Matematica e statistica 10/10/17

(1) (a) $f: \mathbb{N} \to \mathbb{N}$

$f(n) =$ # lettere di n in italiano

$f(0) = 4 \quad f(1) = 3 \quad f(2) = 3 \quad \ldots$

Sì

(b) $f: \mathbb{N} \to \mathbb{N} \qquad f(n) = 2 \cdot n - 3$

No

(c) $f : \mathbb{N} \to \mathbb{N}$ $\quad f(n) = 1 + \dfrac{n^2 - 1}{n+1} = 1 + \dfrac{(n+1)(n-1)}{n+1}$

$= 1 + n - 1 = n$ Sì

(d) $f : \mathbb{N} \to \mathbb{N}$ $\quad f(n) =$ un numero $< n$

No

(e) $f :$ N $\to$ alfabeto

$f(n) =$ iniziale di n in italiano

$f(0) = z \qquad f(1) = u \qquad f(2) = d \ \ldots$

Sì

A) $f : \mathbb{N} \to$ alfabeto

$f(n) =$ IV lettera di n in italiano

$f(0) = o \qquad f(1)$ No

(g) $f: \mathbb{Z} \to \mathbb{Q}$ $\quad f(n) = \dfrac{n^3 - 1}{n^2 + 1}$ Sí

$f(0) = -1 \quad f(1) = 0 \quad f(2) = \dfrac{7}{5}$

$f(-1) = \dfrac{-2}{2} = -1 \quad f(-2) = \dfrac{-9}{5}$ ...

Sí

(h) $f: \mathbb{Q} \to \mathbb{Z}$ $\quad f(x) = \dfrac{x^3 - 1}{x^2 + 1}$ No

$f(2) = \dfrac{7}{5}$

(i) X = popolazione mondo

$f : X \to X$ $\quad f(x)$ = i genitori di x

No

(j) $f : X \to X$ $\quad f(x)$ = la madre di x

Sì

(k) $f : X \to X$ $\quad f(x)$ = il figlio primogenito di x

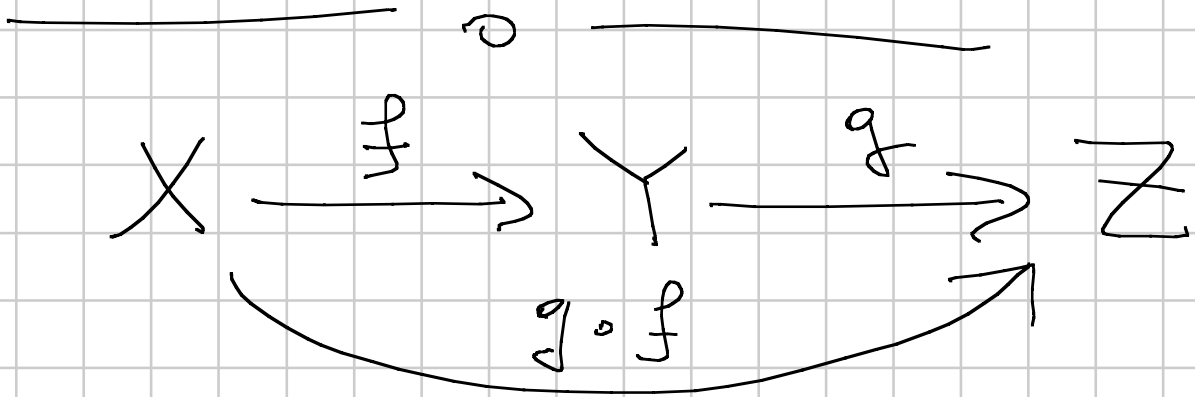
(1) X = la pop. italiana

$f : X \to X$ $\quad f(x) =$ la madre di x

No

No

Def:



$$(g \circ f)(x) = g(f(x))$$

(2a) $f : \mathbb{R} \to \mathbb{R}$ $\quad f(x) = \sqrt[3]{x}$

$g : \mathbb{R} \to \mathbb{R}$ $\quad g(x) = x^2$

$$(g \circ f)(-8) = g(f(-8)) = g(-2) = 4$$

$$(f \circ g)(-8) = f(g(-8)) = f(64) = 4$$

(2b) $f: \mathbb{Z} \to \mathbb{Q}$ $\quad f(n) = \dfrac{7n+3}{2n-5}$

$g: \mathbb{Q} \to \mathbb{Z}$ $\quad g(x) = [x]$

$[11.73] = 11$

$\left[\frac{9}{4}\right] = 2$

$(g \circ f)(4) = g(f(4)) = g\left(\frac{31}{3}\right) = 10$

$(f \circ g)(4) = f(g(4)) = f(4) = \frac{31}{3}$

(4) Dire se f è iniettiva, suriettiva, biiettiva.

$f : X \to Y$

Preso $y \in Y$ mi chiedo quanti $x \in X$ esistono t.c. $f(x) = y$:

→ sempre al massimo uno (f iniettiva)

→ sempre almeno uno (f suriettiva)

→ sempre esattamente uno (f bigettiva)

(b) $f: \mathbb{N} \to \mathbb{N}$ $\quad f(n) = 2n$

iniettiva non surj.

(c) $f: \mathbb{N} \to \mathbb{N}$ $\quad f(n) = \left[\frac{n}{2}\right]$

	0	1	2	3	4	5	6	7	8	...
f:	↓	↓	↓	↓	↓	↓	↓	↓	↓	
	0	0	1	1	2	2	3	3	4	

non iniettiva, suriettiva

(e) $f(n) \to \mathbb{Z}$ $\quad f(n) = (-1)^{n+1} \left[\frac{n+1}{2} \right]$

0	1	2	3	4	5	6	7	8	9	...
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
0	1	-1	2	-2	3	-3	4	-4	5	

biiettiva

(7) $f : \mathbb{N} \to \mathbb{Z}$ $\qquad f(n) = (-1)^{n+1} \left[\frac{n}{3} \right]$

0	1	2	3	4	5	6	7	8	9	10	11	...
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	
0	0	0	1	−1	1	−2	2	−2	3	−3	3	

non iniettiva, surg.

(h) $f : \mathbb{R} \to \mathbb{R}$ $\quad f(x) = 3x - \pi$

Dato $y \in \mathbb{R}$ risolviamo $f(x) = y$ cioè

$3x - \pi = y$; soluz. unica $x = \frac{y+\pi}{3}$

biiettiva.

(i) $f : \mathbb{R} \to \mathbb{R}$ $\quad f(x) = x^3$

$f(x) = y$ $\quad x^3 = y$

soluz. unica $y = \sqrt[3]{x}$ $\Rightarrow$ biiettive.

(j) $f : \mathbb{R} \to \mathbb{R}$ $\quad f(x) = x^2 - 7x + 10$.

Risolviamo $f(x) = y$

I. $x^2 - 7x + 10 = y$

$x^2 - 7x + 10 - y = 0$ $\quad \Delta = 49 - 40 + 4y$

$= 9 + 4y$

$\Delta < 0$ per $y < -9/4$ nessuna soluz.

