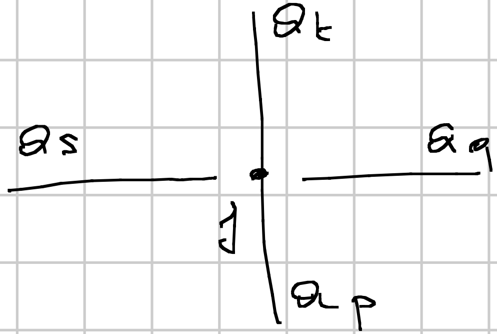
# Teoria dei nodi 19/10/16

$$H_1(S^3 \setminus (K_1 \cup \dots \cup K_n)) = \mathbb{Z}^n$$




$$\pi_1 = \langle x_1 \dots x_{2c} \mid r_1 \dots r_c \rangle$$
$$s_1 \dots s_c$$

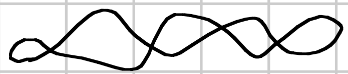
$$s_j : \quad x_p = x_t$$
$$x_q x_p = x_t x_s$$

$$Ab(\pi_1) = \langle x_1 \dots x_{2c} : [\dots] = 1 \quad x_p = x_t \,,\, x_q = x_s \rangle$$

non vede l'inaoais

$\Rightarrow H_1(S^3 \setminus (n)) = H_1(S^3 \setminus$  $)$

$\simeq$

 $n \cdot S^1$

$\Rightarrow H_1 \cong \mathbb{Z}^n$ generato dai

<u>Esercizio</u>: provare che $H_2 \cong \mathbb{Z}^{n-1}$ senza M-V.

$Hf$ ben def $f: M^{(n)} \longrightarrow \mathbb{R}$ nei pti dove $df = 0$

$$\begin{tikzcd}
\mathbb{R}^n_x \arrow[r, "\alpha"] \arrow[rr, bend left, "A"] & M \arrow[r, "f"] & \mathbb{R} \\
\mathbb{R}^n_y \arrow[u, "g"] \arrow[ur, "\beta"] \arrow[urr, bend right, "B"'] & &
\end{tikzcd}$$

$B = A \circ g$

$$\frac{\partial B}{o} = \frac{\partial A}{\partial x_k} \cdot \frac{\partial g_k}{\partial y_i}$$

( $df$ è ben def )

$$\frac{\partial^2 B}{\partial y_i \partial y_j} = \underbrace{\frac{\partial^2 A}{\partial x_k \partial x_h} \cdot \frac{\partial \varphi_k}{\partial y_i} \cdot \frac{\partial \varphi_h}{\partial y_j}} + \underbrace{\frac{\partial A}{\partial x_k}}_{\substack{\| \\ 0 \text{ se } df = 0}} \cdot \frac{\partial^2 \varphi_k}{\partial y_i \partial y_j}$$

