trigonometria

$\boxed{8}$ $\tan\left(\frac{x}{2}\right) = \tan\left(x - \frac{\pi}{2}\right)$



$f(g(x)) = f(h(x))$

$\cancel{\Rightarrow}$ $g(x) = h(x)$

NO

$\frac{x}{2} \neq \frac{\pi}{2} + k\pi$ $\quad x \neq \pi + 2k\pi$

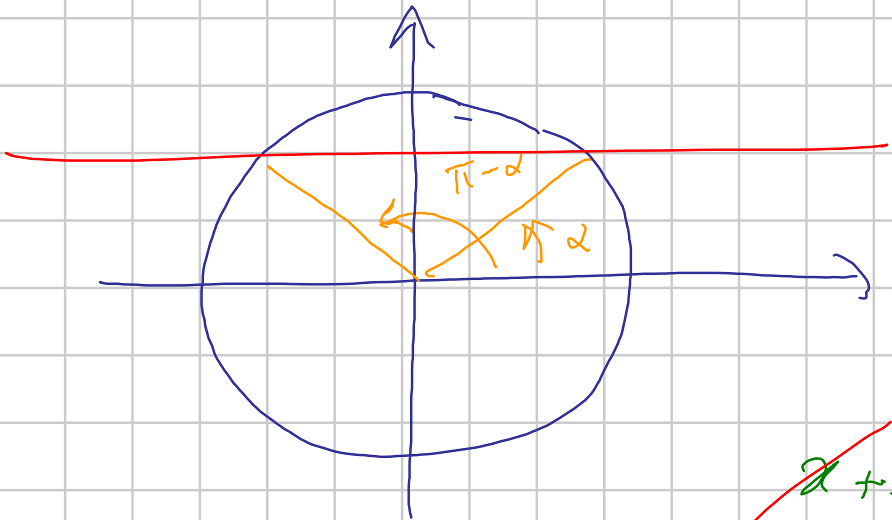
$\frac{x}{2} = x - \frac{\pi}{2} + k\pi \qquad k \in \mathbb{Z}$

$\frac{x}{2} = \frac{\pi}{2} + k\pi \qquad x = \pi + 2k\pi$

non accettabile

$\Longrightarrow$ impossibile.

$\boxed{10}$ $\sin\left(x + \frac{5}{12}\pi\right) = \sin\left(\frac{31}{12}\pi - x\right)$



$x + \frac{5}{12}\pi = \frac{31}{12}\pi - x + 2k\pi$ ...

oppure

$x + \frac{5}{12}\pi = \pi - \left(\frac{31}{12}\pi - x\right) + 2k\pi$

$\cancel{x} + \frac{5}{12}\pi = -\frac{19}{12}\pi + \cancel{x} + 2k\pi$

$2\pi = 2k\pi$ $\quad$ sì

identità $\quad \sin\left(x + \frac{5}{12}\pi\right) = \sin\left(\frac{31}{12}\pi - x\right)$

vale $\forall x \in \mathbb{R}$.

$\boxed{12}$ $\quad (\sqrt{3}+1)\sin(x) + (\sqrt{3}-1)\cos(x) = \sqrt{3}+1$

Eq. lin. in $\sin(x), \cos(x)$.

Pongo $t = \operatorname{tg}\left(\frac{x}{2}\right)$ e sostituisco $\sin(x) = \frac{2t}{1+t^2}$, $\cos(x) = \frac{1-t^2}{1+t^2}$

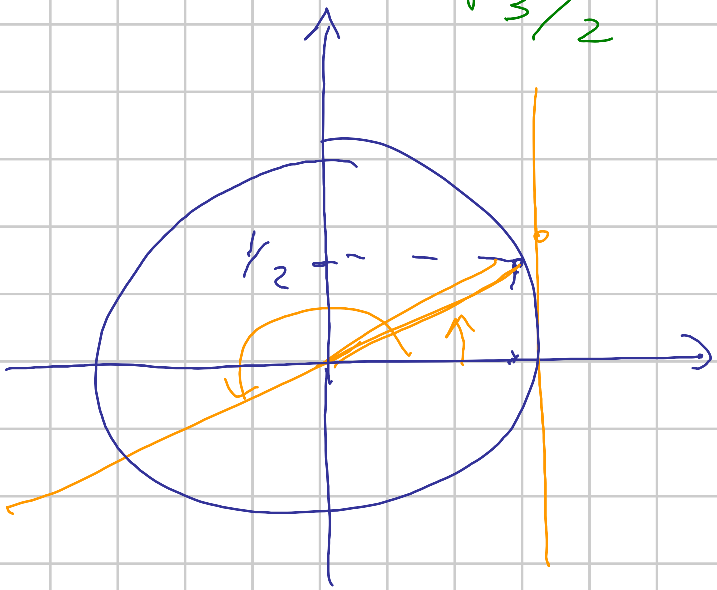
$$(\sqrt{3}+1)\,2t + (\sqrt{3}-1)(1-t^2) = (\sqrt{3}+1)(1+t^2)$$