$\Delta = 0$ per $y = -9/4$ una soluz

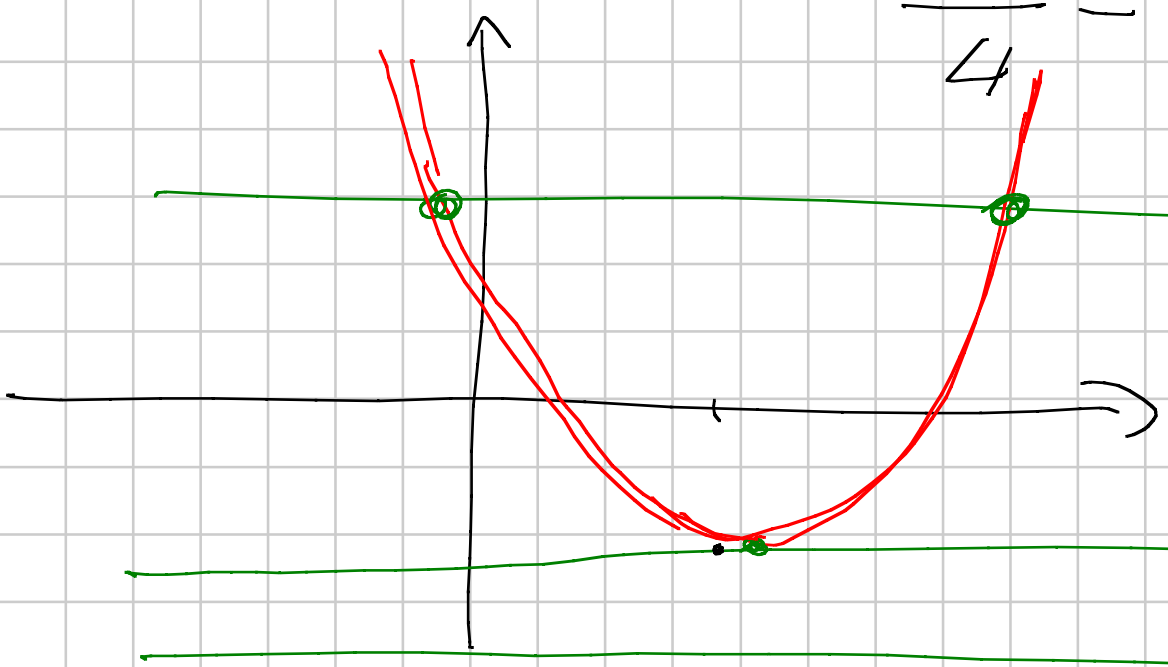
$\Delta > 0$ per $y > -9/4$ due soluz.

né surg. né iniettiva

II. grafico di $x^2 - 7x + 10$ è una parabola verso l'alto di vertice

$(7/2, -9/4)$

$$\frac{49}{4} - \frac{49}{2} + 10 = -\frac{49}{4} + 10 = -\frac{9}{4}$$



$y > -9/4$

$y = -9/4$

$y < -9/4$

(k) $f: \mathbb{R} \to \mathbb{R}$ $\quad f(x) = \dfrac{x-1}{x^2+1}$

Risolviamo $f(x) = y$

$$\frac{x-1}{x^2+1} = y \qquad y \cdot x^2 - x + y + 1 = 0$$

$$\Delta = 1 - 4y(y+1) = 1 - 4y - 4y^2$$

(1) $f : \mathbb{R} \to \mathbb{R}$ $\qquad f(x) = \dfrac{x}{1+|x|}$

$$\frac{x}{1+|x|} = y$$

$y = 0$ sol. unica $x = 0$.

$y > 0$ se c'è soluzione ho $x > 0$; equivale a

$$\begin{cases} \dfrac{x}{1+x} = y \\ x \geq 0 \end{cases} \qquad \begin{cases} x = y + xy \\ x > 0 \end{cases} \qquad \begin{cases} x = \dfrac{y}{1-y} \\ x > 0 \end{cases}$$

$\nexists$ soluz. per $y = 1$

- nessuna soluz. per $y \geq 1$
- una sola $x = \dfrac{y}{1-y}$ per $0 < y < 1$

$y < 0$ analog.:

una sol.t per $-1 < y < 0$

nessuna per $y \leq -1$

Morale: $f(x) = y$ ha

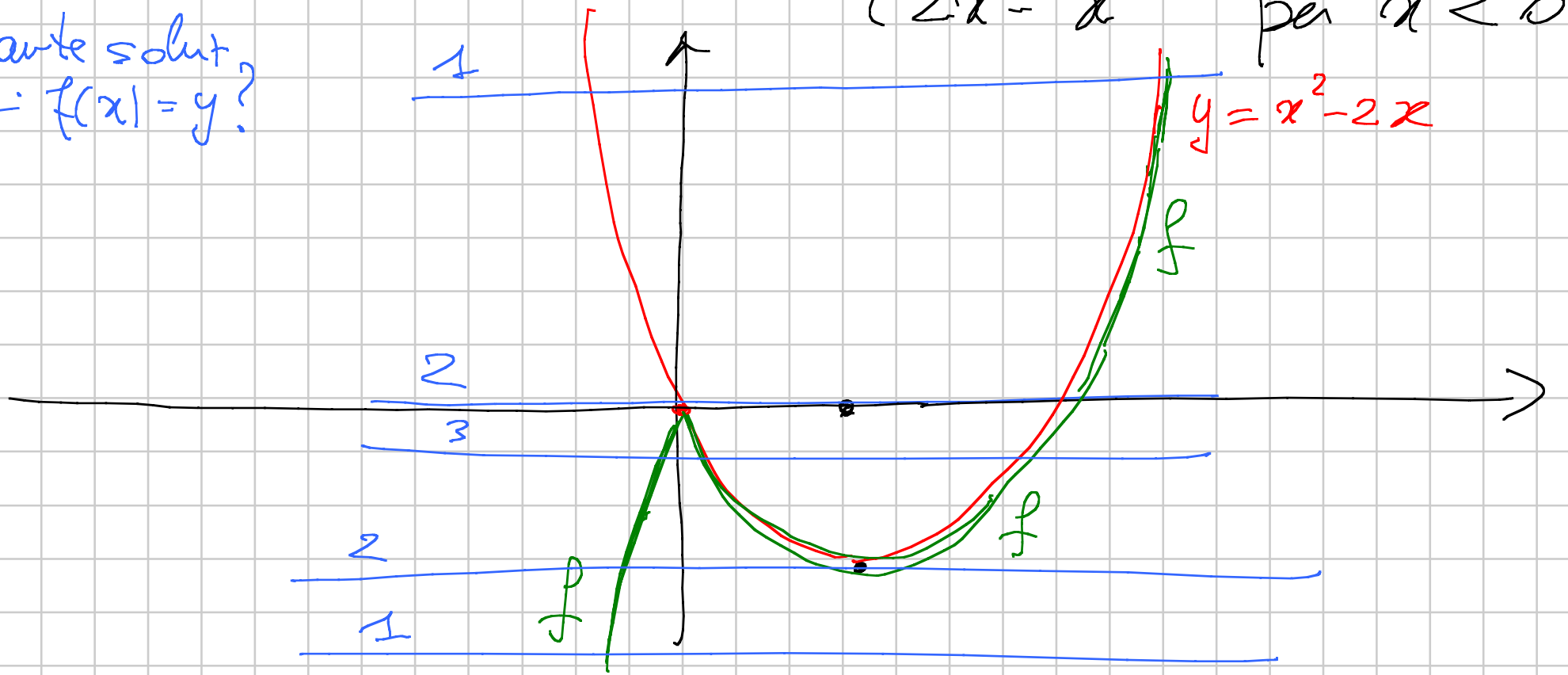
una sol.t. per $|y| < 1$

nessuna per $|y| \geq 1$.

iniettive non surj.

(m) $f : \mathbb{R} \to \mathbb{R}$ $f(x) = \begin{cases} x^2 - 2x & \text{per } x \geq 0 \\ 2x - x^2 & \text{per } x < 0 \end{cases}$