$$(HB)(u,v) = (HA)\big(d\varphi(u), d\varphi(v)\big)$$

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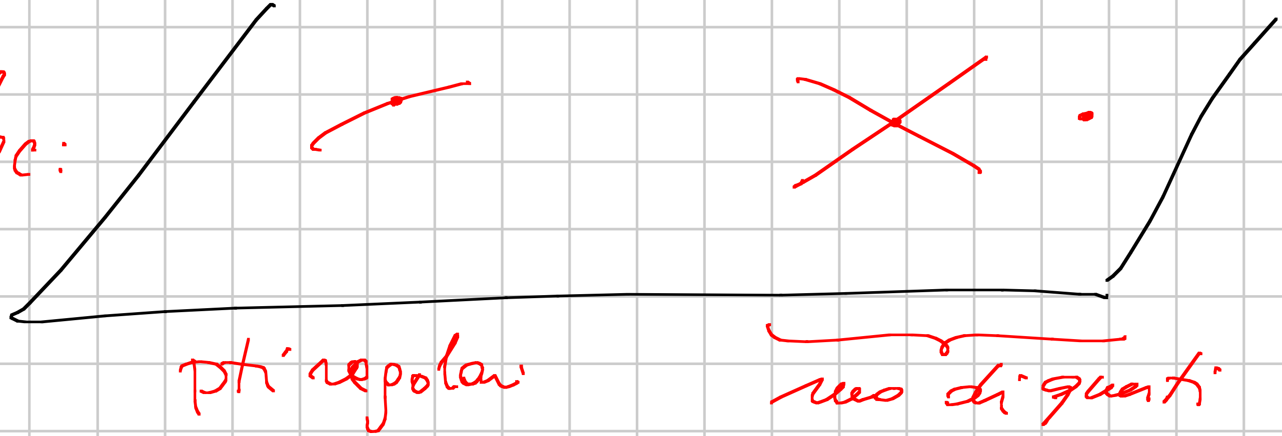
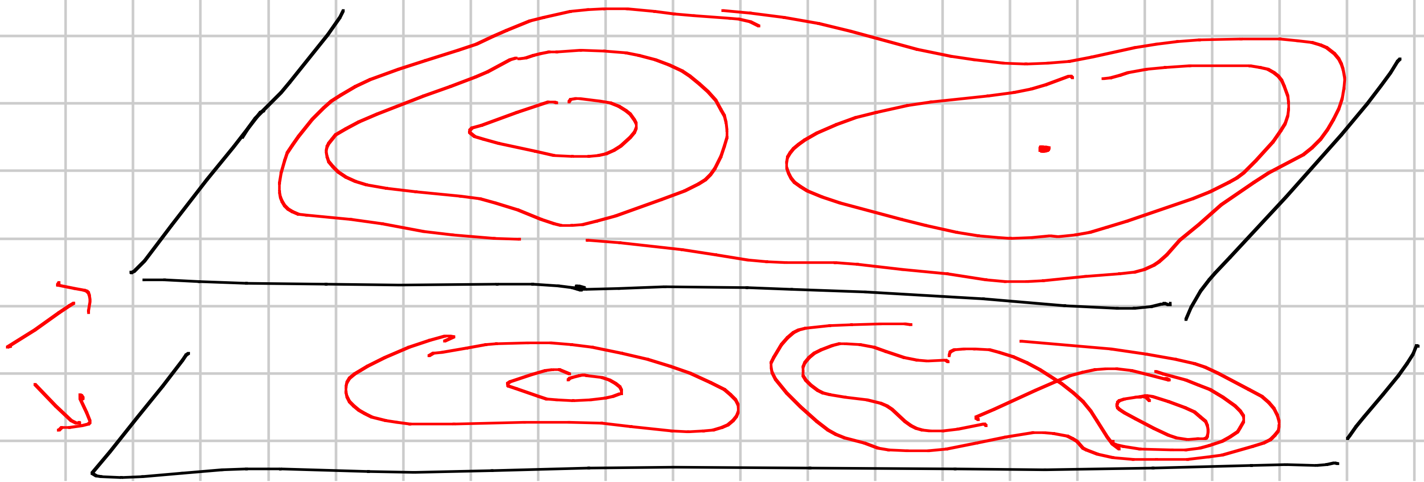
$\Sigma \subset \mathbb{R}^3$ sup. cpt. $\quad z$ altezza t.c. $z|_\Sigma$ Morse con valori critici distinti c valore critico

$(Z/\Sigma_0)^{-1}(c)$

loc:




glob



$$(z|_{\Sigma})^{-1}([c-\varepsilon, c+\varepsilon]) = \text{loc}$$

uno solo di questi

**Prop:** $\Sigma \cong S^2$ liscia o PL, $\Sigma \subset \mathbb{R}^3$

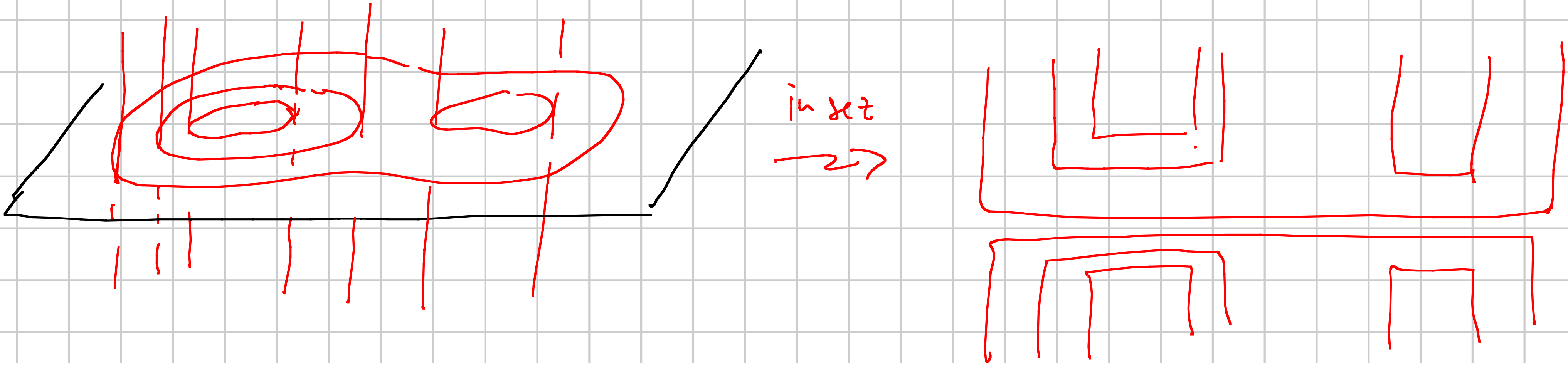
$\Rightarrow \Sigma = \partial B$, $\overline{B} \cong D^3$.

