Geometrie 23/5/18

$\boxed{25}$ 1 $\begin{pmatrix} 26 & -36 \\ 18 & -25 \end{pmatrix}$

$\lambda_1 + \lambda_2 = tr = 1$

$\lambda_1 \cdot \lambda_2 = \det = -26 \cdot 25 + 18 \cdot 36 = -2$

$\lambda_1 = 2 \qquad \lambda_2 = -1$

$v_1 = \begin{pmatrix} x \\ y \end{pmatrix} \quad \begin{cases} 26x - 36y = 2x \\ 18x - 25y = 2y \end{cases} \quad \begin{cases} 24x - 36y = 0 \\ 18x - 27y = 0 \end{cases}$

$v_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}; \quad v_2 = \begin{pmatrix} x \\ y \end{pmatrix} \quad 26x - 36y = -x \quad v_2 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

② Esibire $v \in \mathbb{C}^2$, $\|v\| = 1$, $v \perp \begin{pmatrix} 2-i \\ 1+i \end{pmatrix}$, $v_2 \in \mathbb{R}_-$

$v = \begin{pmatrix} z \\ 1 \end{pmatrix}$: $(2+i) \cdot z + (1-i) = 0$

$$z = -\frac{1-i}{2+i} = -\frac{(1-i)(2-i)}{5} = -\frac{1-3i}{5}$$

$$v = \frac{1}{\sqrt{35}} \begin{pmatrix} 1-3i \\ -5 \end{pmatrix}$$

(3) Pti a ∞ $\{(t, \cos(t), 1-2t) : t \in \mathbb{R}^3\} \subset \mathbb{R}^3$

$[1:0:-2:0]$ o $[1:0:-2]$

(4) Per quali k la $x^2+2xy+ky^2-4x+2y+5=0$ è ellisse?

$$\begin{pmatrix} 1 & 1 & -2 \\ 1 & k & 1 \\ -2 & 1 & 5 \end{pmatrix}$$

$d_1 > 0$

ellisse $\iff$ $d_2 > 0$, $d_3 < 0$

$d_2 = k-1$; $d_3 = 5k - 2 - 2 - 4k - 5 - 1$
$= k - 10$

$\Rightarrow 1 < k < 10$

⑤ Tipo affine di $5y^2 - 8z^2 - 2xy - 6xz - 4yz + 2x - 2y = 0$

$$\left(\begin{array}{ccc|c} 0 & -1 & -3 & 1 \\ -1 & 5 & -2 & -1 \\ -3 & -2 & -8 & 0 \\ \hline 1 & -1 & 0 & 0 \end{array}\right)$$

$d_2 < 0$

$d_3 = -6 - 6 - 45 + 8 < 0$

$$d_4 = \det\left(\begin{array}{c|ccc} 0 & -1 & +3 & 1 \\ -1 & 4 & +2 & -1 \\ -3 & -5 & +8 & 0 \\ \hline 1 & 0 & 0 & 0 \end{array}\right) = 15 + 32 + 10 - 12 \cdot 8$$

$$= 57 - 12 \cdot 8 < 0$$

$(Q)\ A$: $+ \ - \ + \ +$

$$y^2 = 1 + x^2 + z^2 \qquad \text{iperb. 2 falde (ellittico)}$$

⑥ $f(x,y) = (1-y)^3 \cdot (1+x)^2 - \cos(x+y) - 2x + 3y$

ha max/min loc. in $(0,0)$?

