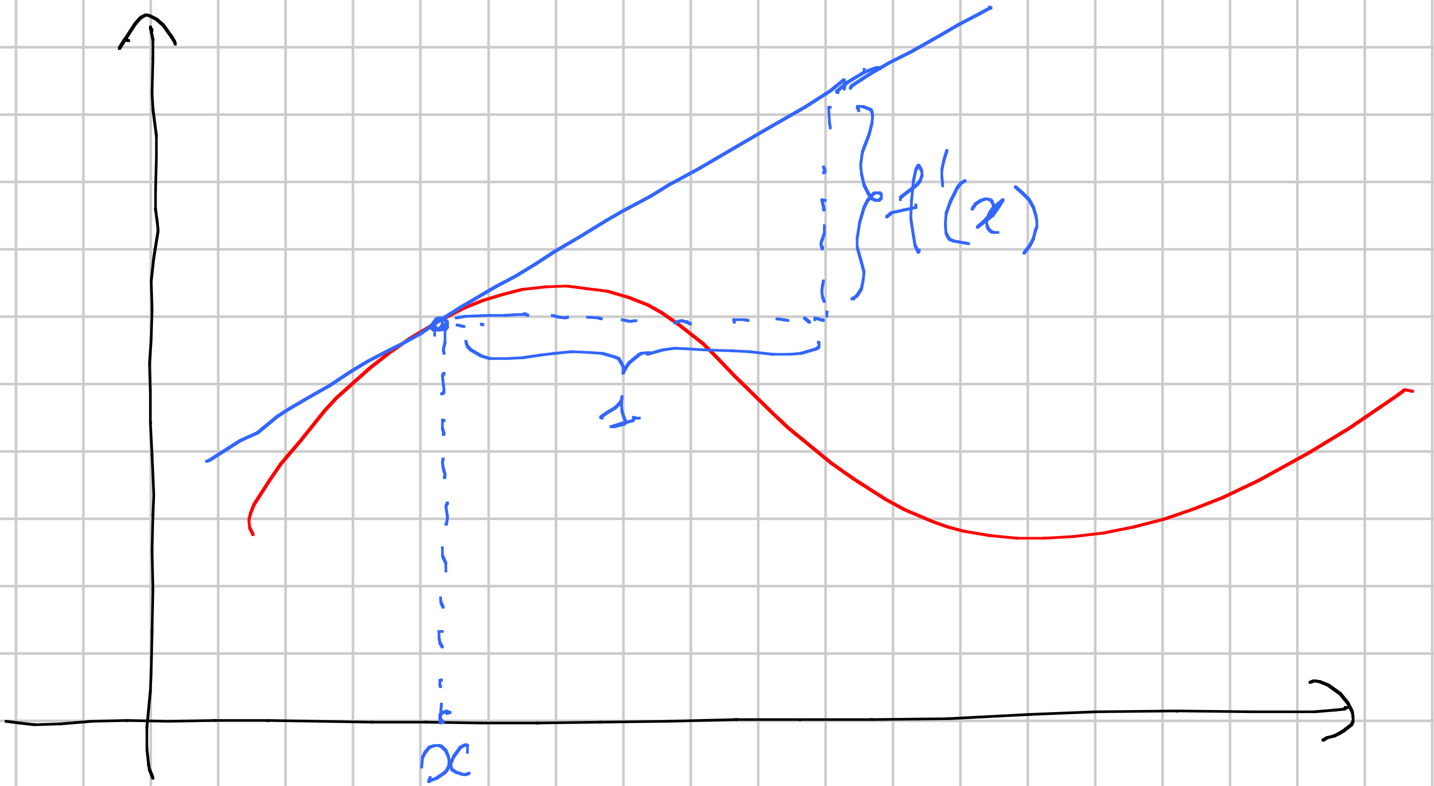
Matematica e statistica 8/11/17

$f : (a, b) \to \mathbb{R}$ derivata

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ se esiste

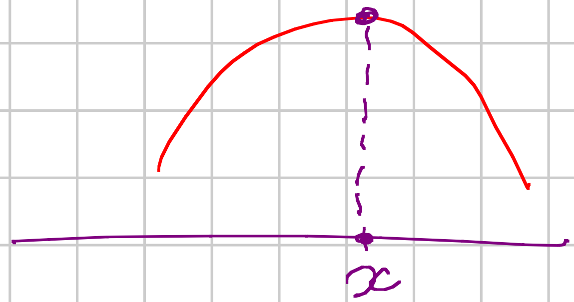
fatto: è la pendenza della retta tangente al grafico:



f'(x)
1
x

- $f'(x) > 0 \Rightarrow f$ crescente vicino a x
  $f'(x) < 0 \Rightarrow f$ decrescente vicino a x

- Se f ha max o min loc. in x
  $\Rightarrow f'(x) = 0$.