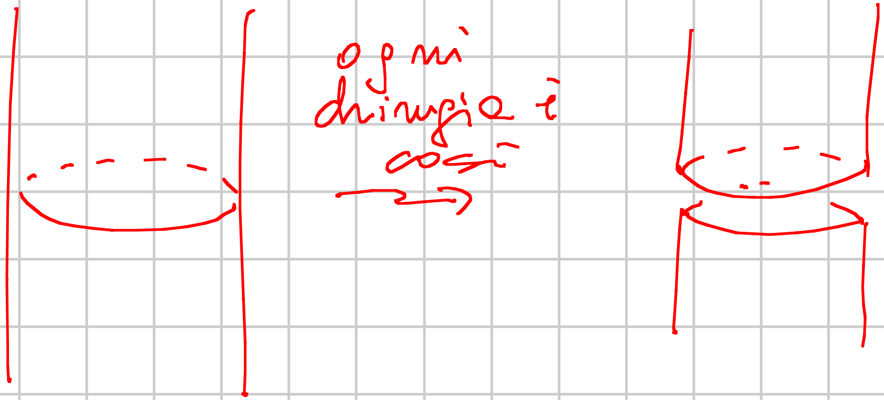
**Dim** (liscia): wlog $z$ sia t.c. $h = z|_{\Sigma}$ Morse con val critici distinti $c_0 < c_1 < \dots < c_m$

Prendo $c_0 < a_1 < c_1 < \dots < a_n < c_n$ dunque $h^{-1}(a_j)$ = unione di circonferenze sul piano $\mathbb{R}^2 \times \{z\}$

$\Longrightarrow$ (J-S 2D) ognuna borda un 2-disco.

Opero chirurgie lungo $\Sigma$ ad ogni altezza $a_j$



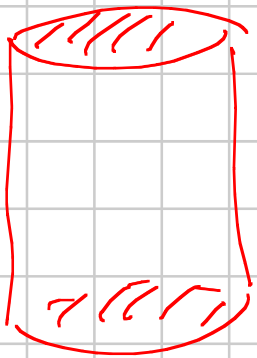



Trovo unione di superfici chiuse $\Sigma_k$

<u>Claim</u>: ciascuna (ignorando le altre) è una sfera $S^2$ che borda un disco $D^3$. Infatti:

Ogni $\Sigma_k$ consiste di alcuni tappi ad altezza

poco più che $a_{j-1}$, alcuni ad altezza poco meno $a_j$
+ parte compresa tra $a_{j-1}$ e $a_j$, la quale consiste di tubi verticali o parti contenenti un pto critico:



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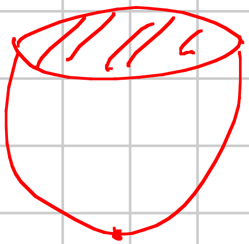

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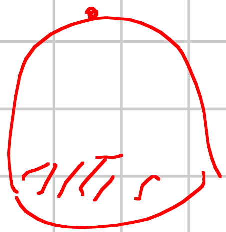

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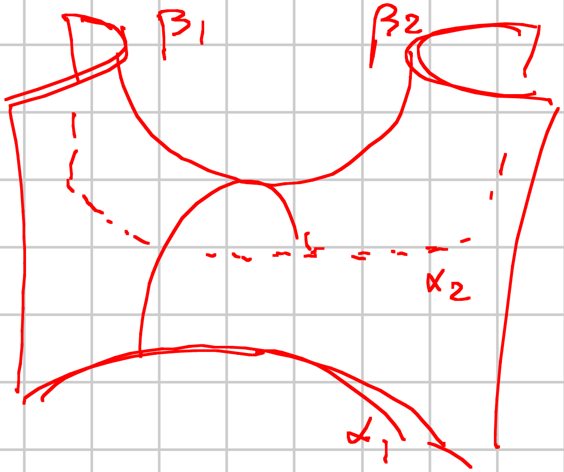
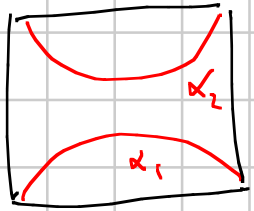


