Geometrie 24/5/18

$\boxed{21/7/15}$ 1

$$\frac{1}{48}\begin{pmatrix} -8 & 45 & 30 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}\begin{pmatrix} 2 \\ -6 \\ 3 \end{pmatrix} = \frac{1}{44}\begin{pmatrix} -186 \\ \cdot \\ \cdot \end{pmatrix} = -2\begin{pmatrix} 2 \\ -6 \\ 3 \end{pmatrix}$$

$$\lambda = -2$$

$$\begin{array}{r} 16 \\ 270 \\ \hline 286 \end{array} - 90 = 186$$

$$\det(A) = -2 \qquad \operatorname{tr}(A) = \frac{1}{49}(-98) = -2$$

$$-2 + \lambda_2 + \lambda_3 = -2 \qquad \lambda_2 + \lambda_3 = 0$$

$$-2 \cdot \lambda_2 \cdot \lambda_3 = -2 \qquad \lambda_2 \cdot \lambda_3 = 1$$

$$\lambda_{2,3} = \pm i$$

(B) $W = v_1^{\perp}$; provare che $A(W) \subset W$.

$v_1 = \begin{pmatrix} 2 \\ -6 \\ 3 \end{pmatrix}$ $\qquad W = \mathrm{Span}\left(\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right)$

Ora calcolo $A \cdot \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \ldots$ e verifico che $\in \mathrm{Span}\left(\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right)$

(C) Provare che $v_2 = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} \in W$ e completarlo a $(v_2, v_3) = \mathcal{B}$ base ortog. di W, con $\|v_3\| = \|v_2\|$

$$(6,3,2)\begin{pmatrix}2\\-6\\3\end{pmatrix} = 12-18+6=0 \checkmark$$

$$\begin{pmatrix}6\\3\\2\end{pmatrix} \wedge \begin{pmatrix}2\\-6\\3\end{pmatrix} = \begin{pmatrix}21\\-14\\-42\end{pmatrix} \rightsquigarrow \begin{pmatrix}-3\\2\\6\end{pmatrix} = v_3$$

