



$$\left( \begin{array}{ccc|c} 0 & -23 & 26 & 150 \\ 1 & 5 & -6 & -27 \\ 0 & -6 & 11 & 56 \end{array} \right)$$

$$\begin{cases} -23y + 26z = 150 \\ -6y + 11z = 5 \end{cases}$$

$$I - 4II$$

$$III - 2I$$

per sostituzione.

$$y = -2$$

$$z = 4$$

$$x = -5y + 6z - 27 = 10 + 24 - 27 = 7$$

$$\text{Sol: } \begin{pmatrix} 7 \\ -2 \\ 4 \end{pmatrix}$$

$$\Rightarrow \text{rk } A = 3$$

6.1.2

$$\begin{cases} 3x - 4y + 2z = 6 \\ 2x - 5y + z = 11 \\ 4x - 3y + 3z = 2 \end{cases}$$

$$\left( \begin{array}{ccc|c} 3 & -4 & 2 & -16 \\ 2 & -5 & 1 & 11 \\ 4 & -3 & 3 & -31 \end{array} \right)$$

$$\left( \begin{array}{ccc|c} -1 & 6 & 0 & -16 \\ 2 & -5 & 1 & 11 \\ -2 & 12 & 0 & -31 \end{array} \right)$$

$$I \rightarrow I - 2II$$

$$II \rightarrow II - 3II$$

incompatibile.

$\Rightarrow$  Non c'è soluzione  $\Rightarrow \text{rk}(A) < 3$

In effetti:  $\text{rk}(A) = 2$ ,  $\text{rk}(A|b) = 3$   
H

6.1.6

$$\begin{cases} 5x - 4y - 3z = -1 \\ -x + 6y + 4z = 1 \\ 7x + 10y + 6z = 1 \end{cases}$$

$$\left( \begin{array}{ccc|c} 5 & -4 & -3 & -1 \\ -1 & 6 & 4 & 1 \\ 7 & 10 & 6 & 1 \end{array} \right)$$

$$I \rightarrow I + 5II$$

$$III \rightarrow III + 7II$$

Si ottiene

$$\left( \begin{array}{ccc|c} 0 & 26 & 17 & 4 \\ -1 & 6 & 4 & 1 \\ 0 & 52 & 34 & 8 \end{array} \right)$$

osservo che  $III = 2II$

$$\begin{cases} 26y + 17z = 4 \\ -x + 6y + 4z = 1 \end{cases}$$

Si risolve per sostituzione.

$$\begin{cases} x = \frac{1}{73}(z-1) \\ y = \frac{(17z-4)}{26} \\ z = z \end{cases}$$

per  $t=0$ , otteniamo:

$$\begin{pmatrix} -\frac{1}{13} \\ \frac{2}{13} \\ 0 \end{pmatrix}$$

sol. particolare.

sol. del sistema omogeneo.

Le sol sono

$$\begin{pmatrix} -\frac{1}{13} \\ \frac{2}{13} \\ 0 \end{pmatrix}$$

+

$$z \begin{pmatrix} \frac{1}{13} \\ -\frac{17}{26} \\ 1 \end{pmatrix}$$

,  $z \in \mathbb{R}$

Per Rouché-Capelli:

$$\text{rk}(A) = \text{rk}(A|b) = 3 - 1 = 2$$

↑  
dim  
dello  
spazio

↑  
dim Ker A

Col where it det.

6 - 2 - 1

(b)

$$\begin{pmatrix} 12 & -7 \\ 5 & -8 \end{pmatrix} = 12 \cdot (-8) - (-7 \cdot 5) = -96 + 35 = -61$$

(c)

$$\begin{pmatrix} 1+t & -1+2t \\ 6+5t & 2-t \end{pmatrix} = (1+t)(2-t) - (6+5t)(2-t) =$$
$$= -11t^2 - 7t + 8$$

(e)

$$\det \begin{pmatrix} -1+t & 2 & -3 \\ \underline{1+2t} & 5 & -1 \\ -2 & 2-3t & 5 \end{pmatrix} = ?$$

$$\text{II} \rightarrow \text{II} - 2\text{I}$$

$$\det \begin{pmatrix} -1+t & 2 & -3 \\ 3 & 1 & 5 \\ -2 & 2-3t & \underline{5} \end{pmatrix} = \det \begin{pmatrix} t-1 & 2 & \underline{-3} \\ 3 & 2 & \underline{5} \\ -5 & 1-3t & 0 \end{pmatrix} \stackrel{\text{III} - \text{II}}{=} =$$

Sviluppo di Laplace rispetto alla III colonna:

$$= -3 \begin{vmatrix} 3 & 1 \\ -5 & 1-3t \end{vmatrix} - 5 \begin{vmatrix} t-1 & 2 \\ -5 & 1-3t \end{vmatrix} =$$

$$= -3 [3(1-3t) + 5] - 5 ((t-1)(1-3t) + 10) =$$

$$= 27t - 24 + 15t^2 - 20t - 45 =$$

$$= 15t^2 + 7t - 69$$