$$\frac{\partial f}{\partial x} = 2(1-y)^3(1+x) + \sin(x+y) - 2 \quad ; \quad \frac{\partial f}{\partial x}(0,0) = 0$$

$$\frac{\partial f}{\partial y} = -3(1-y)^2(1+x)^2 + \sin(x+y) + 3 \quad ; \quad \frac{\partial f}{\partial y}(0,0) = 0$$

$$\frac{\partial^2 f}{\partial x^2} = 2(1-y)^3 + \cos(x+y) \qquad 3$$

$$\frac{\partial^2 f}{\partial x \partial y} = -6(1-y)^2(1+x) + \cos(x+y) \qquad -5$$

$$\frac{\partial^2 f}{\partial y^2} = 6(1-y)(1+x)^2 + \cos(x+y) \qquad 7$$

$$Hf(0) = \begin{pmatrix} 3 & -5 \\ -5 & 7 \end{pmatrix} \qquad \det = -4 < 0$$

autoval $+/-$ $\Longrightarrow$ né max né min loc

⑦ $\omega = \left(kx^2y + e^x\right)dx + \left(x^3 + \sin(y)\right)dy$

Per quali k si ha $\int_\alpha \omega = 0$ $\forall \alpha$ chiusa in $\mathbb{R}^2$.

ω definita su $\mathbb{R}^2$

$$\int_\alpha \omega = 0 \quad \forall \alpha \text{ chiusa} \iff \omega \text{ esatta su } \underbrace{\mathbb{R}^2}_{\text{senza buchi}}$$

$$\iff \omega \text{ chiusa su } \mathbb{R}^2$$

$$\iff \frac{\partial}{\partial y}\left(kx^2y + e^x\right) = \frac{\partial}{\partial x}\left(x^3 + \sin(y)\right) \text{ su } \mathbb{R}^2$$

$$\iff kx^2 = 3x^2 \quad \forall x, y \iff k = 3$$

$\boxed{27}$

(1) per quali k è diago le

$$\begin{pmatrix} 21k^2+12k-8 & -28k^2-16k+12 \\ 15k^2+9k-6 & -20k^2-12k+9 \end{pmatrix} ?$$

$M_{2\times 2}$

$\rightarrow \lambda_1, \lambda_2 \notin \mathbb{R}$ non diago su $\mathbb{R}$

$\rightarrow \lambda_1 \neq \lambda_2$ sì

$\rightarrow \lambda_1 = \lambda_2$ diagonalizzabile $\Leftrightarrow$ già diagonale

$tr = \lambda_1 + \lambda_2 = k^2+1$

$$\det = \lambda_1 \cdot \lambda_2 = \det \begin{pmatrix} 21k^2+12k-8 & -7k^2-4k+4 \\ 15k^2+9k-6 & -5k^2-3k+3 \end{pmatrix}$$

$$= \det \begin{pmatrix} 4 & -7k^2 - 4k + 4 \\ 3 & -5k^2 - 3k + 3 \end{pmatrix} = k^2,$$

$\lambda_1 = 1 \qquad \lambda_2 = k^2$

Se $k \neq \pm 1$ distinti $\Rightarrow$ è diago... zz ...

$k = 1$: $\begin{pmatrix} 25 & -32 \\ 18 & -23 \end{pmatrix}$ non diago $\Rightarrow$ non diago ... zz ...

<u>Verifica</u>: $m.g.(1) = 2 - \underbrace{\text{rank}(1 \cdot I_2 - A)}_{=1} \geq 0$

$\Rightarrow \det(A - I_2) = 0$

$$\det\begin{pmatrix} 24 & -32 \\ 18 & -24 \end{pmatrix} = -24^2 + 18 \cdot 32$$

$$3 \cdot 6 \cdot 8 \cdot 4 = 0$$

$$k = -1 \qquad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{diago.}$$

(2) $X : x_3 = 2x_1 + 3x_2$. Esibire base di X che diagonalizza $f : X \to X$

$$f\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ -1 \end{pmatrix}, \quad f\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ -5 \\ -3 \end{pmatrix}$$