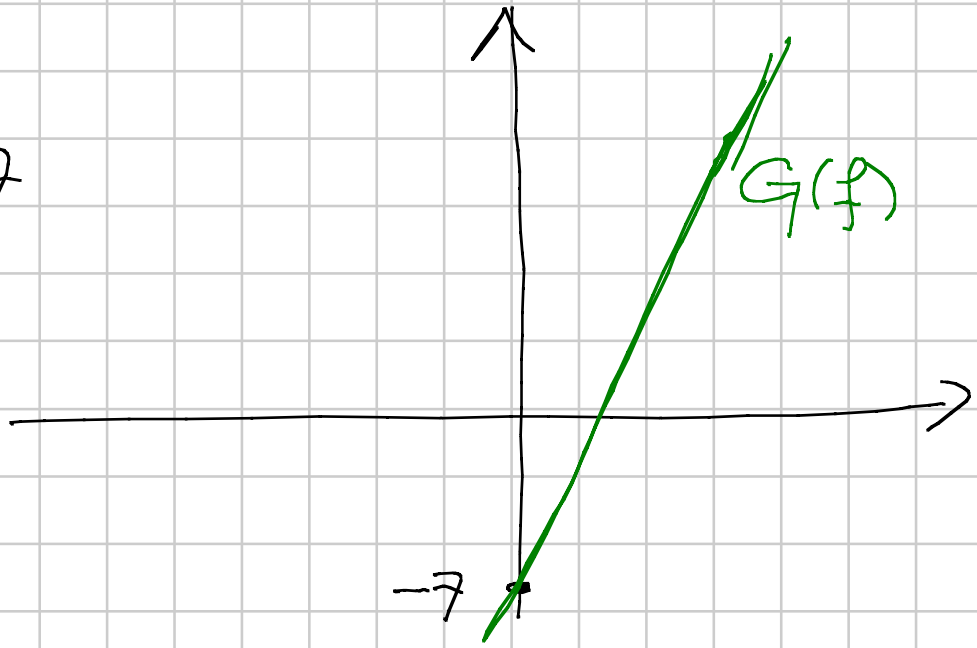
quante soluz. di $f(x) = y$?