(D) Posto $\rho = f\big|_W^W$ trovare $[\rho]_{\mathcal{B}}^{\mathcal{B}}$

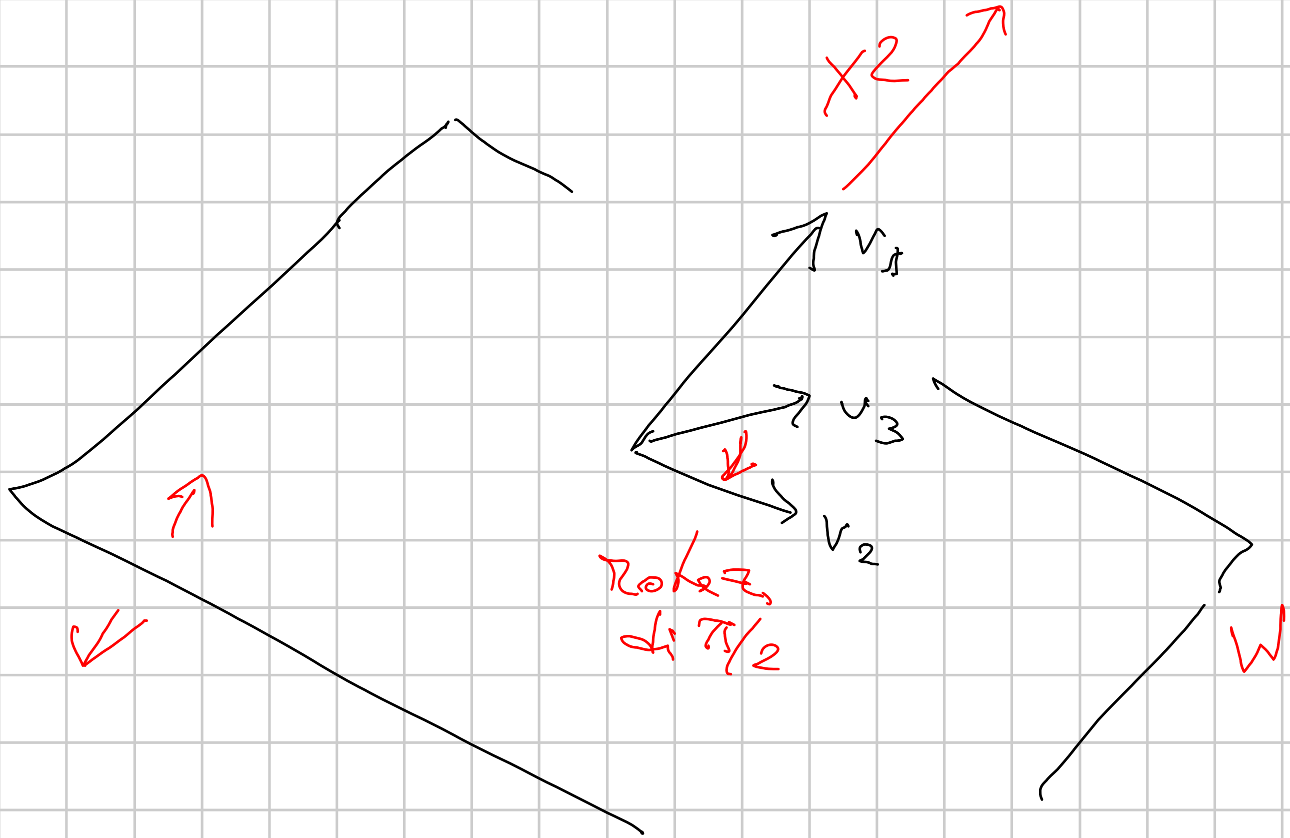
$$\frac{1}{49}\begin{pmatrix}-8 & 45 & 30\\ \cdot & \cdot & \cdot\\ \cdot & \cdot & \cdot\end{pmatrix}\underbrace{\begin{pmatrix}6\\3\\2\end{pmatrix}}_{v_2} = \begin{pmatrix}3\\-2\\-6\end{pmatrix} = -v_3$$

-48

$$\begin{array}{r} 135 \\ 60 \\ \hline 195 \\ 48 \\ \hline 147 \end{array}$$

$$\rightsquigarrow \quad [s]_{\beta}^{\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(E) Disegnare azione A:



② $\alpha(t) = \begin{pmatrix} t^2 - 4t - \log(2t+1) \\ 1 + 2t - 2t^2 \end{pmatrix}$

(A) $D = \left(-\frac{1}{2}, +\infty\right)$

$\alpha'(t) = \begin{pmatrix} 2t - 4 - \frac{2}{2t+1} \\ 2 - 4t \end{pmatrix} \longrightarrow \begin{pmatrix} -6 \\ 2 \end{pmatrix}$

$Y'(t) = 0 \iff t = \frac{1}{2}$; $X'\left(\frac{1}{2}\right) = 1 - 4 - 1 \neq 0$

(B) $K(0)$

$$\alpha''(t) = \begin{pmatrix} 2 + \frac{4}{(2t+1)^2} \\ -4 \end{pmatrix} \longrightarrow \begin{pmatrix} 6 \\ -4 \end{pmatrix}$$

$$K(0) = \frac{\det(\alpha' \ \alpha'')}{\|\alpha'\|^3} = \frac{\det\begin{pmatrix} -6 & 6 \\ 2 & -4 \end{pmatrix}}{(36+4)^{3/2}} = \dots$$

(C) Trovare segno di $K(t)$:

$$\det\begin{pmatrix} 2t-4-\frac{2}{2t+1} & 2+\frac{4}{(2t+1)^2} \\ 2-4t & -4 \end{pmatrix}$$

$$\det\begin{pmatrix} t-2-\frac{1}{2t+1} & 1+\frac{2}{(2t+1)^2} \\ 1-2t & -2 \end{pmatrix}$$

$$(-2t+4)(2t+1)^2+2(2t+1)-(2t+1)^2-2$$
$$+2t(2t+1)^2+4t$$

$$(2t+1)^2\left(-\cancel{2t}+4-1+\cancel{2t}\right)+4t+\cancel{2}-\cancel{2}+4t)$$

$$12t^2+12t+3+8t$$

$$12t^2+20t+3$$

$$\frac{-10\pm\sqrt{100-36}}{12}=\frac{-10\pm 8}{12}=\begin{cases}-3/2\\ -1/6\end{cases}$$

$x(t)<0$ per $-3/2<t<-1/6$ . . . . . .

