9/3/2018

Es: 9.1.8 (g)

$$5x^2-4xy+3y^2-6xz+11z^2-4yz=$$

$$=5\Big(x^2-\frac{4}{5}xy-\frac{6}{5}xz+\underbrace{\frac{4}{25}}y^2+\underbrace{\frac{9}{25}}z^2+\underbrace{\frac{12}{25}yz}\Big)$$

$$-\frac{4}{5}y^2-\frac{9}{5}z^2-\frac{12}{5}yz+11z^2-4yz+3y^2=$$

$$=\underbrace{5\left(x-\frac{2}{5}y-\frac{3}{5}z\right)^2}_{1}+\underbrace{\frac{11}{5}y^2+\frac{46}{5}z^2-\frac{32}{5}yz}_{2}=$$

Studio 2:

$$\frac{1}{5}\begin{pmatrix} \boxed{11} > 0 & -16 \\ -16 & 46 \end{pmatrix} = B \quad , \det B > 0$$

(1) è $\geq 0$ e $= 0 \iff x - \frac{2}{5}y - \frac{3}{5}z = 0$

(2) è $\geq 0$ e $= 0 \iff y = z = 0$

(1)+(2) $\geq 0$ e $= 0 \iff (1) = (2) = 0 \iff x - \frac{2}{5}y - \frac{3}{5}z = 0$ e $y = z = 0 \iff$

$\iff x = y = z = 0$. $\Rightarrow$ la forma quadratica è $> 0$.

Es. 9.2.1 $\quad f: V \times V \longrightarrow \mathbb{R}$

bilineare simmetrica

forma quadratica con! $\quad q: V \longrightarrow \mathbb{R}$

$$q(v) = f(v, v)$$

Vale! $\quad \dfrac{q(v+w) - q(v) - q(w)}{2} = f(v,w)$

(b) $\quad V = \mathbb{R}^2 \quad q(x) = 3x^2 + 2xy - y^2$

$v = (x_1, y_1) \quad f(v,w)$

$w = (x_2, y_2)$

Usando la formula $f(v,w) = q(x_1+x_2, y_1+y_2) - q(x_1,y_1) - q(x_2,y_2)$

(calcolare per esercizio).

$q$ è data dalla matrice simmetrica $A = \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix}$

la forma bilineare è $\quad {}^t v A w$

$$f(v,w) = 3x_1x_2 - y_1y_2 + x_1y_2 + x_2y_1$$

(c) $\quad q(x,y,z) = -2x^2 + y^2 - 5z^2 - xy + 3xz + 2yz$

$v = (x_1, y_1, z_1)\,, \quad w = (x_2, y_2, z_2)$

$$f(v,w) = -2x_1x_2 + y_1y_2 - 5z_1z_2 - \frac{1}{2}x_1y_2 - \frac{1}{2}x_2y_1 + \frac{3}{2}x_1z_2 + \frac{3}{2}x_2z_1 + y_1z_2 + y_2z_1$$

Matrice associata:

$$\begin{pmatrix} -2 & -\frac{1}{2} & \frac{3}{2} \\ -\frac{1}{2} & 1 & 1 \\ \frac{3}{2} & 1 & -5 \end{pmatrix}$$

$H$

(f) $V = \mathbb{R}_{\leq 2}[t]$ $\qquad q(p(t)) = p(1)p(-2)$

forma quadratica

$p, r \in V$

$$2 f(p, r) = q(p+r) - q(p) - q(r) =$$

$$= (p+r)(1)(p+r)(-2) - p(1)p(-2) - r(1)r(-2) =$$

$$= (p(1)+r(1))(p(-2)+r(-2)) - p(1)p(-2) - r(1)r(-2) =$$

$$= \cancel{p(1)p(-2)} + \cancel{r(1)r(-2)} + r(1)p(-2) + p(1)r(-2)$$
$$- \cancel{p(1)p(-2)} - \cancel{r(1)r(-2)}$$

$$f(p, r) = \frac{p(1)r(-2) + p(-2)r(1)}{2}$$

Es. 9.2.3: $\quad V = \mathbb{R}_{\leq 2}[t]$

$\langle p(t) | q(t) \rangle = p(0)q(0) + p(1)q(1) + p(2)q(2)$

prodotto scalare.

Trovare polinomio di norma $\sqrt{5}$ e ortogonale a $1+t$ e $1+t^2$.

$W := \text{Span}(1+t, 1+t^2) \qquad \dim W = 2$

$\dim W^\perp = 1$

$p(t) \in V$

$$0 = \langle p(t), 1+t \rangle = p(0)\cdot 1 + p(1)\cdot 2 + p(2)\cdot 3$$

$$0 = \langle p(t), 1+t^2 \rangle = p(0)\cdot 1 + p(1)\cdot 2 + p(2)\cdot 5$$

$$\iff \quad p(2) = 0 \qquad p(0) + 2p(1) = 0$$

$$p(t) = a + bt + ct^2, \qquad a, b, c \in \mathbb{R}$$

Riscrivo le equazioni:

$$p(2) = a + 2b + 4c = 0$$

$$p(0) + 2p(1) = a + 2(a+b+c) = 0$$

$$\iff \begin{cases} c = a \\ b = -\frac{5}{2}a \end{cases}$$

$$W^{\perp} = \left\{ a - \tfrac{5}{2}at + at^2 \;\middle|\; a \in \mathbb{R} \right\}$$

Cerco gli el. di norma $\sqrt{5}$ in $W^{\perp}$

$$5 = p(0)^2 + p(1)^2 + p(2)^2 = a^2 + \left(-\tfrac{1}{2}a\right)^2 =$$

$$= \frac{5}{4}a^2$$

$$\Rightarrow \quad a^2 = 4 \quad \Longrightarrow \quad a = \pm 2$$

2 sol :

$$2 - 5t + 2t^2$$

$$-2 + 5t - 2t^2.$$