Alg Lin 28.11.2018

6.2.2.

(a) $\det(A) = -3 \qquad A = (v_1, v_2)$

$$B = (3v_1 - 2v_2 \; , \; -5v_1 + 4v_2)$$

$\det(B) = ?$

$$\det(B) = \det(3v_1 - 2v_2 \, , -5v_1) +$$

$$+ \det(3v_1 - 2v_2 \, , 4v_2) =$$

$$= \det(-2v_2 \, , -5v_1) +$$

$$+ \det(3v_1, 4v_2) =$$

$$= -\overbrace{10}^{(-2)(-5)} \det(v_1, v_2) +$$

$$+ 3\cdot v \det(v_1, v_2) = (-10+3)\det(A) =$$

$$= -7\cdot(-3) = 21$$

(c) $\det(A) = -4$

$$B = (v_1 + 2v_2 - 3v_3,\ -v_2 + 2v_3,\ 7v_1 - 4v_2).$$

$$\det(B) = \det(\cancel{v_1} + 2v_2 - 3v_3,\ -v_2 + 2v_3,\ 7v_1) +$$

$$+ \det(v_1 + 2\cancel{v_2} - 3v_3,\ -\cancel{v_2} + 2v_3,\ -4v_2) =$$

$$= \det(2v_2 - 3v_3, -v_2 + 2v_3, 7v_1) +$$

$$+ \det(v_1 - \cancel{3v_3}, 2v_3, -4v_2) =$$

$$= \det(\cancel{2v_2} - 3v_3, -v_2, 7v_1) + \det(2v_2 - \cancel{3v_3}, 2v_3, 7v_1) +$$

$$+ \det(v_1, 2v_3, -4v_2) =$$

$$= (-1)(-3)(-1)(7) \det(v_1, v_2, v_3) +$$

$$+ (2)(2)\cdot(7) \det(v_1, v_2, v_3) +$$

$$+ (-1)(2)(-4) \det(v_1, v_2, v_3) =$$

$$= [-21 + 28 + 8] \det(A) = 15 \cdot (-4) = -60$$

(d) $\det(A) = 2$

$B = (v_3 - v_1 \,,\, v_1 - 3v_4 \,,\, 2v_2 + v_4 \,,\, v_3 - 2v_4)$

$\det(B) = \det(\cancel{v_3} - v_1 \,,\, \cancel{v_1} - 3v_4 \,,\, 2v_2 + \cancel{v_4} \,,\, v_3) +$

$+ \det(v_3 - \cancel{v_1} \,,\, v_1 - \cancel{3v_4} \,,\, 2v_2 + \cancel{v_4} \,,\, -2v_4) =$

$= 6 \det(v_1 \,,\, v_4 \,,\, v_2 \,,\, v_3) +$

$+ (-4) \det(v_3 \,,\, v_1 \,,\, v_2 \,,\, v_4)$

$= (6-4) \cdot \det(B) = 2 \cdot 2 = 4$

6.2.4 $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix}$

$\operatorname{sgn}(4312) = -1$

1

$(4 \quad 1 \quad 3 \quad 2)$

1

$(1 \quad 4 \quad 3 \quad 2) \;—\; (1 \quad 2 \quad 3 \quad 4)$

6.2.5 $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 5 & 2 & 3 \end{pmatrix}$

$\operatorname{sgn}(\sigma) = -1$

1

$(1 \quad 4 \quad 5 \quad 2 \quad 3)$

1

$(1 \quad 2 \quad 5 \quad 4 \quad 3) \;—\; (1 \quad 2 \quad 3 \quad 4 \quad 5)$

6.2.7 $(4\ 2\ 5\ 3\ 6\ 1)$

$(1\ 2\ 5\ 3\ 6\ 4$

$(1\ 2\ 3\ 5\ 6\ 4)$

$(1\ 2\ 3\ 5\ 4\ 6) - (1\ 2\ 3\ 4\ 5\ 6)$

$\operatorname{sgn}(\sigma) = (-1)^4 = +1$

6.2.8. $(5\ 4\ 1\ 7\ 2\ 3\ 6)$

$(1\ 4\ 5\ 7\ 2\ 3\ 6)$

$(1\ 2\ 5\ 7\ 4\ 3\ 6)$

$(1\ 2\ 3\ 7\ 4\ 5\ 6)$

$(1\ 2\ 3\ 4\ 7\ 5\ 6)$

$(1\ 2\ 3\ 4\ 5\ 7\ 6)$

$(1\ 2\ 3\ 4\ 5\ 6\ 7)$

$\text{sgn}(\sigma) = (-1)^6 = 1$

6.2.9 $(7\ 5\ 2\ 8\ 1\ 4\ 3\ 6)$

$(1\ 5\ 2\ 8\ 7\ 4\ 3\ 6)$

$(1\ 2\ 5\ 8\ 7\ 4\ 3\ 6)$

$(1\ 2\ 3\ 8\ 7\ 4\ 5\ 6)$

$(1\ 2\ 3\ 4\ 7\ 8\ 5\ 6)$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 8 & 7 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{pmatrix}$$

$$\operatorname{sgn}(\varepsilon) = (-1)^6 = 1$$

6.2.10 $\det \begin{pmatrix} 3 & 4 \\ -5 & 7 \end{pmatrix} =$

$$= 3 \cdot 7 - 4(-5) = 41$$

6.2.11 $\det \begin{pmatrix} 1+t & -1+2t \\ 6+5t & 2-t \end{pmatrix} =$

$$= (1+t)(2-t) - (-1+2t)(6+5t) =$$

$$= (-t^2 + t + 2) - (10t^2 + 7t - 6) =$$

$$= -11t^2 - 6t + 8$$

6.2.12 $\det \begin{pmatrix} 6 & 1 & -3 \\ 4 & -7 & 2 \\ -2 & 5 & 8 \end{pmatrix} =$ -1

$$= 6 \begin{vmatrix} -7 & 2 \\ 5 & 8 \end{vmatrix} - 1 \begin{vmatrix} 4 & 2 \\ -2 & 8 \end{vmatrix} - 3 \begin{vmatrix} 4 & -7 \\ -2 & 5 \end{vmatrix} = \dots$$

$$\sum a_{1i} \det_2 (A^{1i}) \quad (-1)^{1+i}$$

$(-1)^{1+1}$ $(-1)^{1+2}$ $(-1)^{1+3}$

$$\ldots = 6(-66) - 28 - 3 \cdot 6 =$$

$$= -336 - 28 - 18 = -442$$

6.2.13

$$\det \begin{pmatrix} -1+t & 2 & -3 \\ 1+2t & 5 & -1 \\ -2 & 2-3t & 5 \end{pmatrix} =$$

$$= -3 \begin{vmatrix} 1+2t & 5 \\ -2 & 2-3t \end{vmatrix} + \underbrace{(-1)}_{(-1)^{2+3}} \cdot \underbrace{(-1)}_{a_{23}} \begin{vmatrix} -1+t & 2 \\ -2 & 2-3t \end{vmatrix} +$$

$$+ 5 \begin{vmatrix} -1+t & 2 \\ 1+2t & 5 \end{vmatrix} =$$

$$= -3\left(-6t^2 + t + 2 + 10\right) + \left(-3t^2 + 5t - 2 + 4\right) +$$

$$+ 5\left(-5 + 5t - 2 - 4t\right) =$$

$$= 15t^2 + 7t - 69$$