$\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$ base di X; $\begin{pmatrix} 4 \\ -3 \\ -1 \end{pmatrix}, \begin{pmatrix} 6 \\ -5 \\ -3 \end{pmatrix} \in X$

$$\underbrace{\qquad\qquad}_{\mathcal{B}} \qquad [f]^{\mathcal{B}}_{\mathcal{B}} = \begin{pmatrix} 4 & 6 \\ -3 & -5 \end{pmatrix}$$

$$\lambda_1 + \lambda_2 = -1 \qquad \lambda_1 = -2 \qquad \lambda_2 = 1$$

$$\lambda_1 \cdot \lambda_2 = -20 + 18 = -2$$

autovett della matrice:

$$4x + 6y = -2x \qquad \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad 4x + 6y = x \qquad \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

autovett. di f

$$1 \cdot \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} - 1 \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

$$2 \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

③ Per quali k la $\begin{pmatrix} \sqrt{\pi} & 1-3k \\ 3-k & k^8 \end{pmatrix}$ ammette base ortonorm. di autovettori?

$1-3k = 3-k \qquad k=-1$

④ Trovare $v \in \mathbb{C}^2$, $\|v\|=1$, $v \perp \begin{pmatrix} 1+i \\ 1-2i \end{pmatrix}$, $v_2 \in i\cdot\mathbb{R}$

$v = \begin{pmatrix} z \\ i \end{pmatrix} \qquad (1-i)z + (1+2i)\cdot 1 = 0$

$$z = \frac{2-i}{1-i} = \frac{(2-i)(1+i)}{2} = \frac{3+i}{2}$$

$$\Rightarrow \pm \frac{1}{\sqrt{14}} \begin{pmatrix} 3+i \\ 2i \end{pmatrix}$$

(5) Tipo affine $5x^2+10y^2+z^2-2xy-2xz+6yz$
$-3x-2y+z+1=0$.

$$\left(\begin{array}{ccc|c} 5 & -1 & -1 & -3/2 \\ -1 & 10 & 3 & -1 \\ -1 & 3 & 1 & 1/2 \\ \hline -3/2 & -1 & 1/2 & 1 \end{array}\right)$$

$d_1 > 0 \qquad d_2 > 0$

$d_3 = 50+3+3$
$-10-1-45 = 0$

$\Rightarrow$ Se $d_4 \neq 0$ è parab. ell. $\qquad d_4 = \dots$

⑥ Interaz. tra $\{[t+1 : 3-t : 2t-1] : t \in \mathbb{R}\} \subset \mathbb{P}^2(\mathbb{R})$ e pti a ∞ di $x^2 - yz + 7x - 4z + 1 = 0$.

Equaz. parte a ∞ : $x^2 - yz = 0$

$$\underline{t^2} + \underline{2t} + \underline{1} - \underline{18t} + \underline{9} + \underline{2t^2} - \underline{t} = 0$$

$$3t^2 - 17t + 10 = 0$$

$$(3x - 2)(x - 5) = 0$$

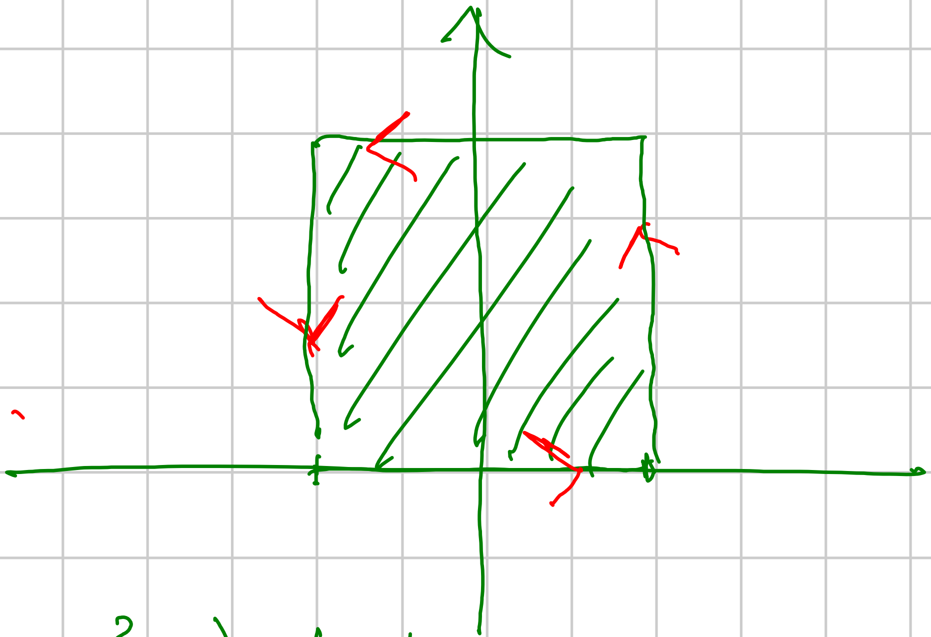
$x = \frac{2}{3}$ $x = 5$ $[5 : 25 : 1]$, $[6 : 4 : 9]$