(D) $\beta = \alpha|_{[0,2]}$

$$\int_\beta e^{y-x}(-dx+dy) = d\left(e^{y-x}\right)$$

$$= \int_0^2 e^{t^2-4t-\log(2t+1)\ \dots}\left(\dots\dots\right)$$

complicato

$$= e^{y-x}\Big|_{\beta(0)}^{\beta(2)} = e^{y-x}\Big|_{(0,1)}^{(-4+\log 5,\,-3)}$$

$$= e^{-3+4-\log 5} - e^{1} = \frac{1}{5}$$

(E) $$\int_\beta \left(y+\frac{1}{2}\right)dx = \int_0^2 \left(\frac{3}{2}+2t-2t^2\right)\left(2t-4-\frac{2}{2t+1}\right)dt$$

$$= \dots$$

$\boxed{15/6/17}$ (1)

$$X_t = \mathrm{Span}\left(\begin{pmatrix} -t \\ t-4 \\ t+10 \end{pmatrix} \begin{pmatrix} t-1 \\ 1-4t \\ 5t-2 \end{pmatrix} \right)$$

(A) Trovare $\bar{t}$ t.c. $X_{\bar{t}}$ non è piano

$$-t + 4t^2$$
$$-4 + 5t - t^2 = 0$$

$$3t^2 + 4t - 4 = 0$$
$$(3t-2)(t+2) = 0$$

$t = -2$ $\qquad \begin{pmatrix} 2 \\ -6 \\ 8 \end{pmatrix} \begin{pmatrix} -3 \\ 9 \\ -12 \end{pmatrix} \to \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$ $\underline{\text{OK}}$

$t = 2/3$ .... $\underline{\text{NO}}$

(B) Matr. proiez. ortog. su $X_{\tilde{t}}$

$$\frac{x-3y+4z}{1+9+16}\begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} = \underbrace{\frac{1}{26}\begin{pmatrix} 1 & -3 & 4 \\ -3 & 9 & -12 \\ 4 & -12 & 16 \end{pmatrix}}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

(C) Trovare matrici A, B delle proiez. su X_1 e X_0.

$$X_1 = \mathrm{Span}\begin{pmatrix} 1 \\ 3 \\ -11 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 3 \\ -11 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 8 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \frac{8x+y+z}{64+1+1}\begin{pmatrix} 8 \\ 1 \\ 1 \end{pmatrix} = \frac{1}{66}\begin{pmatrix} 2 & -8 & -8 \\ -8 & 65 & -1 \\ -8 & -1 & 65 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$x_0 = \ldots$

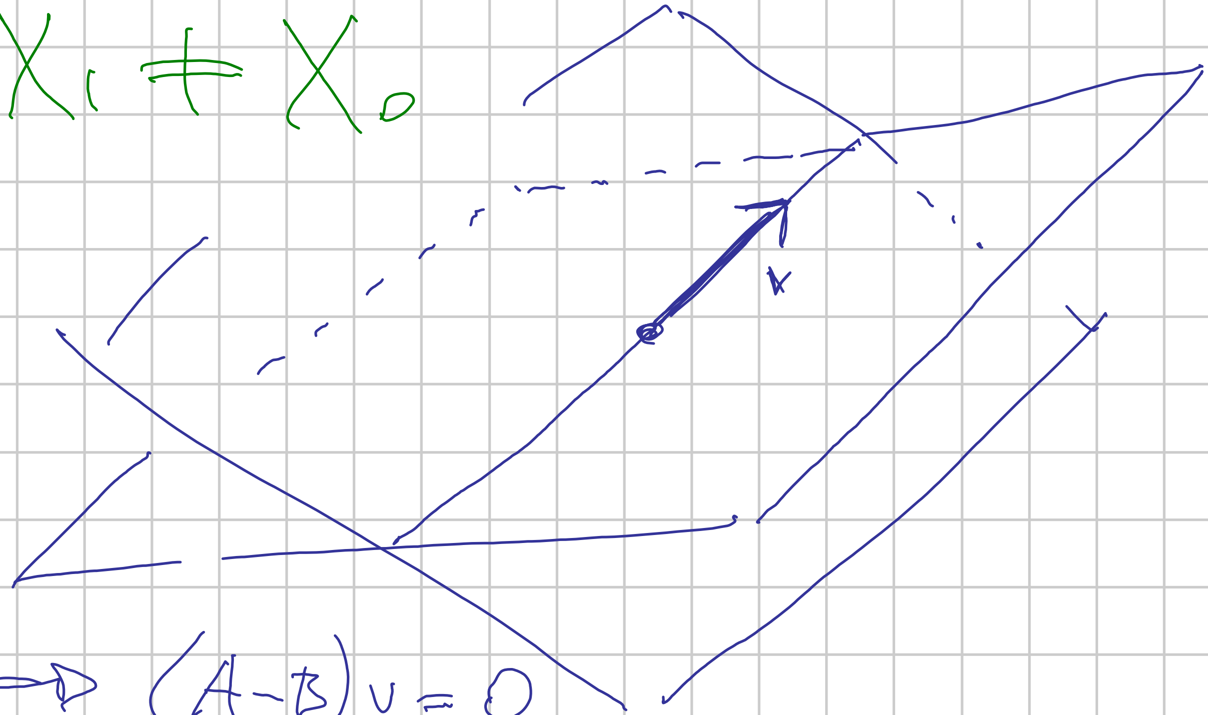
(D) Provare che $A + kB$ è diag. $\forall k \in \mathbb{R}$

A, B simm $\Rightarrow$ $A + kB$ anche $\Rightarrow$ diago.

(E) Provare che $A - B$ ha autoval 0.

Caso 1 : $X_1 = X_0 \Longrightarrow A - B = 0$

Caso 2 : $X_1 \neq X_0$



$Av = Bv = v \Longrightarrow (A - B)v = 0$

②

$$\det\begin{pmatrix} 2-i & 1 & 3 \\ 2+2i & 2i & 3i \\ z-i & 1+iz & z \end{pmatrix}$$

$\mathrm{I} - (2-i)\cdot\mathrm{II}$ $\qquad\qquad$ $\mathrm{III} - 3\cdot\mathrm{II}$

$2+2i-(2-i)2i = 2+2i-4i-2 = -2i$

$(z-i)-(2-i)(1+iz) = z-2iz-z-i-2+i = -2-2iz$

$$= \det \begin{pmatrix} 0 & 1 & 0 \\ -2i & 2i & -3i \\ -2-2iz & 1+iz & (1-3i)z-3 \end{pmatrix}$$

$$= -2i \det \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & -3 \\ 1+iz & 1+iz & (1-3i)z-3 \end{pmatrix}$$

$$= 2i\left((1-3i)z - 3 + 3(1+iz)\right) = 2iz$$

B Sapendo che $p_A(1) = (z-1)(1-i)$ troviamo $p_A(t)$

$$p_A(t) = t^3 - \mathrm{tr}(A)\cdot t^2 + c\cdot t - \det(A)$$

sempre

3 dispari

$$1 - (2+i+z)\cdot 1 + c\cdot 1 - 2iz = z - iz - 1 + i$$

$$c = \dots$$

(C) Sapendo che A ha sempre autoval $\lambda_1 = 2$
trovare altri:

$$2 + \lambda_2 + \lambda_3 = 2 + i + z$$

$$2 \cdot \lambda_2 \cdot \lambda_3 = 2 i z$$

$$\Rightarrow \quad \lambda_2 = i \qquad \lambda_3 = z$$

(D) Stabilire per quali z è diag.

$\lambda_1 = 2 \qquad \lambda_2 = i \qquad \lambda_3 = 2$

Si se $z \neq 2, i$

$$z = 2 \qquad \begin{pmatrix} 2-i & 1 & 3 \\ 2+2i & 2i & 3i \\ 2-i & 1+2i & 2 \end{pmatrix} \rightsquigarrow \begin{pmatrix} -i & 1 & 3 \\ 2+2i & \boxed{\begin{matrix} 2i-2 & 3i \\ 1+2i & 0 \end{matrix}} \\ 2-i & & \end{pmatrix}$$

rank $= 2$

$\Rightarrow$ m.g.$(2) = 1$ $\qquad$ No

$$z = i \qquad \begin{pmatrix} 2-i & 1 & 3 \\ 2+2i & 2i & 3i \\ 0 & 0 & i \end{pmatrix} \rightsquigarrow \begin{pmatrix} 2-2i & 1 & 3 \\ 2+2i & i & 3i \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathrm{II} = i \cdot \mathrm{I} \Longrightarrow \text{rank} = 1$$

$$\Longrightarrow m.g.(i) = 2$$

$\boxed{27/6/17}$ ① Ⓐ $A_t = \begin{pmatrix} 4+i & 2-i \\ 2+i & -3+it \end{pmatrix}$

$\exists$ base ortonormale di autovett. $\Longleftrightarrow A_t \cdot A_t^* = A_t^* \cdot A_t$

$$\begin{pmatrix} 4+i & 2-i \\ 2+i & -3+it \end{pmatrix}\begin{pmatrix} 4-i & 2-i \\ 2+i & -3-it \end{pmatrix} = \begin{pmatrix} 4-i & 2-i \\ 2+i & -3-it \end{pmatrix}\begin{pmatrix} 4+i & 2-i \\ 2+i & -3+it \end{pmatrix}$$

$$|4+i|^2 + |2-i|^2 = |4+i|^2 + |2-i|^2 \quad \checkmark$$

$$\cancel{(4+i)(2-i)}+\cancel{(2-i)}(-3-it)=(4-i)\cancel{(2-i)}+\cancel{(2-i)}(-3+it)$$

$$\cancel{4}+i\cancel{-3}-it=\cancel{4}-i\cancel{-3}+it \qquad 2i=2it$$

$$\boxed{t=1}$$

(B) Trovare autovel A_1

$$A_1=\begin{pmatrix}4+i & 2-i\\ 2+i & -3+i\end{pmatrix}=\underbrace{\begin{pmatrix}4 & 2-i\\ 2+i & -3\end{pmatrix}}_{\text{hermitiana}}+i\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

Oss: gli autoval di $A + k \cdot I_n$ sono gli autoval di $A + k$ (con le stesse m.a. e m.g.)

$$A \cdot v = \lambda \cdot v \qquad (A + kI_n)v = (\lambda + k) \cdot v$$

$tr = 1$

$det = -12 - 5$

$$\Rightarrow \lambda + \frac{1 \pm \sqrt{1+68}}{2}$$

(C) $B_z = \frac{1}{2}(A_z + A_z^*)$: $\exists$ sempre base ortonormale di autovettori

(Qualunque sia A_z)

$B_z^* = \frac{1}{2}(A_z^* + A_z) \Rightarrow B_z$ hermitiana

(D) Calcolare autoval. di B_{-2i} e base ....

$$B_{-2i} = \frac{1}{2}\left(\begin{pmatrix} 4+i & 2-i \\ 2+i & -1 \end{pmatrix} + \begin{pmatrix} 4-i & 2-i \\ 2+i & -1 \end{pmatrix}\right)$$

$$= \begin{pmatrix} 4 & 2-i \\ 2+i & -1 \end{pmatrix}$$

$$\operatorname{tr} = 3 \qquad \det = -4-5 = -9$$

$$\lambda_{1,2} = \frac{3 \pm \sqrt{9+36}}{2} = \frac{3}{2}\left(1 \pm \sqrt{5}\right)$$

$$v_{1,2} = \begin{pmatrix} x \\ y \end{pmatrix} \; : \qquad (2+i)x - y = \frac{3}{2}\left(1 \pm \sqrt{5}\right)y$$

$$2(2+i)x = \left(5 \pm 3\sqrt{5}\right)y$$

$$v_{1,2} = \begin{pmatrix} 5 \pm 3\sqrt{5} \\ 2(2+i) \end{pmatrix}$$

devono già essere ortog tra loro :

$$\left(5+3\sqrt{5}\,,\; 4-2i\right)\begin{pmatrix} 5-3\sqrt{5} \\ 4+2i \end{pmatrix}$$