$$2\sqrt{3}\,t^2 - 2(1+\sqrt{3})\,t + 2 = 0$$

$$t_{1,2} = \frac{1+\sqrt{3} \pm \sqrt{(1+2\sqrt{3}+3) - 4\sqrt{3}}}{2\sqrt{3}} =$$

$$= \frac{(1+\sqrt{3}) \pm (1-\sqrt{3})}{2\sqrt{3}} \left\langle \begin{array}{l} \dfrac{1}{\sqrt{3}} \\[2ex] 1 \end{array} \right.$$

$$1/\sqrt{3} = \frac{1/2}{\sqrt{3}/2} = \frac{\sin}{\cos}$$



$$\tan\left(\frac{x}{2}\right) = \frac{1}{\sqrt{3}}$$

$$\longrightarrow \frac{x}{2} = \frac{\pi}{6} + k\pi$$

$$\boxed{x = \frac{\pi}{3} + 2k\pi}$$

oppure

$$\tan\left(\frac{x}{2}\right) = 1$$

$$\frac{x}{2} = \frac{\pi}{4} + k\pi$$

$$\boxed{x = \frac{\pi}{2} + 2k\pi}$$

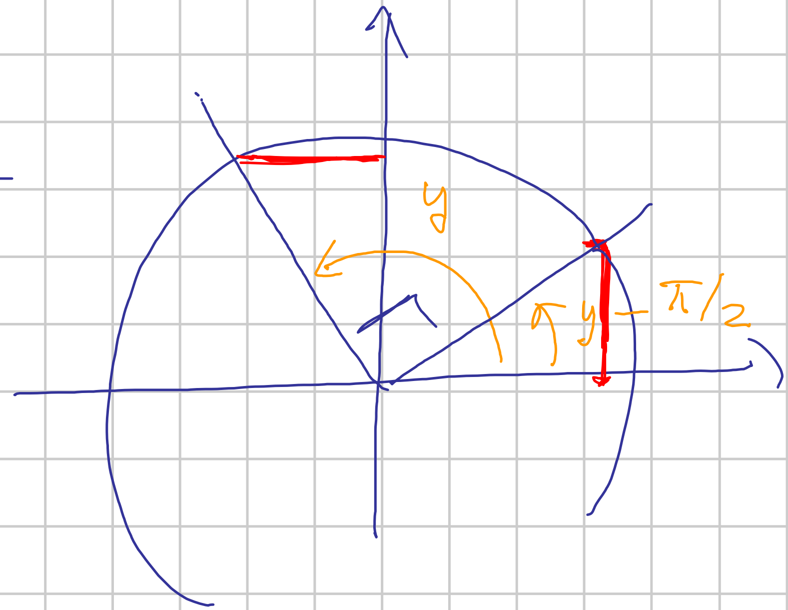
13 $\sqrt{3}\,\sin\left(x + \frac{\pi}{4}\right) + \sin\left(x - \frac{\pi}{4}\right) = 2$.

Usare formule addizione ...

Pongo $y = x + \frac{\pi}{4}$ cioè $x = y - \frac{\pi}{4} \Rightarrow x - \frac{\pi}{4} = y - \frac{\pi}{2}$

$$\sqrt{3}\sin(y) + \sin\left(y - \frac{\pi}{2}\right) = 2$$

$$\sqrt{3}\sin(y) - \cos(y) = 2$$



ottenuto eq. lineare in $\sin(y)$ $\cos(y)$.

Divido per 2:

$$\frac{\sqrt{3}}{2} \cdot \sin(y) - \frac{1}{2}\cos(y) = 1$$

$$\sin\left(y - \frac{\pi}{6}\right) = 1$$

$$y - \frac{\pi}{6} = \frac{\pi}{2} + 2k\pi$$

$$x + \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{2} + 2k\pi$$

$$x = \frac{5}{12}\pi + 2k\pi$$

$\boxed{14}$

$c = \cos(x) \qquad s = \sin(x)$

$$(2c+3s-1)^2 + (c+2)^2 = (c-4s)^2 - 2(3s+1)$$

$$4c^2 + 9s^2 + 1 + 12cs - 4c - 6s$$
$$+c^2 \qquad + 4 \qquad + 4c + 6s$$
$$-c^2 - 16s^2 + 2 + 8cs \qquad = 0$$

$$4c^2 - 7s^2 + 20cs + 7 = 0$$

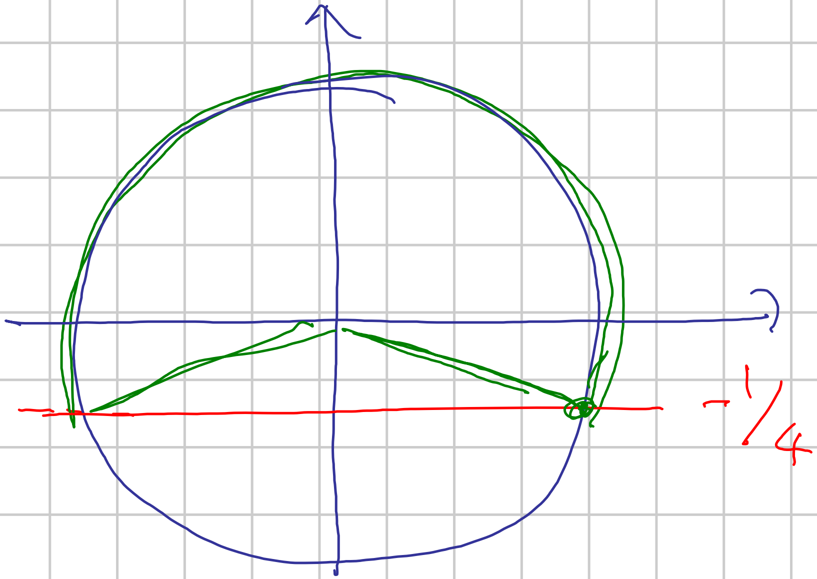
$$\| c^2 + 20cs = 0$$

$c = 0$ oppure $\frac{s}{c} = -\frac{11}{20}$

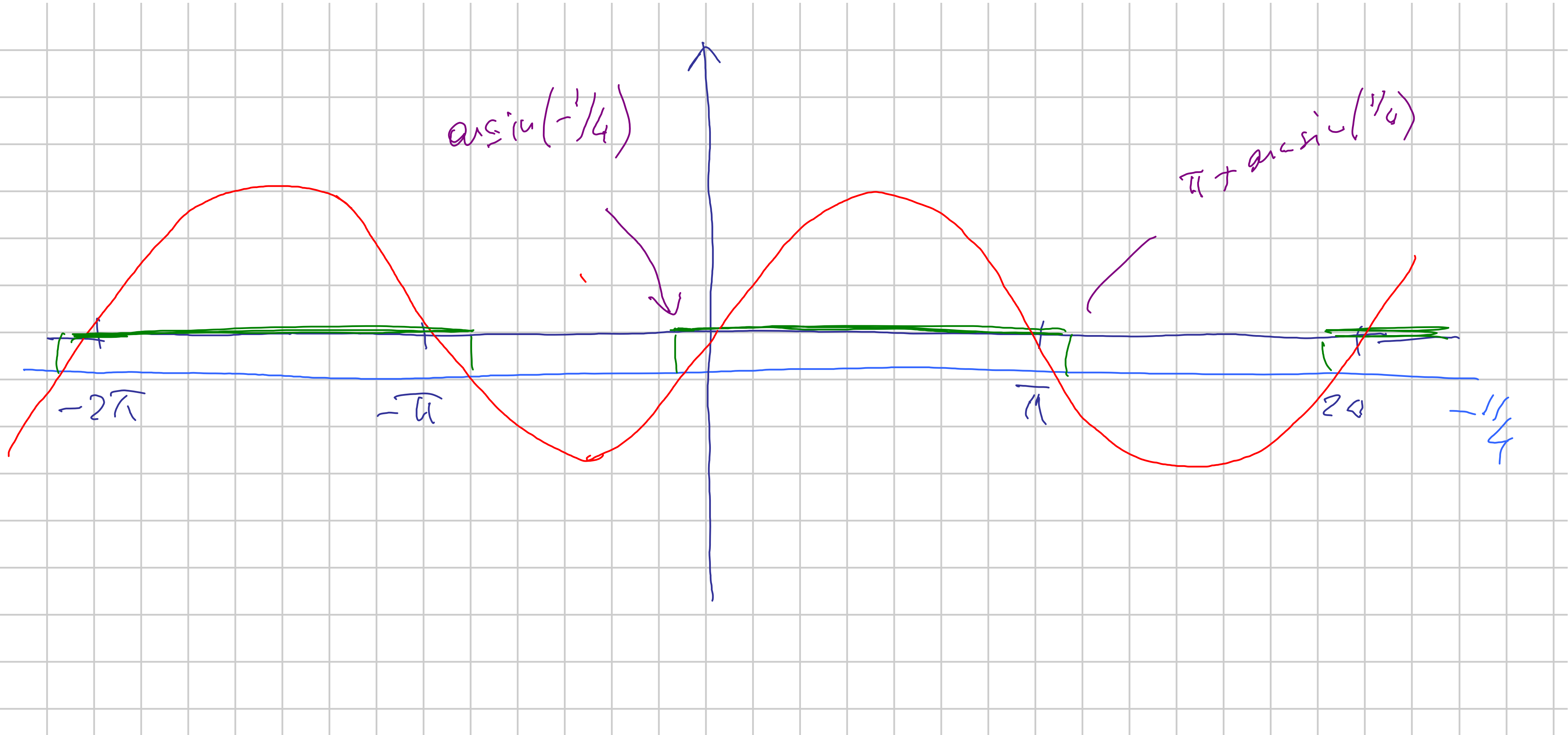
$x = \frac{\pi}{2} + k\pi$ $\qquad$ $x = \arctan\left(-\frac{11}{20}\right) + k\pi$