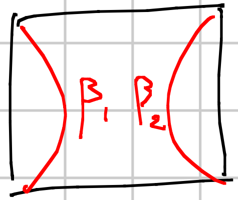

Dall'alto:

$c_j - \varepsilon$




$Q \times \{c_j - \varepsilon\}$

$c_j + \varepsilon$

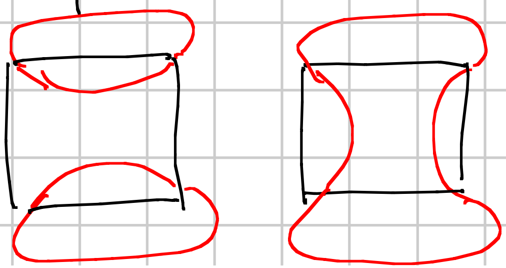



$Q \times \{c_j + \varepsilon\}$

su $(\mathbb{R}^2 \setminus Q) \times [c_j - \varepsilon, c_j + \varepsilon]$
è "un prodotto"

4 possibilità

1.



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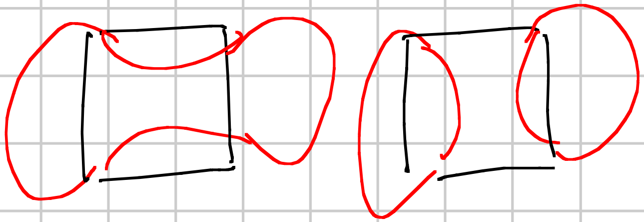
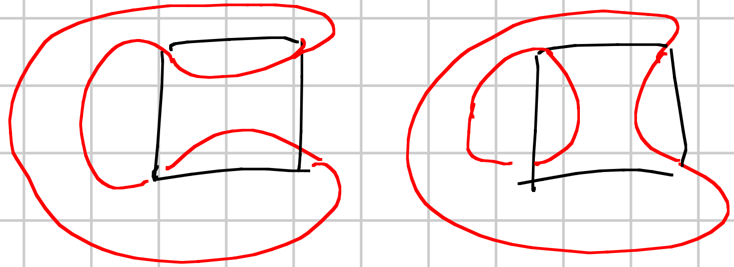
3. 

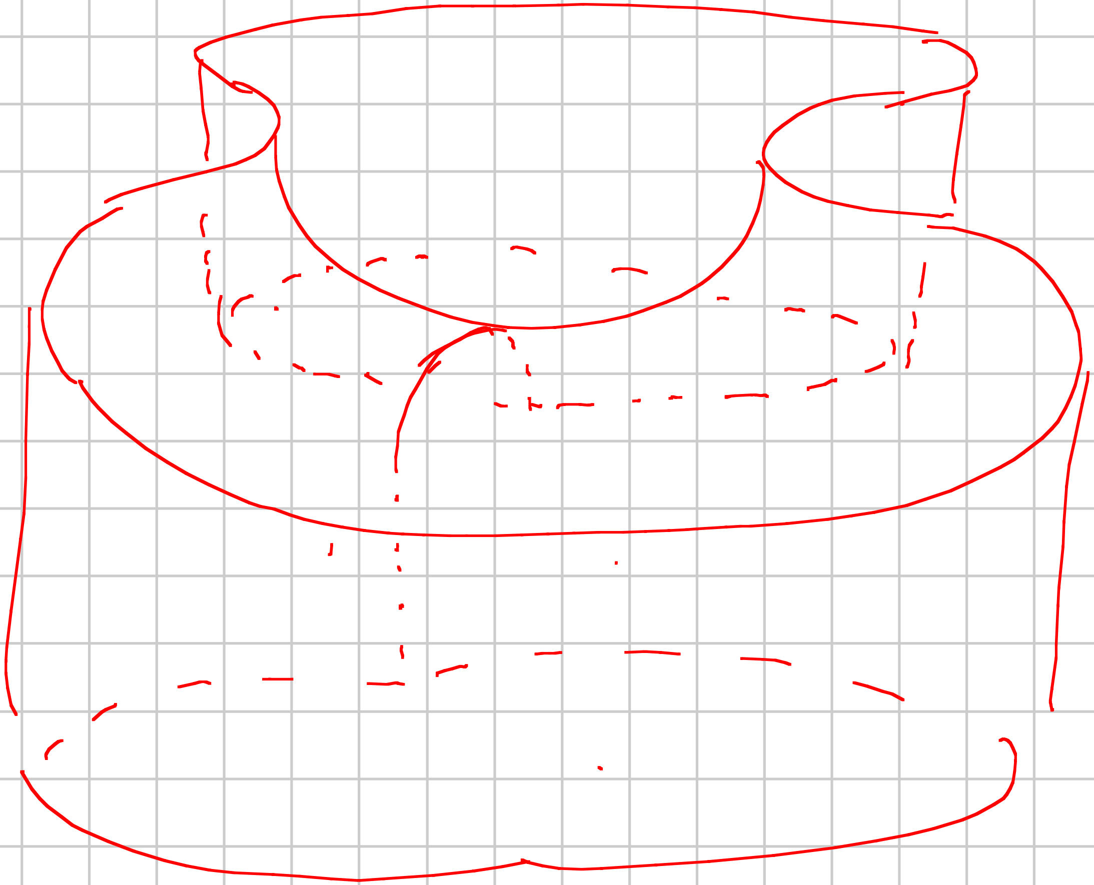
4. 

