23/3/2018

9.3.5

$$f(x,y) = y\cos(x-3y^2) - 2x\sin(y+5x^2)$$

Scrivere lo sviluppo di Taylor di f in $(0,0)$ fino all'ordine 2:

$\underline{v} = (x,y)$

$$f(x,y) = f(0,0) + \frac{\partial f}{\partial x}(0,0)\,x + \frac{\partial f}{\partial y}(0,0)\,y +$$

$$+ \frac{1}{2}\left[\frac{\partial^2 f}{\partial x^2}(0,0)\,x^2 + 2\frac{\partial f}{\partial x \partial y}(0,0)\,xy + \frac{\partial^2 f}{\partial y^2}(0,0)\,y^2\right] + o(\|v\|^2)$$

$$\underline{f(0,0) = 0} \quad , \quad \frac{\partial f}{\partial x}\underline{(0,0) = 0} \quad ,$$

$$\frac{\partial f}{\partial x} = y\,\big(-\sin(x-3y^2)\big) - 2\sin(y+3x^2) - 2x\cos(y+5x^2)\cdot 10x$$

$$\frac{\partial^2 f}{\partial x^2}\Bigg|_{v=(0,0)} = \underbrace{y\cos(x-3y^2)\Big|_{v=(0,0)}}_{0} - 2\cos(y+3x^2)\cdot 10x\Big|_{v=(0,0)}$$

$$+ 0$$

$$\frac{\partial^2 f}{\partial x \partial y}(0,0) = 0 + \left(-2\cos(y+5x^2)\right)\Big|_{v=(0,0)} + 0 = -2$$

$$\frac{\partial f}{\partial y} = \cos(x-3y^2) + y\left(-\sin(x-3y^2)\cdot(-6y)\right) + -2x\cos(y+5x^2)$$

$$\frac{\partial f}{\partial y}(0,0) = 1$$

$$\frac{\partial^2 f}{\partial y^2}\Big|_{(0,0)} = -\sin(x-3y^2)(-6y)\Big|_{(0,0)} + 0 + 0 = 0$$

$$\left.\frac{\partial^2 f}{\partial x \partial y}\right|_{(0,0)} = 0 + 0 - 2 \cos(y + 5x^2)\Big|_{(0,0)} + 0$$

$$= -2$$

$$f(x,y) = 0 + 0 \cdot x + y \cdot 1 + \frac{1}{2} 0 \cdot x^2 + \frac{1}{2} \cdot y^2 + \frac{1}{2}(-4) \cdot xy$$

$$+ o(\|v\|^2) =$$

$$= y + \frac{1}{2} y^2 - 2xy + o(\|v\|^2)$$

9.3.9: V sp. vett. $\mathbb{R}$, $\langle | \rangle$ prodotto scalare

$W \subseteq V$ s. spazio

Scrivere la proiezione $\perp$ $p : V \longrightarrow W$

$V = \mathbb{R}^2$, $\langle | \rangle$ standard

$W = \{4x - 3y = 0\}$ dim $W = 1$

$w = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ é una base ortogonale (!) di W.

$$p : V \longrightarrow W \qquad v = \begin{pmatrix} x \\ y \end{pmatrix} \stackrel{p}{\longmapsto} \frac{\langle v | w \rangle}{\|w\|^2} \cdot w$$

$$\|w\|^2 = 9 + 16 = 25$$

$$\langle v | w \rangle = 3x + 4y$$

$$P\begin{pmatrix}x\\y\end{pmatrix} = \frac{3x+4y}{25}\begin{pmatrix}3\\4\end{pmatrix}$$

9.3.11 : $V = \mathbb{R}^2$, $W = \{4x - 3y = 0\} = \mathrm{Span}\left\{\underbrace{\begin{pmatrix}3\\4\end{pmatrix}}_{w}\right\}$

$\langle \ | \ \rangle_A$ $\quad A = \begin{pmatrix}5 & -2\\-2 & 1\end{pmatrix}$

w è base $\perp$ di W.

$$p(v) = \frac{\langle v | w \rangle_A}{\|w\|_A^2} \, w$$

$$A w = \begin{pmatrix} 3 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}$$

$$\langle v | w \rangle_A = 7x - 2y$$

$$\|w\|_A^2 = 21 - 8 = 13$$

$$p\begin{pmatrix} x \\ y \end{pmatrix} = \frac{7x - 2y}{13} \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

9.3.13 : $\quad V = \mathbb{R}^3, \quad \langle \, | \, \rangle$

$$W = \{ 3x + 2y = 3y - 5z = 0 \} \qquad \dim W = 1$$

W è generato da $w = \begin{pmatrix} -10 \\ 15 \\ 9 \end{pmatrix}$

$v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\langle v | w \rangle = -10x + 15y + 9z$

$\|w\|^2 = 100 + 225 + 81 = 406$

$$p(v) = \frac{-10x + 15y + 9z}{406} \begin{pmatrix} -10 \\ 15 \\ 9 \end{pmatrix}$$

9.3.12

$V = \mathbb{R}^3$ $\langle \, | \, \rangle$ standard

$W = \{ x - 2y + 5z = 0 \}$ $\dim W = 2$

$$\left.\begin{aligned} p &: V \to W \quad \text{proiezione ortogonale} \\ q &: V \to W^{\perp} \quad \text{'' \quad ''} \end{aligned}\right\} \Rightarrow p + q = I_d$$

Scrivo q:

$$W^{\perp} = \text{Span}\left(\underbrace{\begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}}_{w} \right)$$

$$\|w\|^2 = 1 + 4 + 25 = 30$$

$$q\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{x - 2y + 5z}{30} \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$$

$$p\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \frac{x - 2y + 5z}{30} \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$$

9.3.15 $\qquad V = \mathbb{R}^3 \qquad A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix}$

$$W = \{ 3x + 2y = 3y - 5z = 0 \} = \operatorname{Span}\left(\overset{w}{\begin{pmatrix} -10 \\ 15 \\ 9 \end{pmatrix}} \right)$$

$$Aw = \begin{pmatrix} 20 \\ 46 \\ 12 \end{pmatrix} \qquad v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad \langle v | w \rangle_A = 20x + 46y + 12z$$

$$\|w\|^2 = 20 \cdot (-10) + 46 \cdot 15 + 9 \cdot 12 =$$

$$= 598$$

$$p\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{20x+46y+12z}{598}\begin{pmatrix} -10 \\ 15 \\ 9 \end{pmatrix}$$

9.3 - 16

$$V = \mathbb{R}_{\leq 2}[t]$$

$$\langle p | q \rangle = p(-1)\,q(-1) + p(1)\,q(1) + p(2)\,q(2)$$

$W = \text{span}(t, t^2)$ Voglio ricavare una base ortogonale di W:

$f(t) = t \quad , \quad g(t) = t^2$

	-1	1	2
t	-1	1	2
t^2	1	1	4

$\langle t | t^2 \rangle = -1 + 1 + 8 = 8$

$\| t \|^2 = 1 + 1 + 4 = 6$

Sottraggo a t^2 la sua proiezione ortogonale sullo span di t.

$$t^2 - \frac{6}{8} t = t^2 - \frac{3}{4} t$$

$t, \; t^2 - \frac{3}{4} t$ è una base ortogonale di W

$$p(h(t)) = \frac{\langle h(t) | t \rangle}{6} t + \frac{\langle h(t), t^2 - \frac{3}{4} t \rangle}{\| t^2 - \frac{3}{4} t \|^2} \cdot \left(t^2 - \frac{3}{4} t\right)$$