- $(x^n)' = n \cdot x^{n-1}$
  $\Rightarrow (a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots)'$

$$= a_1 + 2a_2 x + 3a_3 x^2 + \dots$$

•) $f(x) = 3x^2 + 10x + 2$

$f'(x) = 6x + 10$ pertanto

$f'(x) > 0$ per $x > -5/3$

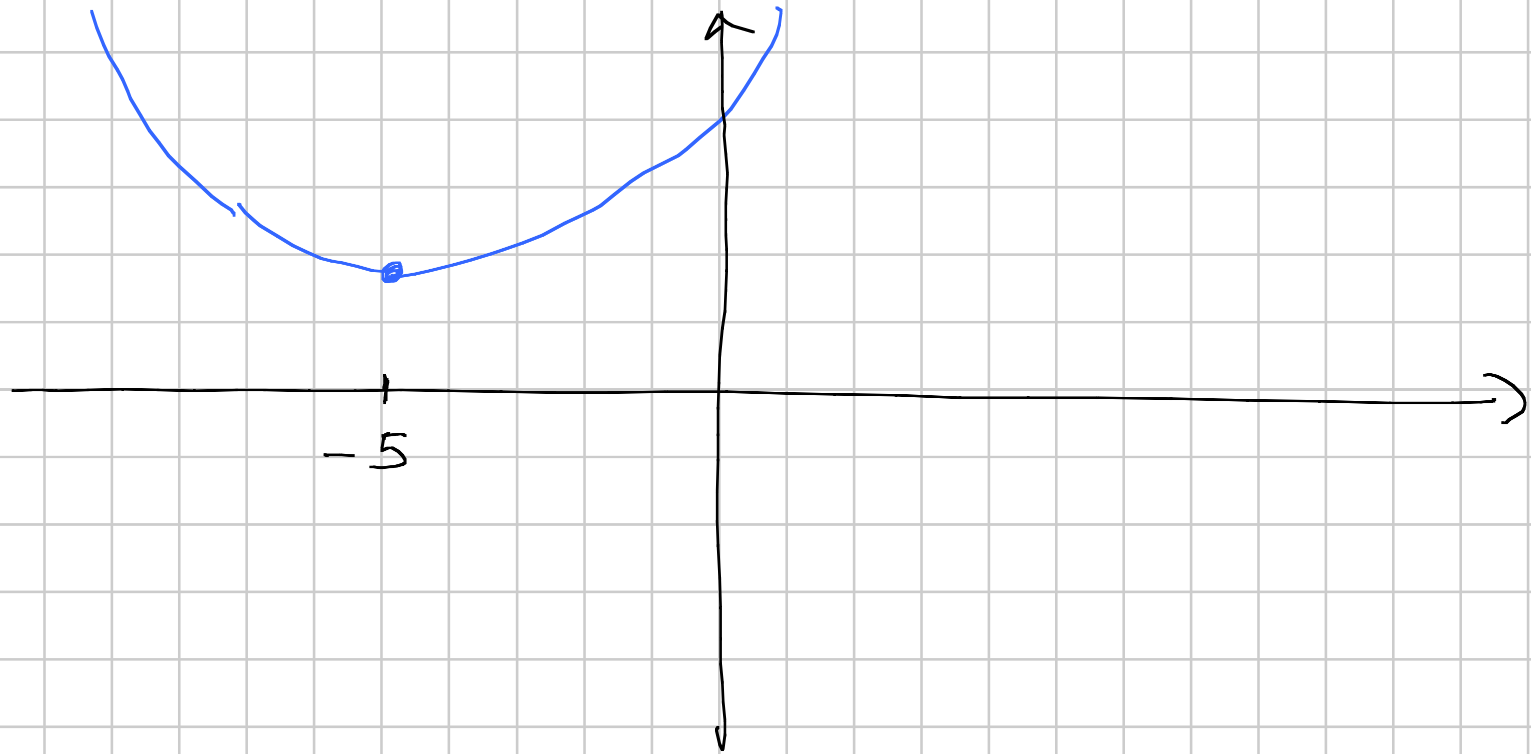
$f'(x) < 0$ per $x < -5/3$

$$f'(-5/3) = 0$$

$$f(-5/3) = 3 \cdot \frac{25}{9} + 10 \cdot \left(-\frac{5}{3}\right) + 2 = \frac{25}{3} - \frac{50}{3} + 2 = -\frac{25}{3} + 2$$

$$= -\frac{19}{3}$$

$$\lim_{x \to +\infty} f(x) = +\infty \qquad \lim_{x \to -\infty} f(x) = +\infty$$



•) $f(x) = 2x^3 + 3x^2 - 36x + 1$

$$\lim_{x \to +\infty} f(x) = +\infty \qquad \lim_{x \to -\infty} f(x) = -\infty$$

$$f'(x) = 6x^2 + 6x - 36 = 6(x^2 + x - 6)$$

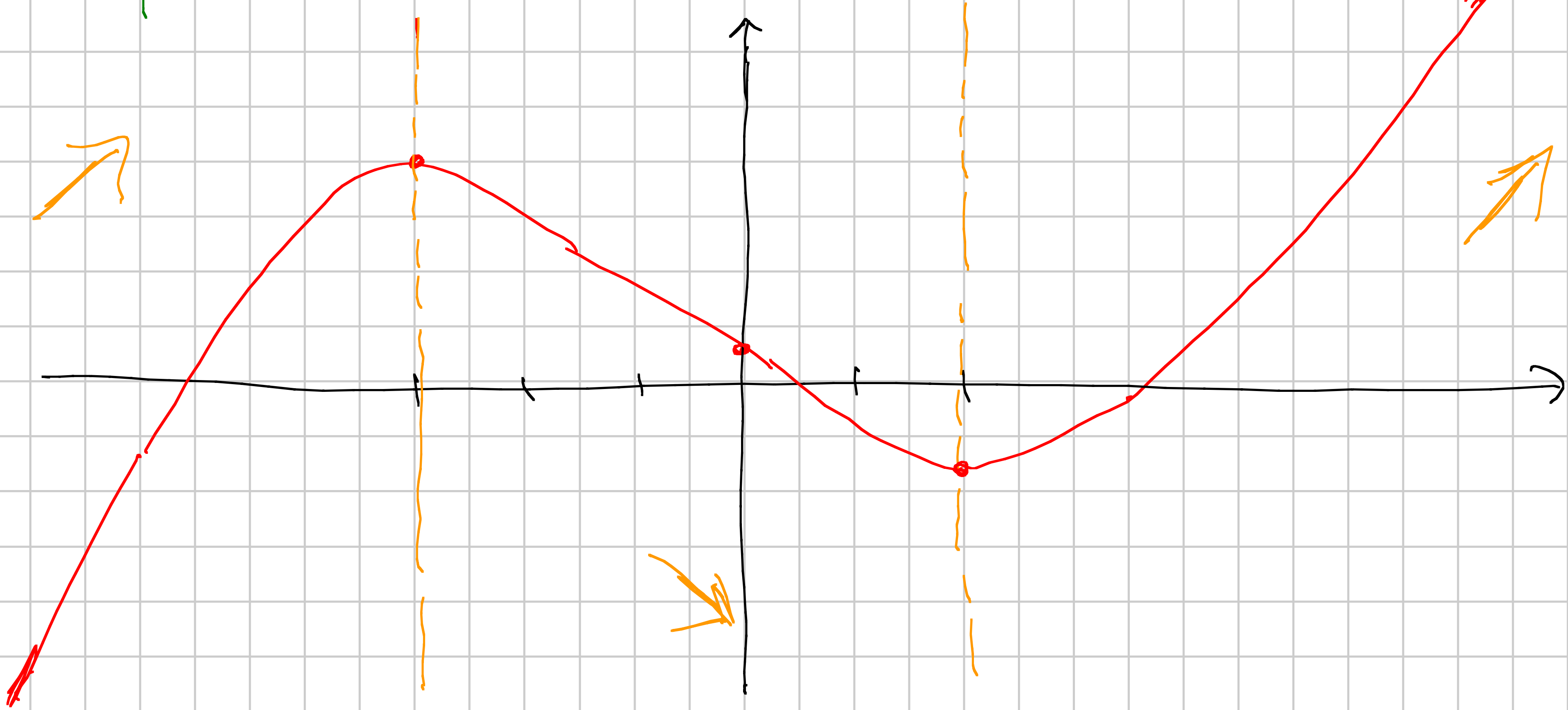
$$= 6(x+3)(x-2)$$

nulla in $x = -3$ e $x = 2$

positiva per $x < -3$ e $x > 2 \Rightarrow f$ crescente

negativa per $-3 < x < 2 \Rightarrow f$ decrescente

$f(-3) = -54 + 27 \; . \; . \; . \; .$

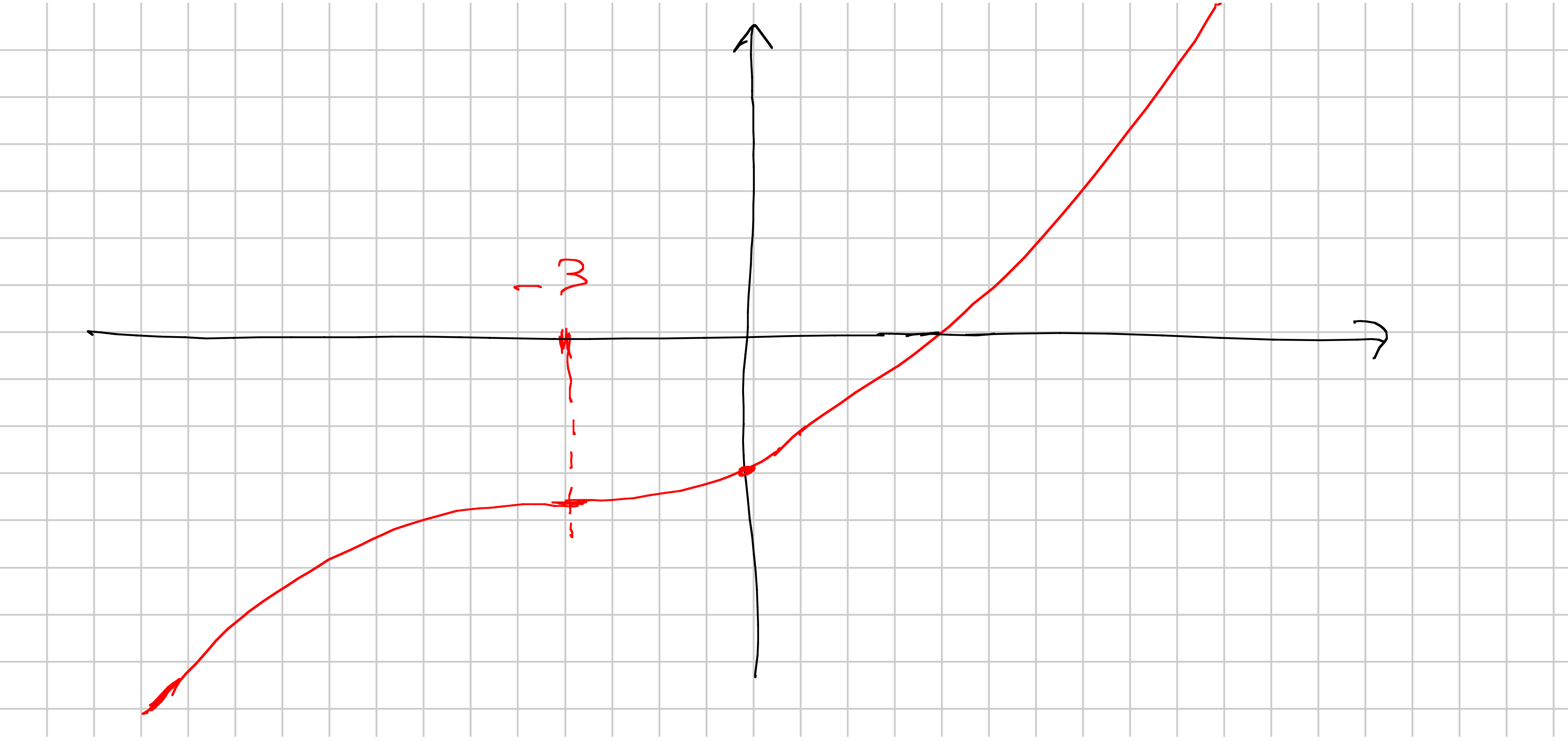