<u>Ora</u>: 3 = 1 a testa in più

4 = 2 a testa in più

Quindi basta provare che $\Sigma_i \cong S^2 \cong \partial D^3$ per 1 e 2

1.



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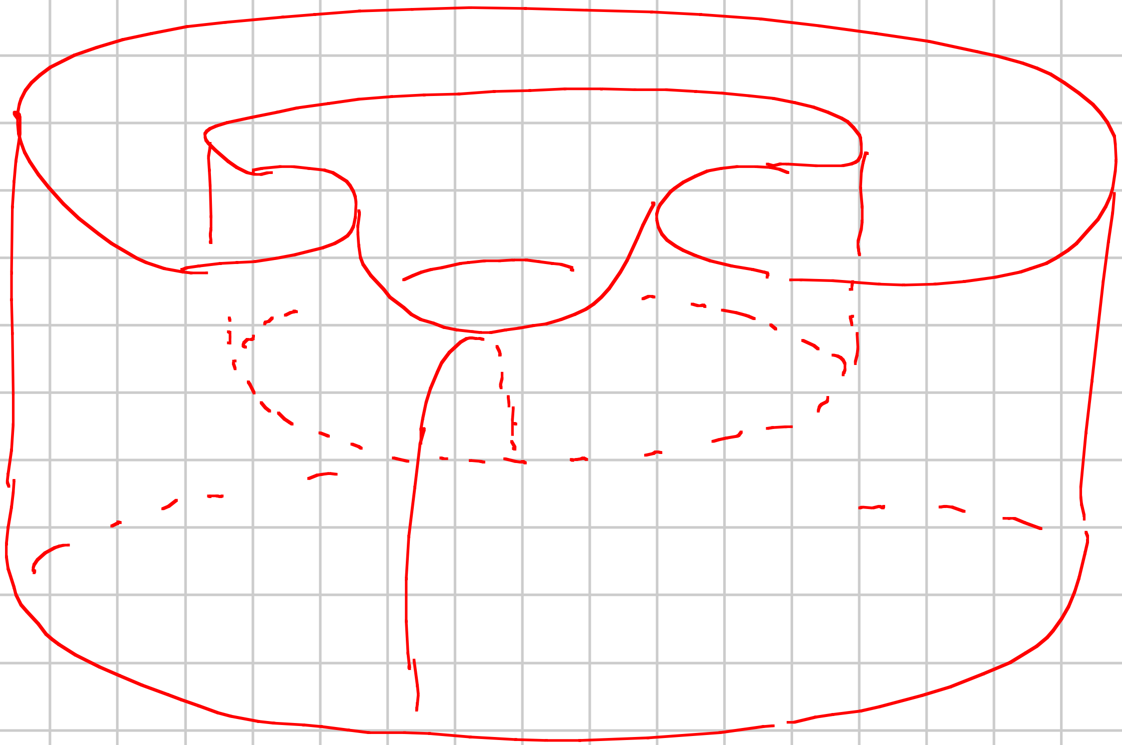
2.



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