$\boxed{15}$ $\quad 4\sin(x) > -1$

$$\sin(x) > -\frac{1}{4}$$

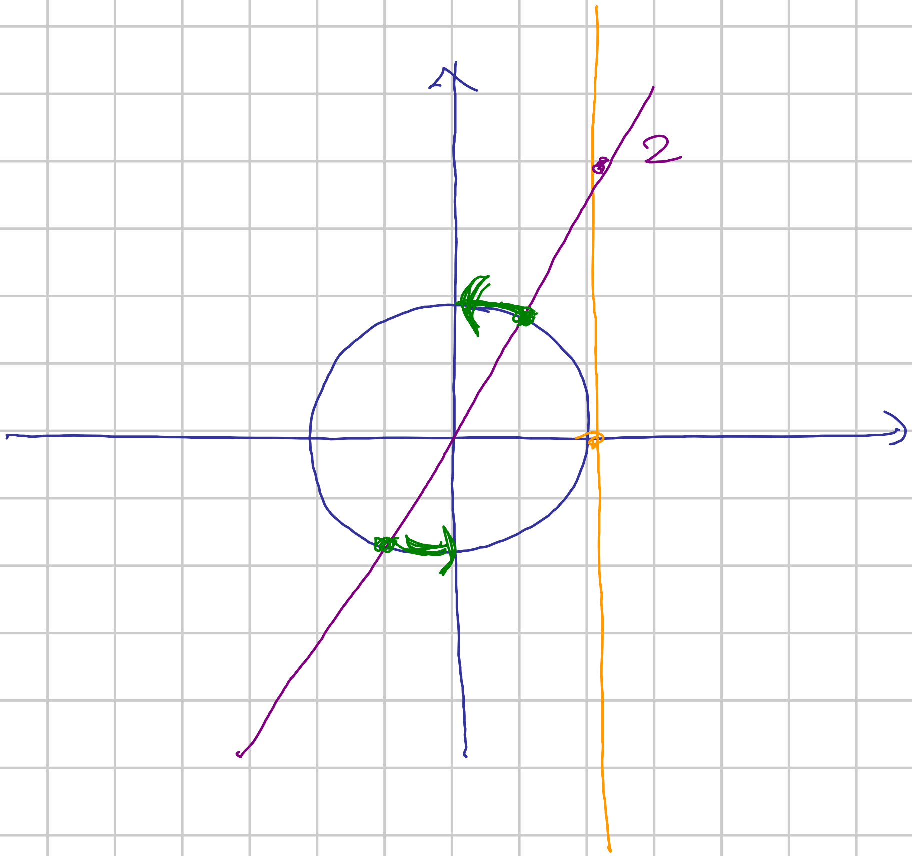


$$\arcsin(-1/4) + 2k\pi < x < \underbrace{\pi - \arcsin(-1/4) + 2k\pi}_{\pi + \arcsin(1/4) + 2k\pi}$$