e) $f(x) = x^3 - 9x^2 + 27x - 5$

$$\lim_{x \to \pm\infty} f(x) = \pm \infty$$

$$f'(x) = 3x^2 - 18x + 27 = 3(x^2 - 6x + 9) = 3(x-3)^2$$

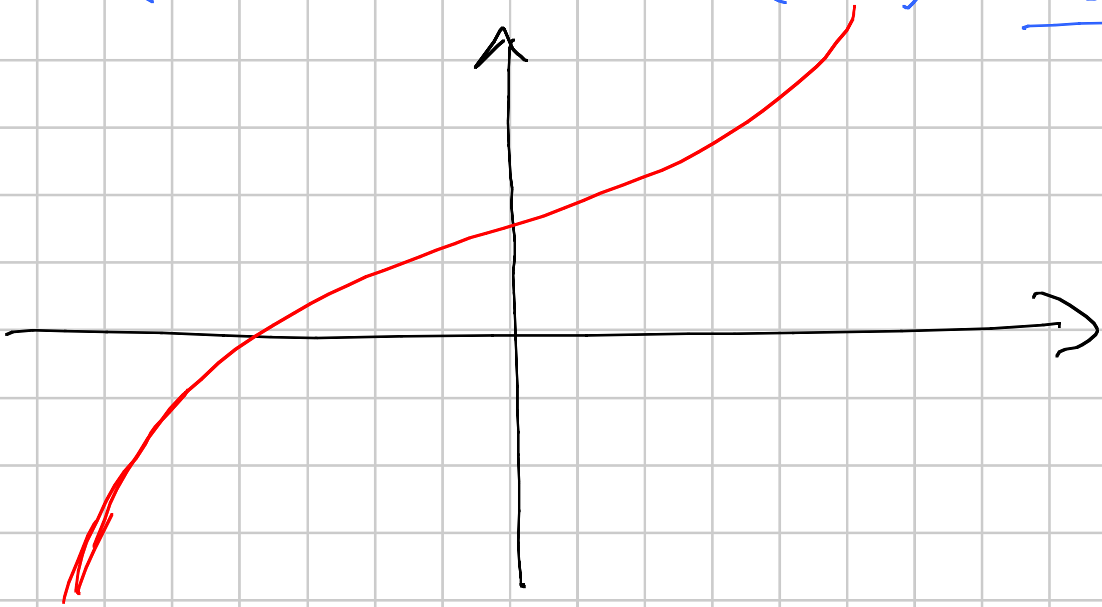
$f'(x) > 0 \quad \forall x \neq -3 \qquad f'(-3) = 0$


−3

•) $f(x) = x^3 - x^2 + x + 2$

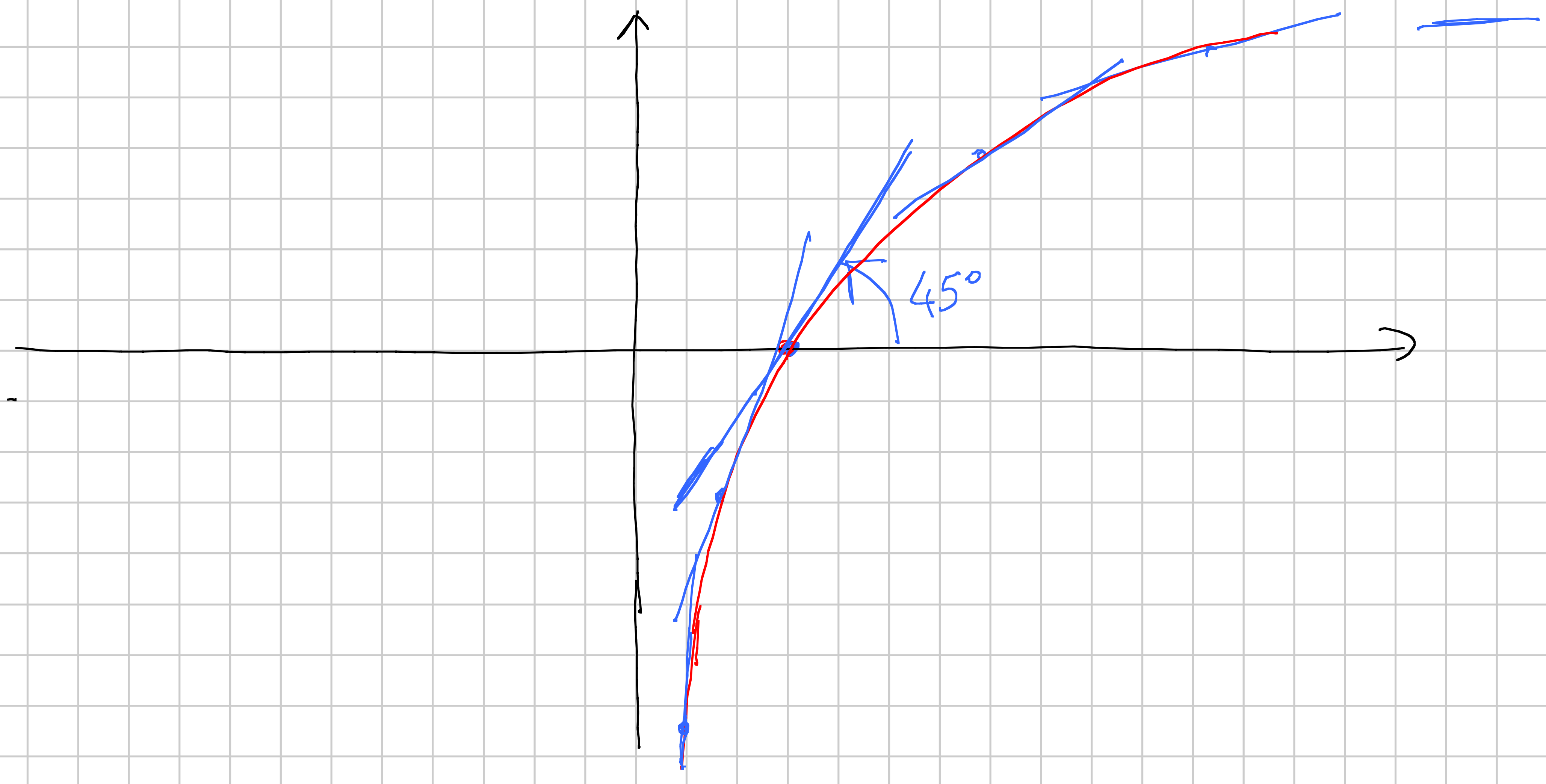
$$\lim_{x \to \pm\infty} f(x) = \pm\infty$$

$f'(x) = 3x^2 - 2x + 1 = 2x^2 + (x-1)^2$ sempre positivo



Regole di derivazione : D = derivata

- $D(x^{\alpha}) = \alpha \cdot x^{\alpha - 1} \quad \forall \alpha \in \mathbb{R} \quad (x > 0)$
- $D(e^{x}) = e^{x}$
- $D(\ln(x)) = \frac{1}{x}$


45°

$$D(\sin(x)) = \cos(x)$$

$$D(\cos(x)) = -\sin(x)$$

- $D(f+g) = D(f) + D(g)$
- $D(k \cdot f) = k \cdot D(f)$
- $D(f \cdot g) = D(f) \cdot g + f \cdot D(g)$

Es: $f(x) = 3x^2 \cdot \sin(x)$

$$f'(x) = 6x \cdot \sin(x) + 3x^2 \cdot \cos(x)$$

Spiegazione: $\dfrac{f(x+h)\cdot g(x+h) - f(x)\cdot g(x)}{h} =$

$$= \frac{f(x+h)\cdot g(x+h) - g(x+h)\cdot f(x) + g(x+h)\cdot f(x) - f(x) g(x)}{h}$$

$$= \frac{f(x+h) - f(x)}{h}\cdot g(x+h) + \frac{g(x+h) - g(x)}{h}\cdot f(x)$$

$$f'(x) \cdot g(x) + g'(x) \cdot f(x)$$

•) $$D\left(\frac{f}{g}\right) = \frac{f' \cdot g - f \cdot g'}{g^2}$$

Es: $$f(x) = \frac{\sin(x)}{3x^2+4x-1}$$

$$f'(x) = \frac{\cos(x) \cdot (3x^2+4x-1) - \sin(x) \cdot (6x+4)}{(3x^2+4x-1)^2}$$

