15/3/2018 Esercitazione.

9.2.5

(g) $V = \mathbb{R}^3$ $\quad \langle x | y \rangle = {}^t y x$

$$v_1 = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}, \quad v_3 = \begin{pmatrix} -3\ell \\ 0 \\ 0 \end{pmatrix}$$

Osservo che v_1, v_2, v_3 sono indipendenti.

$$u_1 = \frac{v_1}{\|v_1\|} \qquad \|v_1\|^2 = 9 + 4 + 1 = 14$$

$$u_1 = \frac{1}{\sqrt{14}} \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$$

$$y_2 = v_2 - \langle v_2 | u_1 \rangle u_1 =$$

$$= v_2 - \frac{\langle v_2 | v_1 \rangle}{14} v_1$$

$$\langle v_2 , v_1 \rangle = -6 + 2 - 3 = -7$$

$$y_2 = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ 2 \\ \frac{5}{2} \end{pmatrix} , \quad \|y_2\|^2 = \frac{1}{4} + 4 + \frac{25}{4} =$$

$$= \frac{13}{2} + 4 = \frac{21}{2}$$

$$u_2 = \frac{y_2}{\|y_2\|} = \sqrt{\frac{2}{21}} \begin{pmatrix} -\frac{1}{2} \\ 2 \\ \frac{5}{2} \end{pmatrix}$$

$$v_3 = \begin{pmatrix} -3e \\ 0 \\ 0 \end{pmatrix}$$ sostituisco v_3 con $\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} = v'_3$

perché questo non cambia il risultato dell'algoritmo:

$$y_3 = v'_3 - \langle v'_3 | u_1 \rangle u_1 - \langle v'_3 | u_2 \rangle u_2$$

$$= v'_3 + \frac{3}{\sqrt{14}} u_1 - u_2 = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} + \frac{3}{14} \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} - \frac{1}{21} \begin{pmatrix} -\frac{1}{2} \\ 2 \\ \frac{5}{2} \end{pmatrix} =$$

$= \ldots$

$$u_3 = \frac{y_3}{\|y_3\|}$$

(i) $\quad V = \mathbb{R}^3 \quad \langle \ | \ \rangle_A \quad A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix}$

$v_1 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$

$$q(x,y,z) = x^2 + 4xy + 5y^2 - 2yz + 3z^2$$

$$\|v_1\|^2 = q(v_1) = 4 + 8 + 5 - 6 + 27 = 38$$

$$u_1 = \frac{1}{\sqrt{38}} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$$y_2 = v_2 - \langle v_2 | u_1 \rangle u_1 =$$

$$= v_2 - \frac{\langle v_1, v_2 \rangle}{\|v_1\|^2} v_1$$

$$\langle v_1, v_2 \rangle = {}^t v_1 A v_2 = (2, 1, 3) \begin{pmatrix} 1 & 2 & 0 \\ 2 & 5 & -1 \\ 0 & -1 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$$

$$= (2, 1, 3) \begin{pmatrix} -1 \\ -6 \\ 8 \end{pmatrix} = -2 - 6 + 24 = 16$$

$$y_2 = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} - \frac{16}{38} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} =$$

$$\|y_2\|^2 = {}^t y_2 A y_2 = q(y_2) \quad , \quad u_2 = \frac{y_2}{\|y_2\|}$$

(m)

$$V = \mathbb{R}[t] \qquad \langle p(t) | q(t) \rangle = \int_0^1 p(t)q(t)\,dt$$

$$p, q \in V$$

$$p_1(t) = 1 + 2t - t^2, \qquad p_2(t) = 2 - t + 3t^2$$

$$\int_0^1 t^m\,dt = \left[\frac{t^{m+1}}{m+1}\right]_0^1 = \frac{1}{m+1}$$

$$p_1^2(t) = (1 + 2t - t^2) = 1 + 4t^2 + t^4 + 4t - 2t^2 - 4t^3 =$$

$$= 1 + 2t^2 - 4t^3 + t^4 + 4t$$

$$\|p_1\|^2 = \int_0^1 p_1(t)^2\, dt = 1 + \frac{2}{3} - 4 \cdot \frac{1}{4} + \frac{1}{5} + \frac{4}{2} =$$

$$= 2 + \frac{2}{3} + \frac{1}{5} = \frac{30 + 10 + 3}{15} = \frac{43}{15}$$

$$u_1 = \frac{p_1}{\|p_1\|} = (1 + 2t - t^2)\frac{\sqrt{15}}{\sqrt{43}}$$

$$y_2 = p_2 - \frac{\langle p_1 | p_2 \rangle}{\|p_1\|^2} p_1$$

$$p_1 \, p_2 = (1+2t-t^2)(2-t+3t^2) = 2+3t+7t^3-3t^4-t^2$$

$$\langle p_1 | p_2 \rangle = \int_0^1 p_1 \, p_2 \, dt = 2+\frac{3}{2}+\frac{7}{4}-\frac{3}{5}-\frac{1}{3} =$$

$$= \frac{120+90+105-36-20}{60} = \frac{259}{60}.$$

$$y_2 = (2-t+3t^2) - \frac{259}{\cancel{60}_{4}} \cdot \frac{\cancel{15}}{43} (1+2t-t^2).$$

$$u_1 = \frac{y_2}{\|y_2\|^2}, \quad \text{dove} \quad \|y_2^2\| = \int_0^1 y_2^2(t)\, dt.$$