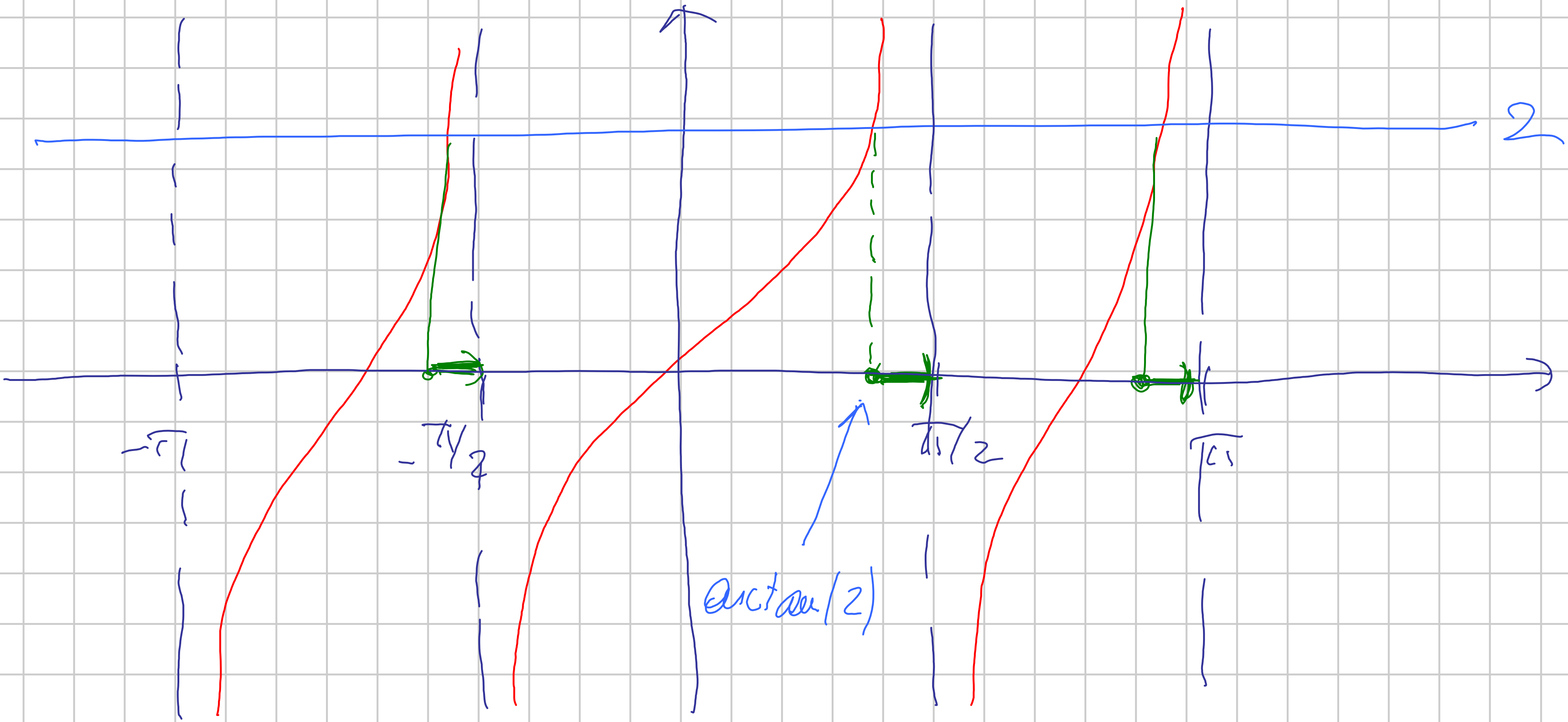


arcsin(-1/4)
π + arcsin(1/4)
-2π
-π
π
2π
-1/4

$\boxed{16}$ $\tan(x) \geq 2$



$$\arctan(2) + k\pi \leq x < \frac{\pi}{2} + k\pi$$


2
−π
−π/2
π/2
π
arctan(2)

Vettori

$\boxed{1b}$ Calcolare $\sqrt{3}\cdot\begin{pmatrix} 7 \\ -4\sqrt{6} \end{pmatrix} - 5\sqrt{2}\cdot\begin{pmatrix} 3\sqrt{6} \\ 4 \end{pmatrix}$

$$= \begin{pmatrix} 7\sqrt{3} \\ -12\sqrt{2} \end{pmatrix} + \begin{pmatrix} -30\sqrt{3} \\ -20\sqrt{2} \end{pmatrix} = \begin{pmatrix} -23\sqrt{3} \\ -32\sqrt{2} \end{pmatrix}$$

$\boxed{2c}$ Calcolare distanza tra $\begin{pmatrix} 2 \\ 9 \\ -4 \end{pmatrix}$ e $\begin{pmatrix} 3 \\ 7 \\ -2 \end{pmatrix}$

$$d(v,w) = \|v-w\|$$

$$\text{dist} = \left\| \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} \right\| = \sqrt{1+4+4} = \sqrt{9} = 3$$