• $D(f \circ g) = ((Df) \circ g) \cdot (Dg)$

<u>Es</u>: $f(x) = \cos(4x^7 - \ln(x))$

$f' = -\sin(4x^7 - \ln(x)) \cdot \left(28x^6 - \frac{1}{x}\right)$

—— ∘ ——

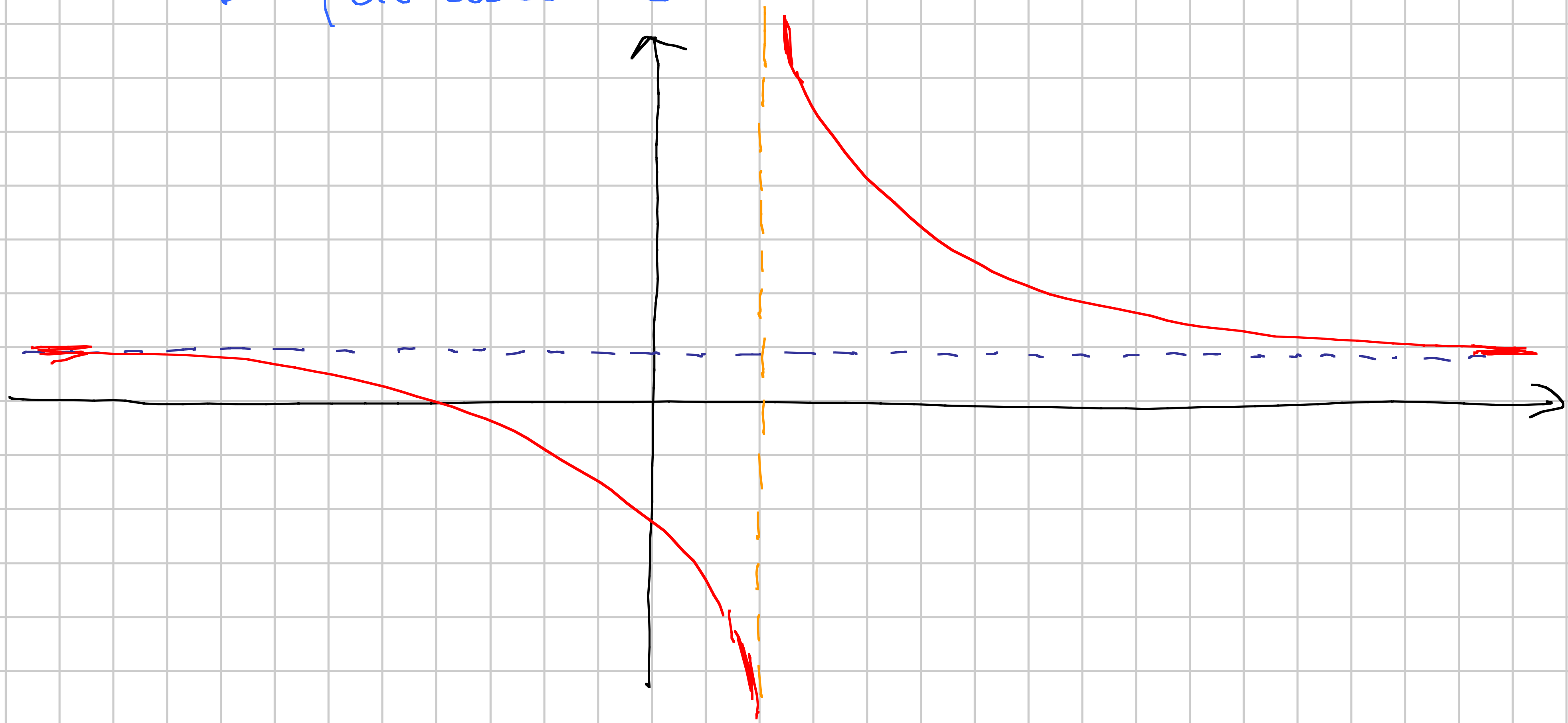
Studio di alcuni quozienti di polinomi:

$$\boxed{f(x) = \frac{x-1}{x-2}}$$

- definita per $x \neq 2$
- $\lim_{x \to \pm\infty} f(x) = 1$
- $\lim_{x \to 2^+} f(x) = +\infty$ $\qquad \lim_{x \to 2^-} f(x) = -\infty$

$$f'(x) = \frac{1 \cdot (x-2) - 1 \cdot (x-1)}{(x-2)^2} = -\frac{1}{(x-2)^2} \text{ sempre } < 0$$

$\Rightarrow$ f decrescente



•) $f(x) = \dfrac{x^2-1}{x-2}$

Definita per $x \neq 2$

$$\lim_{x \to -\infty} f(x) = -\infty \qquad \lim_{x \to +\infty} f(x) = +\infty$$

Oss: $\displaystyle \lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{x^2-1}{x^2-2x} = 1$

dunque $f(x) \simeq x$ vicino a $\pm\infty$

$$\lim_{x \to 2^+} f(x) = +\infty \qquad \lim_{x \to 2^-} f(x) = -\infty$$

$$f'(x) = \frac{2x(x-2) - (x^2-1)\cdot 1}{(x-2)^2} = \frac{2x^2 - 4x - x^2 + 1}{(x-2)^2}$$

$$= \frac{x^2 - 4x + 1}{(x-2)^2}$$

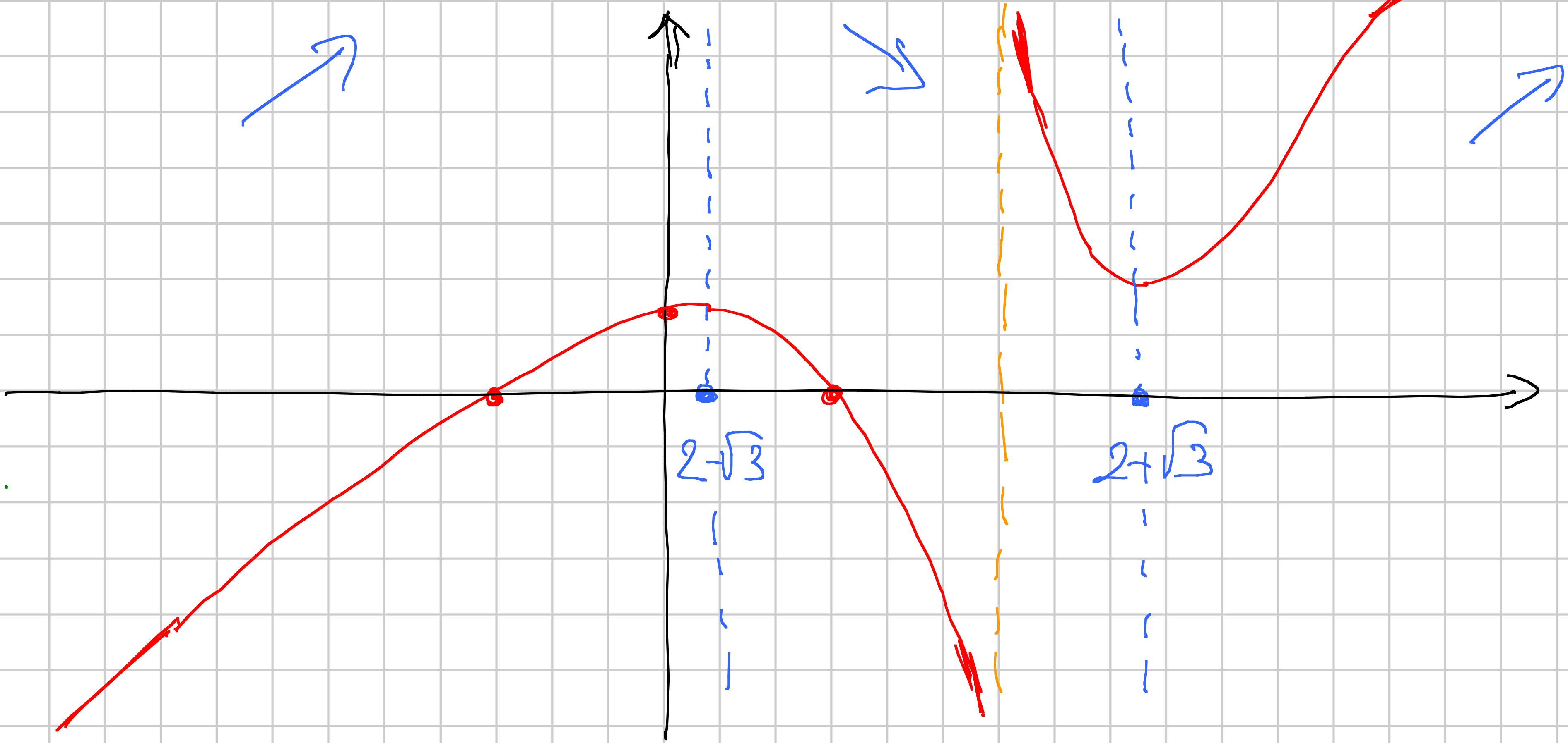
nulla per $x = 2 \pm \sqrt{3}$

positiva per $x < 2 - \sqrt{3}$ e $x > 2 + \sqrt{3}$

negativa per $2 - \sqrt{3} < x < 2 + \sqrt{3}$

Oss: $f(0) = 1/2$ $\quad f(\pm 1) = 0$

$2-\sqrt{3} \simeq 0.28\ldots$ $\quad$ $2+\sqrt{3} \simeq 3.72\ldots$



o) $f(x) = \dfrac{1}{(x-1)(x+2)}$

Def per $x \neq 1$, $x \neq -2$.

