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$$E \subseteq \mathbb{R}^4$$

$$\begin{cases} 2x + 3y - 4z + 5w = 7 \\ 3x - 5y + 7z + 2w = -1 \end{cases}$$

$$\left(\begin{array}{cc|cc|c} 2 & 3 & -4 & 5 & 7 \\ 3 & -5 & 7 & 2 & -1 \end{array} \right)$$

$\nearrow$
 A

$\uparrow$
 b

$$\text{rk } A = 2 \Rightarrow \text{rk}(A|b) = 2$$

$\Rightarrow E$ e un subspațiu
afine de dim 2

$$\begin{cases} 2x + 3y = 7 + 4z - 5w \\ 3x - 5y = -2w - 7z - 1 \end{cases}$$

$$\begin{pmatrix} 2 & 3 \\ 3 & -5 \end{pmatrix}^{-1} = \frac{-1}{19} \begin{pmatrix} -5 & -3 \\ -3 & 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = +\frac{1}{19} \begin{pmatrix} +5 & 3 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} 4z - 5w + 7 \\ -7z - 2w - 1 \end{pmatrix} =$$

$$= +\frac{1}{19} \begin{pmatrix} -z & -31w + 32 \\ 26z & -11w + 23 \end{pmatrix}$$

$$\begin{cases} x = \frac{1}{19} (-t - 31s + 32) \\ y = \frac{1}{19} (26t - 11s + 23) \\ z = t \\ w = s \end{cases}$$

$$E = \left\{ \frac{1}{19} \begin{pmatrix} 32 \\ 23 \\ 0 \\ 0 \end{pmatrix} + \frac{t}{19} \begin{pmatrix} -1 \\ 26 \\ 1 \\ 0 \end{pmatrix} + \frac{s}{19} \begin{pmatrix} 32 \\ -11 \\ 0 \\ 1 \end{pmatrix} \mid t, s \in \mathbb{R} \right\}$$

$$\underline{7 \cdot 1 \cdot 2 \cdot 1}$$

$$\begin{cases} 4x - y + 2z - 5w = 3 \\ 2x + y + 3z - 6w = 3 \\ -x + 5y + 2z - 4w = 4 \end{cases}$$

$$E \subseteq \mathbb{R}^4$$

$$\left(\begin{array}{cccc|c} -1 & 5 & 2 & -4 & 4 \\ 4 & -1 & 2 & -5 & 3 \\ 2 & 1 & 3 & -6 & 3 \end{array} \right) \rightarrow \left(\begin{array}{cccc|c} -1 & 5 & 2 & -4 & 4 \\ 0 & 19 & 10 & -21 & 19 \\ 0 & 11 & 7 & -14 & 11 \end{array} \right)$$

↗ $\det = 23$
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$\text{rk}(A) = 3 \Rightarrow \text{rk}(A|b) = 3 \Rightarrow E \neq \emptyset$ ed è un
 sottospazio affine di
 dimensione 1

$$\begin{pmatrix} 19 & 10 \\ 11 & 7 \end{pmatrix}^{-1} = \frac{1}{23} \begin{pmatrix} 7 & -10 \\ -11 & 19 \end{pmatrix}$$

$$\begin{pmatrix} y \\ z \end{pmatrix} = \frac{1}{23} \begin{pmatrix} 7 & -10 \\ -11 & 19 \end{pmatrix} \begin{pmatrix} -21w + 19 \\ -14w + 11 \end{pmatrix} = \frac{1}{23} \begin{pmatrix} (147 - 140)w + 23 \\ (21 \cdot 11 - 14 \cdot 19)w \end{pmatrix}$$

$$= \begin{pmatrix} \frac{7}{23}w + 1 \\ -\frac{35}{23}w \end{pmatrix}$$

$$\begin{aligned} x &= 5y + 2z - 4w - 4 = 5 \left(\frac{7}{23}w + 1 \right) + 2 \left(-\frac{35}{23}w \right) - 4w - 4 \\ &= \left(-\frac{35}{23} - 4 \right)w + 1 = -\frac{127}{23}w + 1 \end{aligned}$$

$$E = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -127 \\ 7 \\ -35 \\ 23 \end{pmatrix} \mid t \in \mathbb{R} \right\}$$

La generatrice di E è $\text{Span} \left\{ \begin{pmatrix} -127 \\ 7 \\ -35 \\ 23 \end{pmatrix} \right\}$

⇐

7.1.22

$$\begin{cases} x = 1 + 2t \\ y = -2 + 5t \\ z = 3 + 4t \\ w = 3 - 7t \end{cases}$$

$$E \subseteq \mathbb{R}^4$$

$$E = \left\{ \begin{pmatrix} 1 \\ -2 \\ 3 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ 5 \\ 4 \\ -7 \end{pmatrix} \mid t \in \mathbb{R} \right\}$$

$$\dim E = 1$$

$$\text{generatore di } E =$$

$$= \text{Span} \left\{ \begin{pmatrix} 2 \\ 5 \\ 4 \\ -7 \end{pmatrix} \right\}$$

$$t = \frac{1}{2}(x-1)$$

$$y = -2 + \frac{5}{2}(x-1) = \frac{5}{2}x - \frac{9}{2}$$

$$z = 3 + 2(x-1) = 2x + 1$$

$$w = 3 - \frac{7}{2}(x-1) = -\frac{7}{2}x + \frac{13}{2}$$

$$5x - 2y = 9$$

$$2x - z = -1$$

$$+ 7x + 2w = 13$$

$$\left(\begin{array}{ccc|c} 5 & -2 & 0 & 0 & 9 \\ 2 & 0 & -1 & 0 & -1 \\ 7 & 0 & 0 & 2 & 13 \end{array} \right)$$

$\det \neq 0$

e quindi $\text{rk}(A) = \text{rk}(A|b) = 3$
come previsto

$$\underline{7 \cdot 1 \cdot 24}$$

$$\begin{array}{l} H \\ \left\{ \begin{array}{l} x = -2 + t + 3s \\ y = 4 + st - 2s \\ z = -3 + 4t + 5s \\ w = 1 + 3t - s \end{array} \right. \end{array}$$

$$E \subseteq \mathbb{R}^4$$

generare di E : Span $\left\{ \begin{pmatrix} 1 \\ 5 \\ 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ 5 \\ -1 \end{pmatrix} \right\}$ ← *dim 2.*

Considero:

$$\begin{cases} t + 3s = x + 2 \\ 3t - s = w - 1 \end{cases}$$

Risolvenolo si ottiene:

$$t = \frac{1}{10}(x + 3w - 1)$$

$$s = \frac{1}{10}(3x - w + 7)$$

Sostituendo queste espressioni nella 2^a e 3^a eq:

$$z = -3 + \frac{4}{10} (x + 3w - 1) - \frac{2}{10} (3x - w + 7) = \dots$$

$$w = 1 + \frac{3}{10} (x + 3w - 1) - \frac{1}{10} (3x - w + 7) = \dots$$

$\Rightarrow$ obtenemos 2 eq. condiciones independientes: