Algebra Lineare 28/11/17

$$\begin{pmatrix} 2 & 1 & 5 \\ -3 & 4 & 7 \\ -2 & -1 & 6 \end{pmatrix}^{-1} = \frac{1}{121} \begin{pmatrix} 31 & \cdot & \cdot \\ 4 & \cdot & \cdot \\ 11 & - & \cdot \end{pmatrix}$$

$A \qquad\qquad M$

$$A \cdot M = \frac{1}{121} \begin{pmatrix} 62+4+55 & \cdot & \cdot \\ - & \cdot & \cdot \\ - & - & - \end{pmatrix} = \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ - & - & \cdot \end{pmatrix}$$

$\boxed{6.2.12}$

$$\det\begin{pmatrix} 6 & 1 & -3 \\ 4 & -7 & 2 \\ -2 & 5 & 8 \end{pmatrix} = \det\begin{pmatrix} 0 & 1 & 0 \\ 46 & -7 & -19 \\ -32 & 5 & 23 \end{pmatrix} \leftarrow \text{svil}$$

uso

$$= -\det\begin{pmatrix} 46 & -19 \\ -32 & 23 \end{pmatrix} = \cdots$$

$\boxed{6.2.13}$

$$\det\begin{pmatrix} -1+t & 2 & -3 \\ 1+2t & 5 & -1 \\ -2 & 2-3t & 5 \end{pmatrix} \leftarrow \text{uso}$$

$$= \det\begin{pmatrix} -4-5t & -13 & 0 \\ 1+2t & 5 & -1 \\ 3+10t & 27-3t & 0 \end{pmatrix} = +\det\begin{pmatrix} -4-5t & -13 \\ 3+10t & 27-3t \end{pmatrix}$$

$\uparrow$ svil

$= \dots$

$\boxed{6.2.15}$

$$\det \begin{pmatrix} 5 & 4 & 6 & 2 \\ -1 & 2 & 2 & 3 \\ 2 & -7 & -3 & 1 \\ 3 & 1 & 4 & -5 \end{pmatrix} \leftarrow \text{svil}$$

$\uparrow$

uso

$$= \det \begin{pmatrix} 5 & 14 & 16 & 17 \\ -1 & 0 & 0 & 0 \\ 2 & -3 & 1 & 7 \\ 3 & 7 & 10 & 4 \end{pmatrix}$$

$$= \det \begin{pmatrix} 14 & 16 & 17 \\ -3 & 1 & 7 \\ 7 & 10 & 4 \end{pmatrix} = \dots$$

$\boxed{6.2.16}$

$$\det \begin{pmatrix} 0 & t & -1 & 2 \\ 1-t & 2 & 3 & -6 \\ 2 & -2t & 1 & -2 \\ 3 & 0 & 2 & 1+t \end{pmatrix} \longleftarrow \text{svil}$$

$\uparrow$ uso

$$= \det \begin{pmatrix} 0 & 0 & -1 & 0 \\ 1-t & 2+3t & 3 & 0 \\ 2 & -t & 1 & 0 \\ 3 & 2t & 2 & 5+t \end{pmatrix}$$

$$= -\det\begin{pmatrix} 1-t & 2+3t & 0 \\ 2 & -t & 0 \\ 3 & 2t & 5+t \end{pmatrix}$$

$$= -(5+t)\det\begin{pmatrix} 1-t & 2+3t \\ 2 & -t \end{pmatrix} = \dots$$

AVVISO: recuperi giov 30/11 B.2.4 11:30 (1h)

giov 7/12 B.2.4 11:30 (1h)

Determinante di Vandermonde. Se $n \geq 2$

dati $a_1, \dots, a_n \in \mathbb{R}$ si ha

$$\det\begin{pmatrix} 1 & 1 & \cdots & 1 \\ a_1 & a_2 & & a_n \\ a_1^2 & a_2^2 & & a_n^2 \\ a_1^3 & a_2^3 & & a_n^3 \\ \vdots & \vdots & & \vdots \\ a_1^{n-1} & a_2^{n-1} & & a_n^{n-1} \end{pmatrix} = \prod_{1\le i<j\le n} (a_j - a_i)$$

Svolg. 1 : Per induz su n

$n = 2$: $\det\begin{pmatrix} 1 & 1 \\ a_1 & a_2 \end{pmatrix} = a_2 - a_1$ ✓

$n = 3$ NON SERVE

$$\det\begin{pmatrix} 1 & 1 & 1 \\ a_1 & a_2 & a_3 \\ a_1^2 & a_2^2 & a_3^2 \end{pmatrix} \overset{?}{=} (a_2 - a_1)\cdot(a_3 - a_1)\cdot(a_3 - a_2)$$