$= 25 - 45 + 16 + 4 = 0$ ✓

$\boxed{18/7/17}$ ②

$$\alpha(s) = \begin{pmatrix} s + \cos(2s) \\ s^3 - s^2 + s \\ s^2 + \sin(3s) \end{pmatrix}$$

Ⓐ Provare che è semplice e regolare

α semplice significa iniettiva, cioè che se $\alpha(s_1) = \alpha(s_2)$ si ha per forza $s_1 = s_2$.

$$\alpha'(t) = \begin{pmatrix} 1-2\sin(2t) \\ 3t^2-2t+1 \\ 2t+3\cos(3t) \end{pmatrix}$$

$$Y'(t) = 2t^2 + (t-1)^2 > 0$$

$\Longrightarrow$ regolare e semplice

($Y' > 0 \Longrightarrow Y$ crescente

$\Longrightarrow Y$ iniettiva $\Longrightarrow \alpha$ anche

$$\alpha(t) = \begin{pmatrix} t^2 \\ \sin(t) \end{pmatrix}$$

$$\alpha(t_1) = \alpha(t_2)$$

$$\Longleftrightarrow \begin{cases} t_1^2 = t_2^2 \\ \sin(t_1) = \sin(t_2) \end{cases}$$

$$\Longleftrightarrow \begin{cases} t_2 = t_1 \\ \end{cases} \quad o \quad \begin{cases} t_2 = -t_1 \\ \sin(t_1) = -\sin(t_1) \end{cases}$$

$$\Longleftrightarrow \begin{cases} t_2 = t_1 \\ \end{cases} \quad o \quad \begin{cases} t_2 = -t_1 \\ \sin(t_1) = 0 \end{cases}$$

$t_1 = \pi$, $t_2 = -\pi$ non semplice.

(B) + (C) $\quad t, n, b, K, \tau$ in $s = 0$

$\alpha|_{(-\pi,\pi)}$ è semplice

$$\alpha'(s) = \begin{pmatrix} 1 - 2\sin(2s) \\ 3s^2 - 2s + 1 \\ 2s + 3\cos(3s) \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$$

$$\alpha''(s) = \begin{pmatrix} -4\cos(2s) \\ 6s - 2 \\ 2 - 9\sin(3s) \end{pmatrix} \longrightarrow \begin{pmatrix} -4 \\ -2 \\ 2 \end{pmatrix}$$

noto che è $\perp \alpha'$

$$\alpha'''(s) = \begin{pmatrix} -8\sin(2s) \\ 6 \\ -27\cos(3s) \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ 6 \\ -27 \end{pmatrix}$$

$$t = \frac{1}{\sqrt{11}} \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \qquad n = \frac{1}{\sqrt{6}} \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$$

$$b = t \wedge n = \frac{1}{\sqrt{66}} \begin{pmatrix} 4 \\ -7 \\ 1 \end{pmatrix} \qquad \alpha' \wedge \alpha'' = 2 \cdot \begin{pmatrix} 4 \\ -7 \\ 1 \end{pmatrix}$$

$$\kappa = \frac{\| \alpha' \wedge \alpha'' \|}{\| \alpha' \|^3} = \frac{2\sqrt{66}}{11\sqrt{11}} = \frac{2}{11}\sqrt{6}$$

$$\tau = \frac{\langle \alpha' \wedge \alpha'' \mid \alpha''' \rangle}{\| \alpha' \wedge \alpha'' \|^2} = \frac{2 \left\langle \begin{pmatrix} 4 \\ -7 \\ 1 \end{pmatrix} \middle| \begin{pmatrix} 0 \\ 6 \\ -27 \end{pmatrix} \right\rangle}{4 \cdot 66} = \frac{2 \cdot (-42 - 27)}{4 \cdot 66} = \cdots$$