14/3/2018 – Esercitazione.

Ricevimento: venerdì 14.30, nel mio studio (dipartimento di matematica).

Es. 9.2.1 (e)

$V = M_{3\times 3}(\mathbb{R})$

$$q(A) = \sum_{j=1}^{3} (A^2)_{j,4-j} = (A^2)_{1,3} + (A^2)_{2,2} + (A^2)_{3,1}$$

forma quadratica.

Cerco la forma bilineare simm: $f: V \times V \longrightarrow \mathbb{R}$

da cui proviene q. $A, B \in V$

$$f(A, B) = \frac{1}{2}\left[q(A+B) - q(A) - q(B)\right]$$

$$(A+B)^2 = (A+B)(A+B) = A^2 + AB + BA + B^2$$

$$(A+B)^2 - A^2 - B^2 = AB + BA$$

$$q(A+B) - q(A) - q(B) = \sum_{j=1}^{3} \left((A+B)^2\right)_{j,4-j} - \sum_{j=1}^{3} (A^2)_{j,4-j}$$

$$- \sum_{j=1}^{3} (B^2)_{j,4-j} =$$

$$= \sum_{j=1}^{3} \left[(A+B)^2 - A^2 - B^2 \right]_{j,4-j} = \sum_{j=1}^{3} (BA + AB)_{j,4-j} =$$

$$= 2 f(A, B)$$

$\dashv$

9.2.4 (b)

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}, \quad y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$d : \mathbb{R}^n \times \mathbb{R}^n \longrightarrow \mathbb{R}$$
$$(x, y) \longrightarrow \sum_{i=1}^{n} |x_i - y_i|$$

Mostrare che questa è una distanza:

d Deve verificare:

$\forall x, y$

(1) $d(x,y) \geq 0$ ✓

$d(x,y) = 0$ se e solo se $x = y$

$$d(x,y) = \sum_{j=1}^{n} |x_j - y_j| = 0 \iff |x_j - y_j| = 0 \quad \forall j = 1, \dots, n$$

$$\iff x_j - y_j = 0 \quad \forall j = 1, \dots, n \iff x = y.$$ ✓

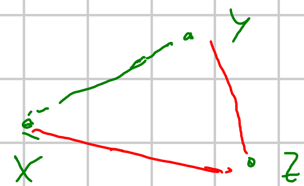
(2) $d(x,y) = d(y,x) \quad \forall y, x$ ✓

(simmetria)

(3) disuguaglianza triangolare:

$\forall x, y, z$: $$d(x,y) \leq d(x,z) + d(y,z)$$

Idea:



$$|x_j - y_j| = |(x_j - z_j) + (z_j - y_j)| \leq \qquad \forall j = 1, \dots, n$$

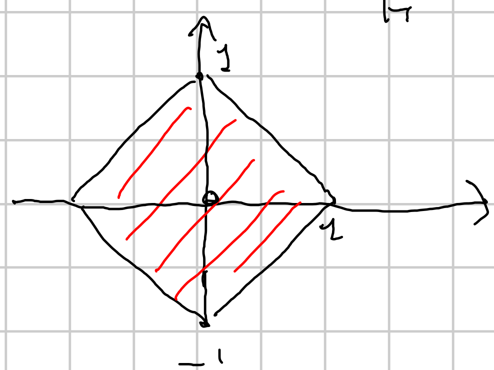
$$\leq |x_j - z_j| + |z_j - y_j|$$

$$d(x,y) = \sum_{j=1}^{n} |x_j - y_j| \leq \sum_{j=1}^{n} \left(|x_j - z_j| + |z_j - y_j| \right) =$$

$$= \sum_{j=1}^{n} |x_j - z_j| + \sum_{j=1}^{n} |y_j - z_j| = d(x,z) + d(y,z)$$

$\mapsto$

$n = 2$



Palla di centro 0 e raggio 1

$$\{(x,y) \in \mathbb{R}^2 \mid |x| + |y| \leq 1\}$$

$\longmapsto$

9.2.5 : (b) $V = \mathbb{R}^2$, $\langle | \rangle$ standard

$$v_1 = \begin{pmatrix} -2 \\ 5 \end{pmatrix}, \quad v_2 = \begin{pmatrix} -e^2 \\ 1 \end{pmatrix}$$

$$u_1 = \frac{v_1}{\|v_1\|} \qquad \|v_1\|^2 = (-2)^2 + 5^2 = 29$$

$$\|v_1\| = \sqrt{29}$$

$$u_1 = \frac{1}{\sqrt{29}}\, v_1 = \frac{1}{\sqrt{29}} \begin{pmatrix} -2 \\ 5 \end{pmatrix}.$$

$$y_2 = v_2 - \langle v_2 | u_1 \rangle u_1 = \begin{pmatrix} -e^2 \\ 1 \end{pmatrix} - \left\langle v_2, \frac{v_1}{\sqrt{29}} \right\rangle \frac{v_1}{\sqrt{29}} =$$

$$= \begin{pmatrix} -e^2 \\ 1 \end{pmatrix} - \langle v_2, v_1 \rangle \frac{1}{29} \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} -e^2 \\ 1 \end{pmatrix} - \frac{2e^2+5}{29} \begin{pmatrix} -2 \\ 5 \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{25}{29} e^2 + \frac{10}{29} \\ \frac{24}{29} - \frac{10 e^2}{29} \end{pmatrix}$$

$$u_2 = \frac{y_2}{\|y_2\|} \qquad \|y_2\|^2 = \frac{1}{(29)^2}\left[(-25e^2+10)^2 + (24-10e^2)^2\right]$$

$\dots$

(d) $\langle x | y \rangle = {}^t x A y$ $\qquad V = \mathbb{R}^2$

$$A = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix} \qquad v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\|v_1\|^2 = 5 \quad , \qquad \|v_2\|^2 = 1$$

$$\langle v_1 | v_2 \rangle = -2$$

$$u_1 = \frac{v_1}{\|v_1\|} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$y_2 = v_2 - \langle v_2 | u_1 \rangle u_1 = v_2 - \frac{1}{5} \langle v_2 | v_1 \rangle v_1 =$$

$$= \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \frac{1}{5} (-2) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{2}{5} \\ 1 \end{pmatrix}$$

$$u_2 = \frac{y_2}{\|y_2\|}$$

$$\|y_2\|^2 = 5\cdot\left(\frac{2}{5}\right)^2 - 4\cdot\frac{2}{5} + 1 = \qquad A = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$$

$$= \frac{4}{5} - \frac{8}{5} + 1 = \frac{1}{5} \qquad \|y_2\| = \frac{1}{\sqrt{5}}$$

$$u_2 = \sqrt{5}\begin{pmatrix} \frac{2}{5} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \sqrt{5} \end{pmatrix}$$

(e) $V = \mathbb{R}^2$ $\langle\,|\,\rangle$ associato a $A = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$

$$v_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad v_2 = \begin{pmatrix} -1 \\ 5 \end{pmatrix} \qquad q(x,y) = 3x^2 - 2xy + 2y^2$$

$$\|v_1\|^2 = 3\cdot 4 - 2\cdot 6 + 2\cdot 9 = 18$$

$$\|v_2\|^2 = 3\cdot 1 + 10 + 2\cdot 25 = 63$$

$$\langle v_1 | v_2 \rangle = (2,\ 3)\begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 5 \end{pmatrix} = (2,\ 3)\begin{pmatrix} -8 \\ 11 \end{pmatrix} = 17$$

$$u_1 = \frac{1}{\sqrt{18}}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

. Finire a casa .