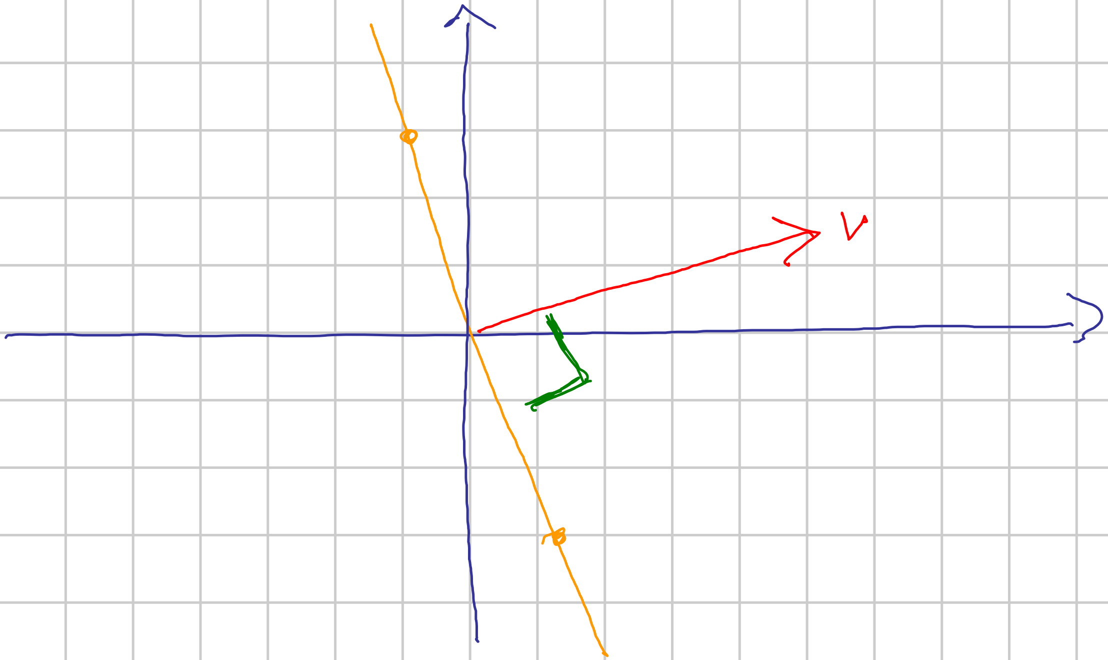
$\boxed{3c}$ Calcolare angolo tra $\begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix}$ e $\begin{pmatrix} 8 \\ 1 \\ -7 \end{pmatrix}$.

$$\cos(\text{angolo tra } v \text{ e } w) = \frac{\langle v | w \rangle}{\|v\| \cdot \|w\|}.$$

$$\Rightarrow \arccos\left(\frac{4 \cdot 8 + (-2) \cdot 1 + 3 \cdot (-7)}{\sqrt{16+4+9} \cdot \sqrt{64+1+49}} \right) = \arccos\left(\frac{9}{\sqrt{29} \cdot \sqrt{114}} \right)$$

$\boxed{4b}$ Trovare tutti i vett. del piano

ortog. a $\begin{pmatrix} \sqrt{6} \\ 5\sqrt{3} \end{pmatrix}$ e di norma 1.



$$v = \begin{pmatrix} \sqrt{6} \\ 5\sqrt{3} \end{pmatrix} = \sqrt{3} \begin{pmatrix} \sqrt{2} \\ 5 \end{pmatrix}$$

cerco $w = \begin{pmatrix} x \\ y \end{pmatrix}$ t.c.

$w \perp v$ cioè

$\langle w | v \rangle = 0$ cioè

$$\sqrt{2}\,x + 5y = 0 .$$

Le soluzioni sono i vettori multipli di $\begin{pmatrix} 5 \\ -\sqrt{2} \end{pmatrix}$;

ha norma $\sqrt{25+2} = 3\sqrt{3}$; i vettori cercati

$$\pm \frac{1}{3\sqrt{3}} \begin{pmatrix} 5 \\ -\sqrt{2} \end{pmatrix} .$$

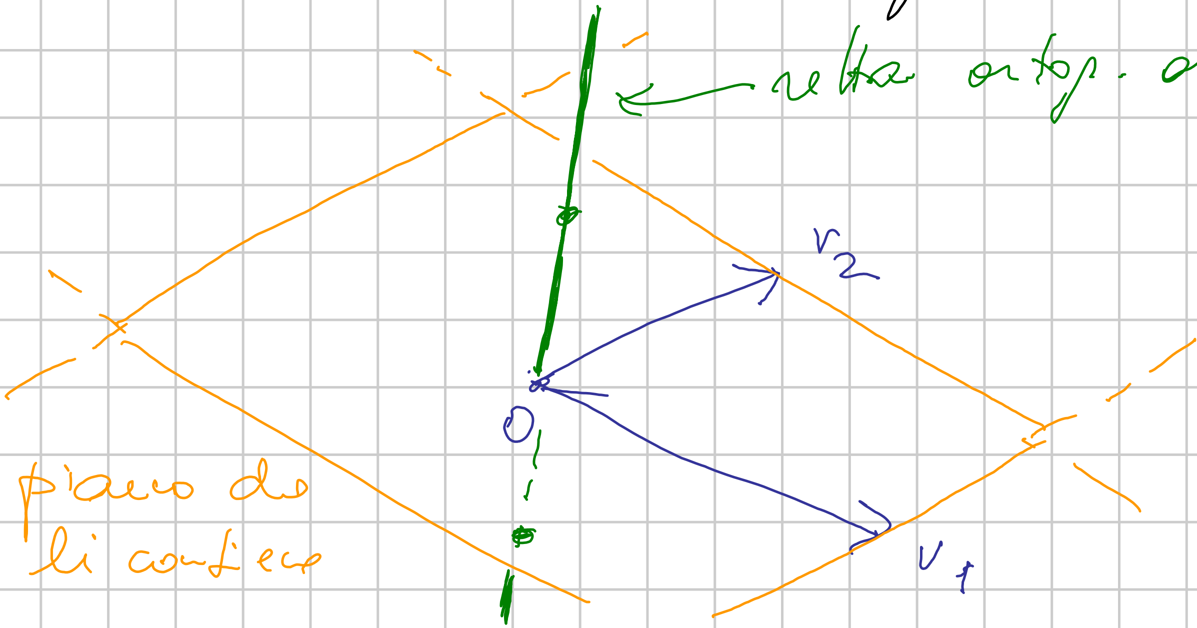
$\boxed{5a}$ Dati $v_1 = \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 5 \\ 7 \end{pmatrix}$ trovare:

– i vettori dello spazio ortogonali a v_1 e v_2

– area parallelogramma con lati v_1 e v_2

– i vettori dello spazio ortog. a v_1 e v_2 di norma 1.

retta ortog. al piano : multipli di



$$v_1 \times v_2 = \begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 5 \\ 7 \end{pmatrix}$$

$$= \begin{pmatrix} (-5)\cdot 7 - 3\cdot 5 \\ -4\cdot 7 + 3\cdot 1 \\ 4\cdot 5 - (-5)\cdot 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -50 \\ -25 \\ 25 \end{pmatrix} = -25 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

Verifico che $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ sia $\perp$ a v_1 e v_2:

$$2 \cdot 4 + 1 \cdot (-5) + (-1) \cdot 3 = 0 \quad \checkmark$$

$$2 \cdot 1 + 1 \cdot (5) + (-1) \cdot 7 = 0 \quad \checkmark$$

R1: i multipli di $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

R2 : $\|v_1 \times v_2\| = \left\| (-25) \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \right\| = 25 \left\| \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \right\| =$

$= 25\sqrt{6}$.

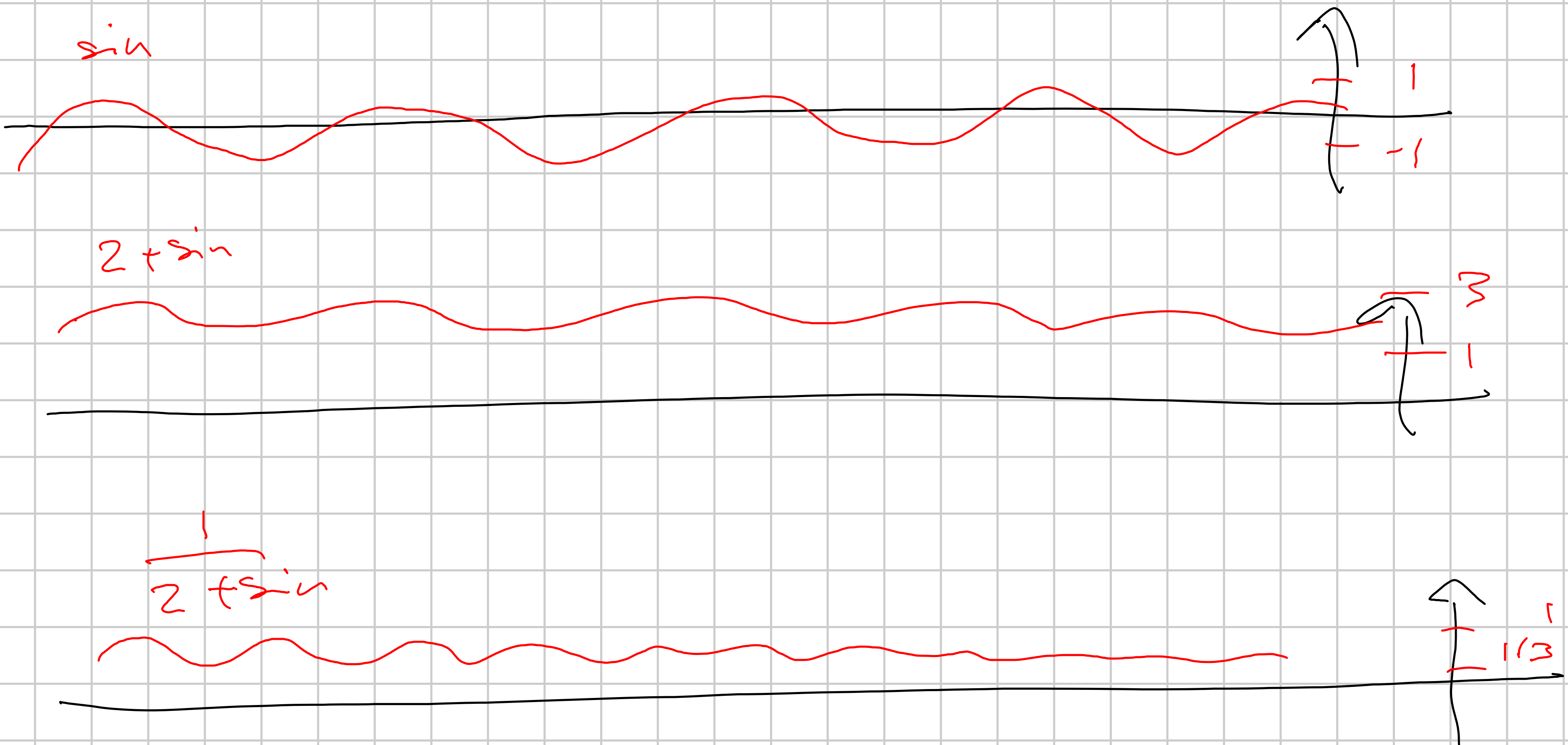
R3 : $\pm \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$

