29/11/17

Recupero: domani in B24 alle 11.30.

6.2.17

lavoro sulle colonne

(c)

$$\det \begin{pmatrix} 2 & 3 & 0 & 1 \\ 1 & -1 & -3 & 0 \\ 0 & 2 & 3 & -1 \\ -1 & 1 & 2 & 3 \end{pmatrix}$$

$II \longrightarrow II + I$

$$= \det \begin{pmatrix} 2 & 5 & 0 & 1 \\ 1 & 0 & -3 & 0 \\ 0 & 2 & 3 & -1 \\ -1 & 0 & 2 & 3 \end{pmatrix} =$$

Sviluppo di Laplace rispetto alla II colonna:

$$= -5 \det \begin{pmatrix} 1 & -3 & 0 \\ 0 & 3 & -1 \\ -1 & 2 & 3 \end{pmatrix} - 2 \det \begin{pmatrix} 2 & 0 & 1 \\ 1 & -3 & 0 \\ -1 & 2 & 3 \end{pmatrix} =$$

$$= -5\left(1\cdot(9+2) - 3\right) - 2\left(2 \det \begin{pmatrix} -3 & 0 \\ 2 & 3 \end{pmatrix} + \det \begin{pmatrix} 1 & -3 \\ -1 & 2 \end{pmatrix}\right) =$$

$$= -40 - 2(-18 - 1) = -40 + 38 = -2$$

(d)

$$\det \begin{pmatrix} 4 & -1 & 2 & -8 \\ -3 & 2 & 3 & -2 \\ 2 & 3 & -5 & 3 \\ 5 & 4 & -3 & -5 \end{pmatrix}$$

lavoro sulle colonne:

$\mathrm{I} \longrightarrow \mathrm{I} + \mathrm{III}$

$\mathrm{II} \longrightarrow \mathrm{II} + \mathrm{IV}$

sviluppo rispetto alla II riga

$$= \det \begin{pmatrix} 6 & -9 & 2 & -8 \\ 0 & 0 & 3 & -2 \\ -3 & 6 & -5 & 3 \\ 2 & -1 & -3 & -5 \end{pmatrix} = -3 \det \begin{pmatrix} 6 & -9 & -8 \\ -3 & 6 & 3 \\ 2 & -1 & -5 \end{pmatrix} - 2 \det \begin{pmatrix} 6 & -9 & 2 \\ -3 & 6 & -5 \\ 2 & -1 & -3 \end{pmatrix}$$

$$= -9 \det \begin{pmatrix} 6 & -9 & -8 \\ -1 & 2 & 1 \\ 2 & -1 & -5 \end{pmatrix} - 2 \det \begin{pmatrix} -12 & -9 & 2 \\ 9 & 6 & -5 \\ 0 & -1 & -3 \end{pmatrix} =$$

$I \to I + III$, $II \to II - 2\,III$

$I_c \to I + 2\,II$

$$= -9 \det \begin{pmatrix} -2 & 7 & -8 \\ 0 & 0 & 1 \\ -3 & 9 & -5 \end{pmatrix} - 2 \left(-12 \det \begin{pmatrix} 6 & -5 \\ -1 & -3 \end{pmatrix} - 9 \det \begin{pmatrix} -9 & 2 \\ -1 & -3 \end{pmatrix} \right) =$$

$$= -9\left[-\det\begin{pmatrix}-2 & 7\\ -3 & 9\end{pmatrix}\right] - 2\left[-12(-23) - 9\cdot 29\right] =$$

$$= 9(-18+21) - 2\left[276 - 261\right] = +27 - 30 = -3$$

$6 \cdot 2 \cdot 18$

(c) $\quad n = 3 \qquad A = (v_1, v_2, v_3) \qquad \det A = -7$

$$B = (2v_1 + v_3,\ 5v_1 - v_2 + 3v_3,\ 4v_2 - v_3)$$

$\det B$? $\qquad$ Usiamo il teorema di BINET.

[Se ho matrici quadrate B, C della stessa taglia
Vale $\det(BC) = \det(B)\det(C)$]

Cerco C 3×3 t.c.: $B = AC$

$$C = \begin{pmatrix} 2 & 5 & 0 \\ 0 & -1 & 4 \\ 1 & 3 & -1 \end{pmatrix} \qquad \det C = 2\det\begin{pmatrix} -1 & 4 \\ 3 & -1 \end{pmatrix} + \det\begin{pmatrix} 5 & 0 \\ -1 & 4 \end{pmatrix} =$$

$$= 2\cdot(-11) + 20 = -2$$

$$\det(B) = \det(A)\det(C) = -7\cdot(-2) = 14$$

(d) $n = 4$ $\qquad A = (v_1, v_2, v_3, v_4)$, $\det A = 6$

$$B = (v_3 - v_1 \,,\, v_1 - 3v_4 \,,\, 2v_2 + v_4 \,,\, v_3 - 2v_4)$$

Scrivo $B = A\,C$, dove:

$$C = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & -3 & 1 & -2 \end{pmatrix} \qquad \det C = -2 \det \begin{pmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & -3 & -2 \end{pmatrix} =$$

$$= 2 \det \begin{pmatrix} 0 & 1 \\ -3 & -2 \end{pmatrix} + 2 \det \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} =$$

$$= 6 - 4 = 2$$

$$\det B = \det(A) \cdot \det(C) = 6 \cdot 2 = 12.$$

4

6.3.1

(e) $$A = \begin{pmatrix} 4 & -2 & +1 \\ \boxed{3} & 1 & 2 \\ -5 & 6 & +1 \end{pmatrix} \qquad A^{-1}\ ?$$

$$= \det \begin{pmatrix} 0 & 0 & 1 \\ -5 & 5 & 2 \\ -9 & 8 & 1 \end{pmatrix} = \det \begin{pmatrix} -5 & 5 \\ -9 & 8 \end{pmatrix} = -5 \det \begin{pmatrix} 1 & 1 \\ 9 & 8 \end{pmatrix} = 5 \neq 0$$

$$A^{-1} = \frac{1}{5} \begin{pmatrix} -11 & 8 & -5 \\ -13 & 9 & -5 \\ 23 & -14 & 10 \end{pmatrix}$$

$$(\det A)\, A^{-1} = B$$

$$b_{11} = \det\begin{pmatrix} 1 & 2 \\ 6 & 1 \end{pmatrix}$$

$$b_{12} = -\det\begin{pmatrix} -2 & 1 \\ 6 & 1 \end{pmatrix} = 8$$

$$b_{13} = +\det\begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix} = -5$$