//

$$\underbrace{a_2a_3^2} + \underbrace{a_3a_1^2} + \underbrace{a_1a_2^2}$$
$$\underbrace{-a_2a_1^2} \underbrace{-a_1a_3^2} \underbrace{-a_3a_2^2}$$

$$(a_2-a_1)(a_3^2-a_3a_2-a_1a_3+a_1a_2)$$
$$= \underbrace{a_2a_3^2} \underbrace{-a_3a_2^2} \cancel{-a_1a_2a_3}$$
$$\underbrace{+a_1a_2^2} \underbrace{-a_1a_3^2} \cancel{+a_1a_2a_3}$$
$$\underbrace{+a_3a_1^2} \underbrace{-a_2a_1^2}$$

Passo induttivo: suppongo vera la tesi per $n-1$.

$$\det \begin{pmatrix} 1 & 1 & \cdots & 1 \\ a_1 & a_2 & & a_n \\ a_1^2 & a_2^2 & & a_n^2 \\ \vdots & & & \\ a_1^{n-3} & a_2^{n-3} & & a_n^{n-3} \\ a_1^{n-2} & a_2^{n-2} & & a_n^{n-2} \\ a_1^{n-1} & a_2^{n-1} & & a_n^{n-1} \end{pmatrix}$$

fabbrico zeri nei posti dopo il I sulla I colonna:

- sostituire ultima riga con sé stessa − a_1 volte la penultima
- sostituire penultima riga con sé stessa − a_1 volte la terzultima

...   ...

- $III \rightsquigarrow III - a_1 \cdot II$
- $II \rightsquigarrow II - a_1 \cdot I$

$$
\det \begin{pmatrix}
1 & 1 & \cdots & 1 \\
0 & a_2 - a_1 & & a_n - a_1 \\
0 & a_2^2 - a_1 a_2 & & a_n^2 - a_1 a_n \\
\vdots & & & \\
0 & a_2^{n-3} - a_1 \cdot a_2^{n-4} & & a_n^{n-3} - a_1 \cdot a_n^{n-4} \\
0 & a_2^{n-2} - a_1 \cdot a_2^{n-3} & & a_n^{n-2} - a_1 a_n^{n-3} \\
0 & a_2^{n-1} - a_1 a_2^{n-2} & \cdots & a_n^{n-1} - a_1 a_n^{n-2}
\end{pmatrix}
$$

svil

$$= \det \begin{pmatrix} a_2 - a_1 & \cdots & a_n - a_1 \\ a_2(a_2 - a_1) & & a_n(a_n - a_1) \\ \vdots & & \vdots \\ a_2^{n-3}(a_2 - a_1) & & a_n^{n-3}(a_n - a_1) \\ a_2^{n-2}(a_2 - a_1) & \cdots & a_n^{n-2}(a_n - a_1) \end{pmatrix}$$

$$= \underbrace{(a_2 - a_1) \cdots (a_n - a_1)}_{\prod_{j=2}^{n} (a_j - a_1)} \cdot \underbrace{\det \begin{pmatrix} 1 & \cdots & 1 \\ a_2 & & a_n \\ \vdots & & \vdots \\ a_2^{n-3} & & a_n^{n-3} \\ a_2^{n-2} & & a_n^{n-2} \end{pmatrix}}_{\text{Matrice di Vandermonde}}$$

$(n-1)\times(n-1)$ für $a_2, \dots, a_n$

ip. id.

$$\underbrace{\prod_{2\le i<j\le n} (a_j - a_i)}$$

$$\prod_{1\le i<j\le n} (a_j - a_i)$$

$\blacksquare$

II Sviluppo: considero

$$\det \begin{pmatrix} 1 & 1 & \cdots & 1 \\ a_1 & a_2 & \cdots & a_n \\ \vdots & & & \vdots \\ a_1^{n-1} & a_2^{n-1} & \cdots & a_n^{n-1} \end{pmatrix} \in \mathbb{R}[a_1, \dots, a_n]$$

polinomio nelle indeterminate $a_1, \dots, a_n$

- grado: $0 + 1 + 2 + \dots + n-1$

$$= \frac{n(n-1)}{2}$$

- inoltre se $a_j = a_i$ la matr. ho 2 col uguali

$\Longrightarrow$ fa 0 $\longrightarrow$ per Ruffini è divisibile per $a_j - a_i$

$\Longrightarrow \det\left(\ \dots \ \right)$ è multiplo di $\prod_{1 \le i < j \le n} (a_j - a_i)$

$\deg = \frac{n(n-1)}{2}$

$\deg = \binom{n}{2} = \frac{n(n-1)}{2}$

$\Longrightarrow$

$$\det\left(\ \cdots \ \right) = c \cdot \prod_{1 \le i < j \le n} (a_j - a_i) \qquad c \in \mathbb{R}$$

Se esamino il coeff di

$$1 \cdot a_2 \cdot a_3^2 \cdot \ldots \, a_n^{n-1}$$

a sin fa 1 ... e anche a dx

$\Longrightarrow c = 1.$ $\blacksquare$