<u>Limiti</u> $\boxed{1}$

(a) $$\lim_{x \to \pi} \left(7 \cdot \left(\frac{x}{\pi}\right)^3 - 4 \cdot \sin\left(\frac{x}{2}\right) \right)$$

$$7 \cdot \left(\frac{\pi}{\pi}\right)^3 - 4 \cdot \sin\left(\frac{\pi}{2}\right) = 7 \cdot 1^3 - 4 \cdot 1 = 3$$

(b) $$\lim_{x \to -\infty} \frac{1}{2 + \sin(x)}$$



NON ESISTE

$\boxed{c}$ $\lim_{x \to 0} \dfrac{1 - \sin(x)}{1 + \cos(x)}$

$$\frac{1 - \sin(0)}{1 + \cos(0)} = \frac{1 - 0}{1 + 1} = \frac{1}{2}$$

$\boxed{d}$ $\lim_{x \to 0} \dfrac{1}{x^4} = +\infty$

$\boxed{e}$ $\displaystyle\lim_{x \to -1} \frac{1}{x^2+3x+2}$

$$\frac{1}{1-3+2} = \frac{1}{0} = ?$$

$$\lim_{x \to -1} \frac{1}{(x+1)(x+2)}$$

NON ESISTE

$\boxed{f}$ $\lim\limits_{x \to (-1)^-} \dfrac{1}{x^2+3x+2}$

$= \lim\limits_{x \to (-1)^-} \dfrac{1}{\underbrace{(x+1)}_{0^-}\underbrace{(x+2)}_{1}} = -\infty$

$\boxed{g}$ $\lim\limits_{x \to -\infty} \dfrac{1+2x-x^3}{3-x+x^2} = \lim\limits_{x \to -\infty} \dfrac{-x^3}{x^2}$

$= \lim\limits_{x \to -\infty} (-x) = +\infty$

$\boxed{h}$ $$\lim_{x \to +\infty} \frac{7+4x-9x^3}{1+5x^2+6x^3}$$

$$= \lim_{x \to +\infty} \frac{-9x^3}{6x^3} = \lim_{x \to +\infty} \left(-\frac{3}{2}\right) = -\frac{3}{2}$$

$\boxed{i}$ $$\lim_{x \to -\infty} \frac{6x^7 - x^3 + 1}{8|x|^7 + 5x^4 - 3}$$

$$= \lim_{x \to -\infty} \frac{6x^7 - x^3 + 1}{8(-x)^7 + 5x^4 - 3}$$

$$= \lim_{x \to -\infty} \frac{6x^7}{-8x^7} = -\frac{3}{4}$$

$\boxed{j}$ $$\lim_{x \to +\infty} \frac{15x + \cos(2x-1)}{12x - \sin(1-3x)}$$

$$= \lim_{x \to +\infty} \frac{15x}{12x} = \frac{5}{4}$$

$\boxed{k}$ $$\lim_{x \to 2^+} \frac{x^3 - 7x + 5}{2x^2 - 5x + 2}$$

$$\frac{8 - 14 + 5}{8 - 10 + 2} = ?$$

$$(2x^2-5x+2) = (x-2)(2x-1)$$

$$\lim_{x\to 2^+} \frac{\overbrace{x^3-7x+5}^{-1}}{\underbrace{(x-2)}_{0^+}\underbrace{(2x-1)}_{3}} = -\infty .$$