⑦ $R = [-1,1] \times [0,2]$

$$\omega = \left(5y^2 + e^{\cos^3(x)}\right) dx + \left(\sin(y^2) + x^3 y\right) dy$$

$$\int_{\partial R} \omega = \, ?$$

$$\int_{\partial R} \omega = \int_{\ell_1} + \int_{\ell_2} + \int_{\ell_3} + \int_{\ell_4} \cdots$$



$$\int_{\partial R} \omega = \int_R d\omega = \int_R \left(-10y + 3x^2 y\right) dx\,dy$$

$$= \int_{-1}^{1} \left(\int_{0}^{2} (-10y + 3x^2 y)\, dy \right) dx$$

$$= \int_{-1}^{1} \left(-5y^2 + \frac{3}{2} x^2 y^2 \right) \Big|_{0}^{2} dx = \int_{-1}^{1} (-20 + 6x^2)\, dx$$

$$= -20x + 2x^3 \Big|_{-1}^{1}$$

$$= -40 + 4 = -36$$

$\boxed{29}$ 2 Per quali k è diago

$$\begin{pmatrix} k & 0 & 0 \\ k+1 & -1 & 0 \\ k-1 & k & k^2 \end{pmatrix} \quad ?$$

$\lambda_1 = k \quad \lambda_2 = -1 \quad \lambda_3 = k^2$

Se sono distinti allora è diago.

$\lambda_1 = \lambda_2$ per $k = -1$

$\lambda_1 = \lambda_3$ per $k = 0,\ k = 1$

$\lambda_2 = \lambda_3$ mai

Dunque: per $k \neq 0, \pm 1$ diago-

$$\begin{pmatrix} k & 0 & 0 \\ k+1 & -1 & 0 \\ k-1 & k & k^2 \end{pmatrix}$$

$$k=0 \quad \begin{pmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$\lambda_1 = \lambda_3 = 0$

rank $= 2$

m.g. $= 1$ No

$$k=1 \quad \begin{pmatrix} \overset{0}{\cancel{1}} & 0 & 0 \\ 2 & \overset{-2}{\cancel{-1}} & 0 \\ 0 & 1 & \overset{0}{\cancel{1}} \end{pmatrix}$$

$\lambda_1 = \lambda_3 = 1$

rank $= 2$

m.g. $= 1$ no

$$k=-1 \quad \begin{pmatrix} \overset{0}{\cancel{-1}} & 0 & 0 \\ 0 & \overset{0}{\cancel{-1}} & 0 \\ -2 & -1 & \overset{2}{\cancel{1}} \end{pmatrix}$$

$\lambda_1 = \lambda_2 = -1$

rank $= 1$

m.g. $= 2$ sì

(3) Intersez. in $\mathbb{P}^2(\mathbb{R})$ di

$$\{ [3+2t : t+1 : -2-t] : t \in \mathbb{R} \} \subset \mathbb{P}^2(\mathbb{R})$$

$$\{ [1-t : 2-t : 2t-3] : t \in \mathbb{R} \} \subset \mathbb{P}^2(\mathbb{R})$$

$$\text{rank} \begin{pmatrix} \boxed{3+2t \quad 1-s} \\ t+1 \quad 2-s \\ -2-t \quad 2s-3 \end{pmatrix} = 1$$

$$2st - 3t + 2s - 3 \qquad\qquad st - t + 1 = 0$$

$$-st + 2t - 2s + 4 = 0 \qquad\qquad t = \frac{1}{1-s}$$

$$\begin{aligned} &6s - 9 + 4st - 6t \\ &-2s + 2 - st + t = 0 \end{aligned}$$

$$3st - 5t + 4s - 7 = 0$$

$$(3s - 5) + (1 - s)(4s - 7) = 0$$

$$\therefore \quad 2s^2 - 7s + 6 = 0$$

$$(2s - 3)(s - 2) = 0$$

$$s = \frac{3}{2} \qquad s = 2$$

$$t = -2 \qquad t = -1$$

$$\begin{pmatrix} -1 & -1/2 \\ -1 & 1/2 \\ 0 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & -1 \\ 0 & 0 \\ -1 & 1 \end{pmatrix} \qquad [1 : 0 : -1]$$

$\boxed{30}$ (1) $X: x_1 + x_2 = x_3$

$$f(x) = \begin{pmatrix} 3x_1 + x_2 \\ x_3 - 2x_1 \\ x_1 + x_2 + x_3 \end{pmatrix} \; ; \quad P_f(t) = ?$$

$$x_1 + x_2 + x_3 = (3x_1 + x_2) + (x_3 - 2x_1) \qquad \forall x \in \mathbb{R}^3$$

(bastava per $x \in X$)

$$\mathcal{B} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$f\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ \cancel{2} \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 0 \\ \cancel{1} \end{pmatrix} - 1 \begin{pmatrix} 0 \\ 1 \\ \cancel{1} \end{pmatrix}$$

$$f\begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\2\end{pmatrix} = 1\begin{pmatrix}1\\0\\1\end{pmatrix} + 1\begin{pmatrix}0\\1\\1\end{pmatrix}$$

$$[f]^{\alpha}_{\beta} = \begin{pmatrix}3 & 1\\-1 & 1\end{pmatrix}$$

$$p_f(t) = t^2-4t+4 = (t-2)^2$$

$\boxed{32}$ (1) Proiez ortog. di $\underset{v}{\begin{pmatrix}1\\3\\-2\end{pmatrix}}$ su $\underbrace{\mathrm{Span}\left(\begin{pmatrix}2\\-1\\0\end{pmatrix}\begin{pmatrix}0\\2\\-5\end{pmatrix}\right)}_{W}$.

$$v = P_W(v) + P_{W^\perp}(v)$$

$\Rightarrow P_W(v) = v - P_{W^\perp}(v)$ $\qquad W^\perp = \text{Span}\begin{pmatrix} 5 \\ 10 \\ 4 \end{pmatrix}$

$$= \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} - \frac{5+30-8}{25+100+4}\begin{pmatrix} 5 \\ 10 \\ 4 \end{pmatrix} = \dots$$

(2) Se $A \in M_{n\times n}(\mathbb{C})$ e ${}^tA = A$

posso concludere che A ha autoval reali?

So: $A \in M_{n\times n}(\mathbb{R})$ e ${}^tA = A$ ok

$A \in M_{n\times n}(\mathbb{C})$ e ${}^t\bar{A} = A$ ok

Non deduco nulla

$$A = \begin{pmatrix} z & w \\ w & u \end{pmatrix} \leftarrow \text{analisi generale complicata.}$$

Provo con $A = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

$$p_A(t) = t^2 + 1 \qquad \text{autoval } \pm i \notin \mathbb{R}.$$

$\boxed{\text{no}}$