$$\lim_{x \to \pm\infty} f(x) = 0$$

$$\lim_{x \to -2^-} f(x) = +\infty$$

$$\lim_{x \to -2^+} f(x) = -\infty$$

$$\lim_{x \to 1^-} f(x) = -\infty$$

$$f(x) = \frac{1}{x^2 + x - 2}$$

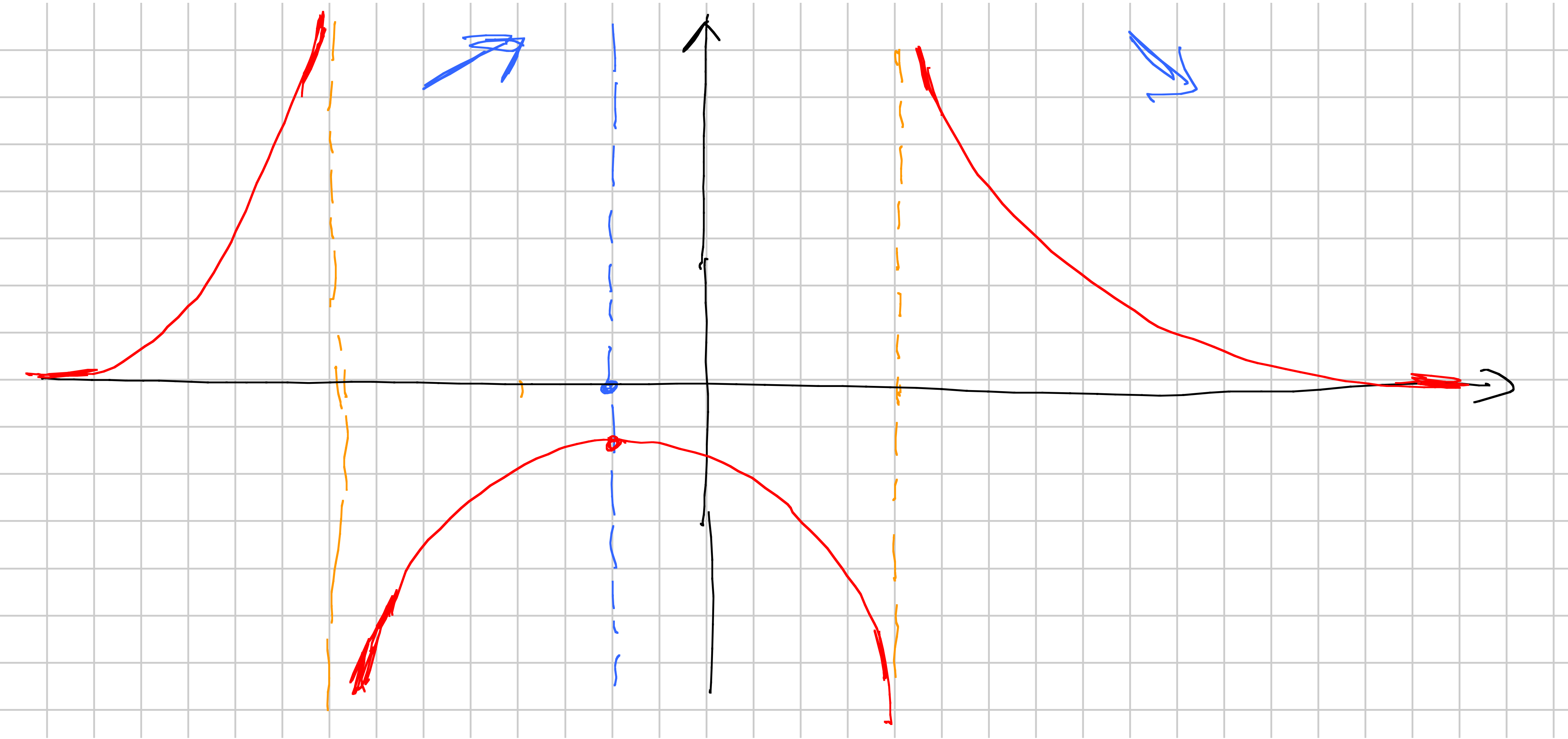
$$\lim_{x \to 1^+} f(x) = +\infty$$

$$f'(x) = -\frac{2x+1}{(x^2+x-2)^2}$$

nulla in $x = -1/2$

positiva per $x < -1/2$

negativa per $x > -1/2$



Ese exp/log

② $5^{x+1} \cdot 25^{x} = \left(\frac{1}{49}\right)^{2x} \cdot 7^{3x+1}$

$5^{x+1} \cdot 5^{2x} = 7^{-4x} \cdot 7^{3x+1}$

$5^{3x+1} = 7^{1-x}$

$$3x+1 = (1-x)\log_5 7$$

$$x(3+\log_5 7) = 1+\log_5 7$$

$$x = \frac{1+\log_5 7}{3+\log_5 7}$$

③ $$10^{2x} + 4\cdot 10^{x} - 5 = 0$$

$$t = 10^{x} \qquad t^2 + 4t - 5 = 0$$

$$(t+5)(t-1)=0$$

$t=-5$ o $t=1$

$10^x=-5$ mai

$10^x=1$

$x=0$

$$\boxed{x=0}$$

(6) Dire quante soluz. ha al variare di $a \in \mathbb{R}$

$$2a^2(1-2^{x-1})+(\sqrt{2})^{6x} = 2^{x+1}(a-2)+4(4^x-a)$$

$$\underline{2a^2} - \underline{a^2 \cdot 2^x} + \underline{2^{3x}} - \underline{2a \cdot 2^x} + \underline{4 \cdot 2^x} - \underline{4 \cdot 2^{2x}} + \underline{4a} = 0$$

$$(2^x)^3 - 4 \cdot (2^x)^2 + (4-2a-a^2)2^x + 4a + 2a^2 = 0$$

$t = 2^x$

$$t^3-4t^2+(4-2a-a^2)t+4a+2a^2=0$$

Osservo che $t=2$ è una soluz:

$$8-16+8-4a-2a^2+4a+2a^2 \quad \checkmark$$

$$(t-2)(t^2-2t-2a-a^2)=0$$

$$t_1=2 \qquad t_{2,3}=1\pm\sqrt{1+2a+a^2}$$

$$=1\pm(1+a)=\begin{cases} a+2 \\ -a \end{cases}$$

Quante soluz. in x ? $\quad (2^x = t)$

Devo vedere se $x_1 = 1 \quad x_2 = \log_2(a+2) \quad x_3 = \log_2(-a)$
sono accettabili e se sono distinte :

x_2 accettabile per $\quad a > -2$
x_3 accettabile per $\quad a < 0$

$x_1 = x_2$ se $\quad 2 = a+2$ cioè $\quad a = 0$

$x_1 = x_3$ se $2 = -a$ cioè $a = -2$

$x_2 = x_3$ se $a + 2 = -a$ cioè $a = -1$

CONCLUSIONE:

$a < -2$	x_1, x_3	2 soluz
$a = -2$	x_1	1 soluz
$-2 < a < -1$	x_1, x_2, x_3	3 soluz
$a = -1$	$x_1, x_2 = x_3$	2 soluz
$-1 < a < 0$	x_1, x_2, x_3	3 soluz

$a = 0$ $\quad x_1 \quad$ 1 soluz

$a > 0$ $\quad x_1, x_2 \quad$ 2 soluz

13 $\quad \left(\left(\frac{1}{3} \right)^x - 1 \right)^2 \leq \frac{1}{9^x}$

$$\cancel{\left(\frac{1}{3} \right)^{2x}} - 2 \left(\frac{1}{3} \right)^x + 1 \leq \cancel{\left(\frac{1}{3} \right)^{2x}}$$

$$1 \leq 2 \cdot \left(\frac{1}{3} \right)^x$$

$$3^x \leq 2$$

$$x \leq \log_3(2)$$

15 $$(9^x - 1)(3^{x+1} + 2) \leq 88$$

$$(3^{2x} - 1)(3 \cdot 3^x + 2) - 88 \leq 0$$

$$3 \cdot 3^{3x} + 2 \cdot 3^{2x} - 3 \cdot 3^x - 90 \leq 0$$

$t = 3^x$

$$3t^3 + 2t^2 - 3t - 90 \leq 0$$

cerco soluz. equaz. assoc. tentativi ....

$t = 3$: $81 + 18 - 9 - 90$ ✓

$$(t-3)(\underbrace{3t^2 + 11 \cdot t + 30}) \leq 0$$

$$\Delta = 121 - 360 < 0$$

$t \leq 3$

$\downarrow$

$3^x \leq 3$

$$\boxed{x \leq 1}$$