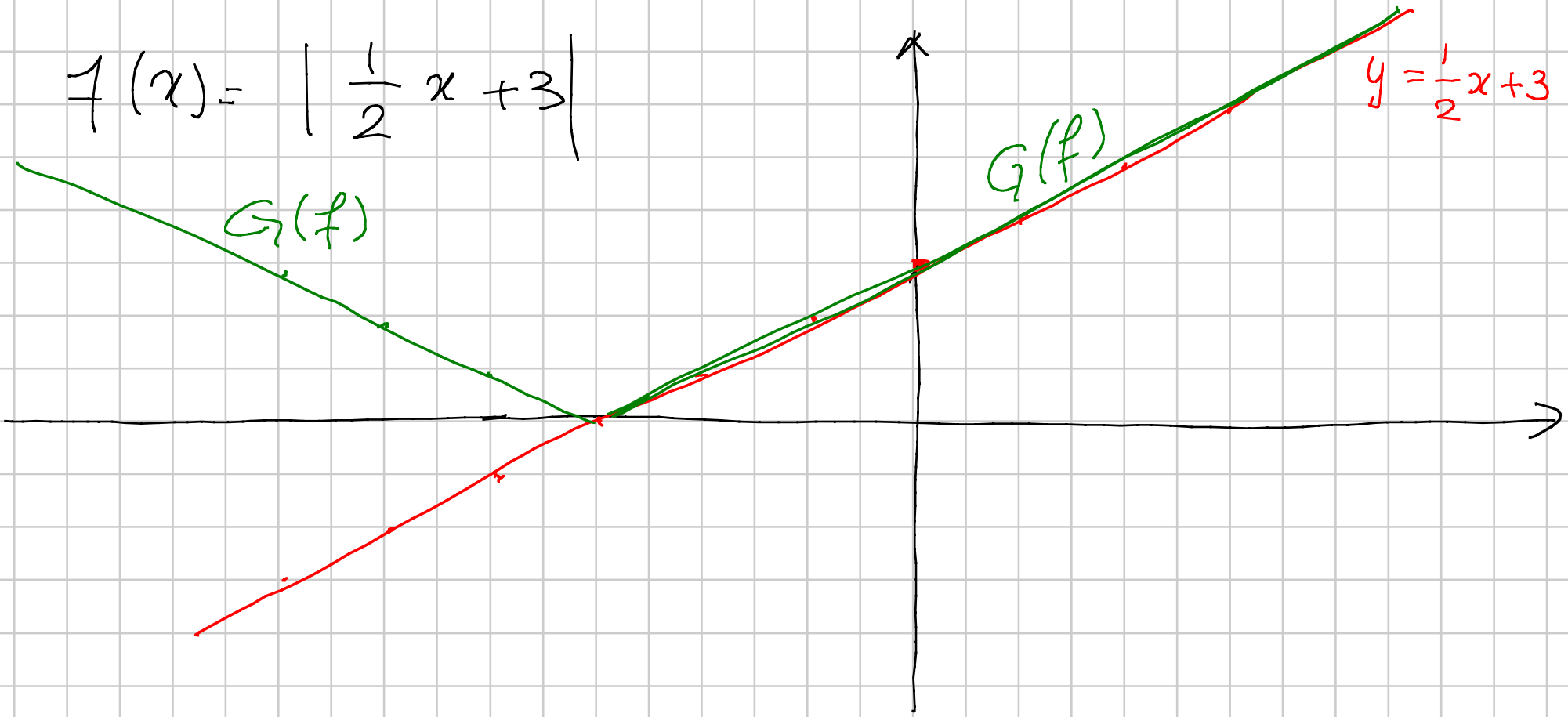
non iniettive, suriettive

⑤ Disegnare grafico di f

(a) $f : \mathbb{R} \to \mathbb{R}$ $\quad f(x) = 2x - 7$

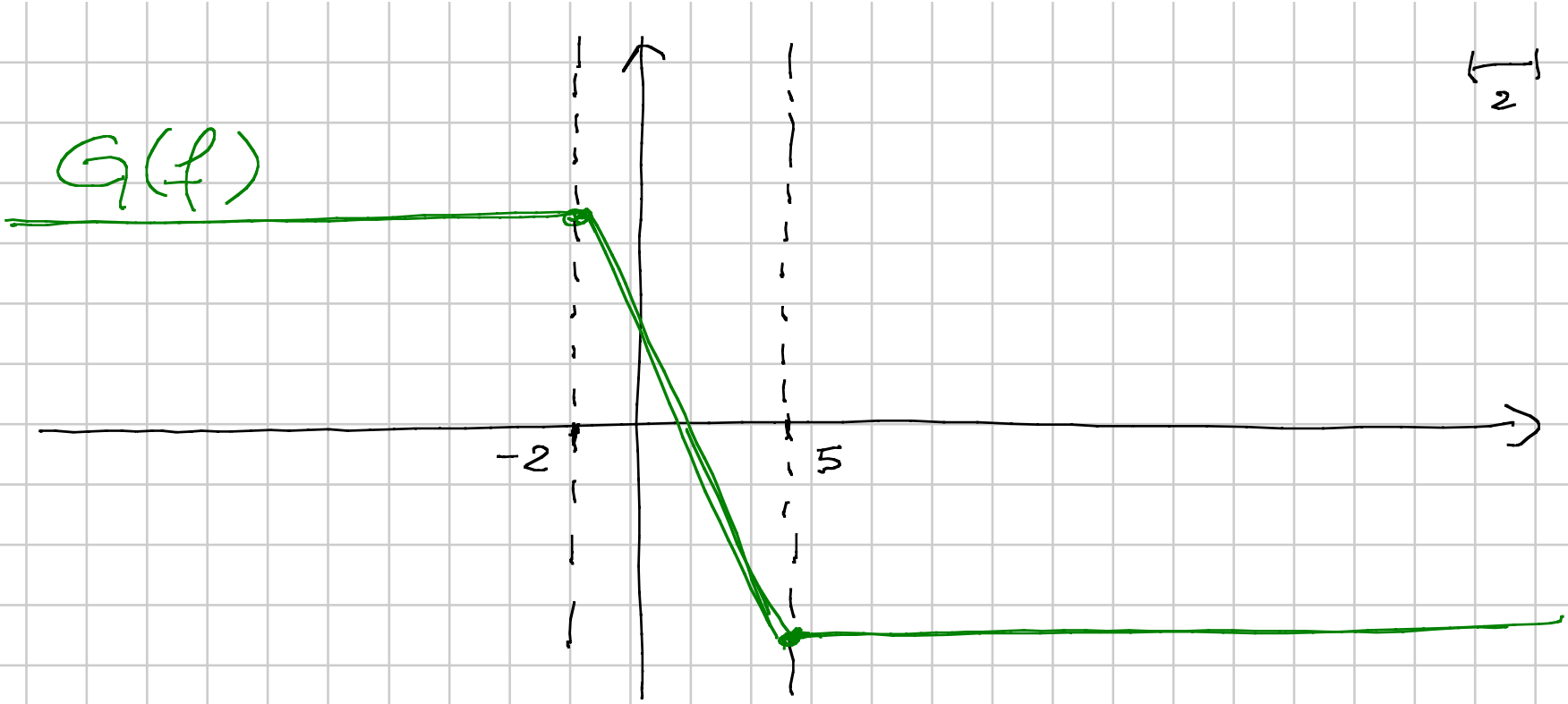


(b) $f(x) = \left| \frac{1}{2}x + 3 \right|$

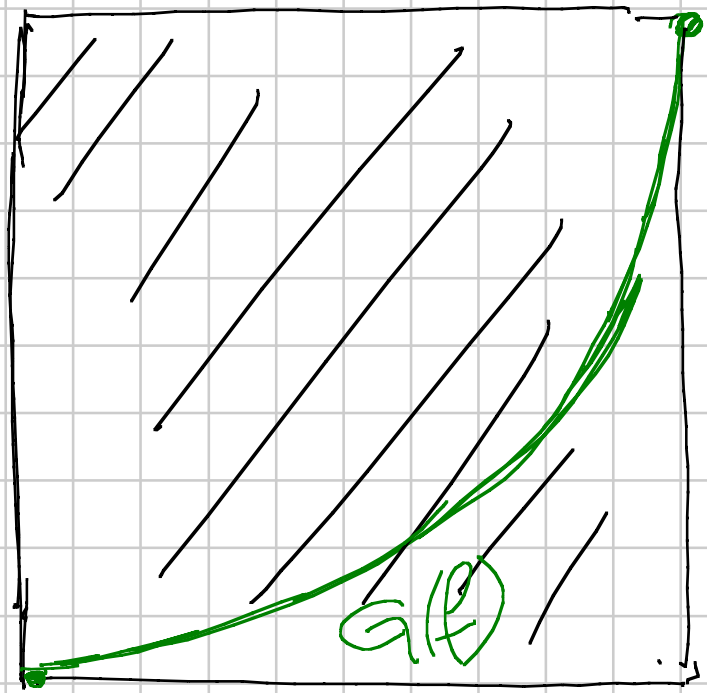


(c) $f(x) = |x-5| - |x+2|$

$$f(x) = \begin{cases} (5-x) - (-x-2) = 7 & \text{pu} \quad x \leq -2 \\ (5-x) - (x+2) = 7-2x & \text{pu} \quad -2 < x \leq 5 \\ (x-5) - (x+2) = -7 & \text{pu} \quad x > 5 \end{cases}$$


G(f)
2
-2
5

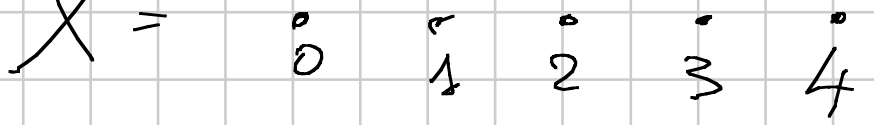
(i) $X = [0,1]$ $f : X \to X$ $f(x) = x^2$



$[0,1] \times [0,1]$

(i) $X = \{0,1,2,3,4\}$

$f : X \to X$ $\quad f(x) =$ resto della div. $(x+2)^2 : 5$

$X =$ 

$X \times X = \{(0,0),(0,1),\ldots (1,0),(1,1)\ldots (2,0),(2,1\ldots\}$

$f(x) =$ resto di $(x+2)^2 : 5$

$X \times X =$