Geom 28 / 4 / 2017

10.2.10 forma canonica in $\mathbb{R}$ (se ha senso).

in $\mathbb{C}$

$$\frac{1}{\sqrt6}\begin{pmatrix} 2 & 0 & \sqrt2 \\ 1 & \sqrt3 & -\sqrt2 \\ -1 & \sqrt3 & \sqrt2 \end{pmatrix} = A$$

$$\det A = \left(\frac{1}{\sqrt6}\right)^3 \left(2 \cdot 2\sqrt6 + \sqrt2 \cdot 2\sqrt3\right) = \frac{1}{6\sqrt6}\, 6 \cdot \sqrt6 = 1$$

A è ortogonale. Per quanto visto a teoria esiste M ortogonale t.c.

${}^tM\,A\,M$ = diagonale a blocchi.

blocchi di tipo 1×1 (± 1)

di tipo 2×2 $\boxed{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = R_\theta}$

<u>OSS</u> se fosse tutto

a blocchi 1×1

almeno due uguali $\left(\begin{array}{cc|c} \varepsilon_1 & 0 & \\ 0 & \varepsilon_2 & \\ \hline & & \varepsilon_2 \end{array}\right)$

$\begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$ o $\begin{pmatrix} -1 & \\ & -1 \end{pmatrix}$ è blocco 2×2 di rotazione

$\Rightarrow$ ${}^tMAM$ può sempre essere scritto nella forma

$$\begin{pmatrix} R_\theta & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & \pm 1 \end{pmatrix}$$ — Per capire se $+1$ o -1 basta guardare $\det A$

$$\det A = \left(2 \cdot 2\sqrt{6} + \sqrt{2} \cdot 2\sqrt{3}\right)\frac{1}{6\sqrt{6}} =$$

$$= +1 \quad \Rightarrow \quad \text{ho } \boxed{+1}$$

Per trovare θ: $\quad tr(A) = tr({}^tM A M) = 2\cos\theta + 1$

$$\theta = \arccos\left(\frac{1}{2}\left(tr(A) - 1\right)\right) = \arccos\left(\frac{1}{2}\left(1+\sqrt{2}+\sqrt{3}\right)\right)$$

$$ {}^{t}M\,A\,M = \left(\begin{array}{cc|c} \cos\theta & -\sin\theta & \\ \sin\theta & \cos\theta & \\ \hline & & +1 \end{array}\right) $$

$$ \frac{1}{\sqrt{6}} \begin{pmatrix} 2 & 0 & \sqrt{2} \\ 1 & \sqrt{3} & -\sqrt{2} \\ -1 & \sqrt{3} & \sqrt{2} \end{pmatrix} = \begin{pmatrix} 2/\sqrt{6} & 0 & \sqrt{2}/\sqrt{6} \\ 1/\sqrt{6} & \sqrt{3}/\sqrt{6} & -\sqrt{2}/\sqrt{6} \\ -1/\sqrt{6} & \sqrt{3}/6 & \sqrt{2}/\sqrt{6} \end{pmatrix} $$

10.2.11

$$\begin{pmatrix} 3 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & 3 \end{pmatrix} \quad \text{è simmetrica} \implies \exists\, M \text{ ortogonale t.c.}$$

$$ {}^{t}M A M = \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix}$$

$$P_A(t) = \det \begin{pmatrix} t-3 & 1 & -2 \\ 1 & t & -1 \\ -2 & -1 & t-3 \end{pmatrix} =$$

$$= t^3 - 6t^2 + 9t + 2 +$$
$$- 4t + 2$$
$$- t + 3$$

$$-t+3=$$

$$= t^3 - 6t^2 + 3t + 10 =$$

$$= (t+1)(t^2 - 7t + 10) = (t+1)(t-2)(t-5)$$

$$\begin{pmatrix} -1 & & \\ & 2 & \\ & & 5 \end{pmatrix}$$

<u>10.2.12</u>

$$A = \begin{pmatrix} 0 & 2 & -1 \\ -2 & 0 & -3 \\ 1 & 3 & 0 \end{pmatrix}$$ è antisimmetrica

$\Rightarrow$ esiste M ortogonale t.c.

$$ {}^tMAM = \left(\begin{array}{cc|c} 0 & -\lambda & \\ \lambda & 0 & \\ \hline & & 0 \end{array}\right) $$

e gli autovalori di A sono $\pm i\lambda$

$$ p_A(t) = \det \begin{pmatrix} t & -2 & 1 \\ 2 & t & 3 \\ -1 & -3 & t \end{pmatrix} = t^3 + t + 9t + 4t + 6 - 6 $$

$$ = t(t^2 + 14) = $$

$$ = t(t + i\sqrt{14})(t - i\sqrt{14}) $$

$$ \Longrightarrow \lambda = \sqrt{14} $$

10.2.13 $A = \begin{pmatrix} 2 & 1+3i \\ 1-3i & -1 \end{pmatrix}$ hermitiana

$\Rightarrow$ esiste M unitaria t.c. ${}^t\bar{M} A M = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$

$\lambda_1, \lambda_2 \in \mathbb{R}$

$\lambda_1 + \lambda_2 = 1 = \operatorname{tr} A$

$\lambda_1 \cdot \lambda_2 = -12 = \det A$ $\Rightarrow$ $\lambda_1 = 4$

$\lambda_2 = -3$

$p_A(t) = t^2 - \operatorname{tr} A \, t + \det A$

10.2.14

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \qquad A^* A = I$$

$\Rightarrow A$ unitaria

$\exists\, M$ unitaria tc

$$M^{-1} A M = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$

$$\lambda_1 + \lambda_2 = 2/\sqrt{2} = \sqrt{2}$$

$$\lambda_1 \cdot \lambda_2 = 1$$

$$\lambda_1 = \frac{1+i}{\sqrt{2}}$$

$$\lambda_2 = \frac{1-i}{\sqrt{2}}$$

10.2.15

$$A = \begin{pmatrix} -i & i+\sqrt{3} \\ i-\sqrt{3} & 2i \end{pmatrix} \qquad A^* = -A$$

$\Rightarrow A^* A = A A^* \Rightarrow A$ è normale $\Rightarrow$

$\Rightarrow A$ diagonalizzabile con M unitaria

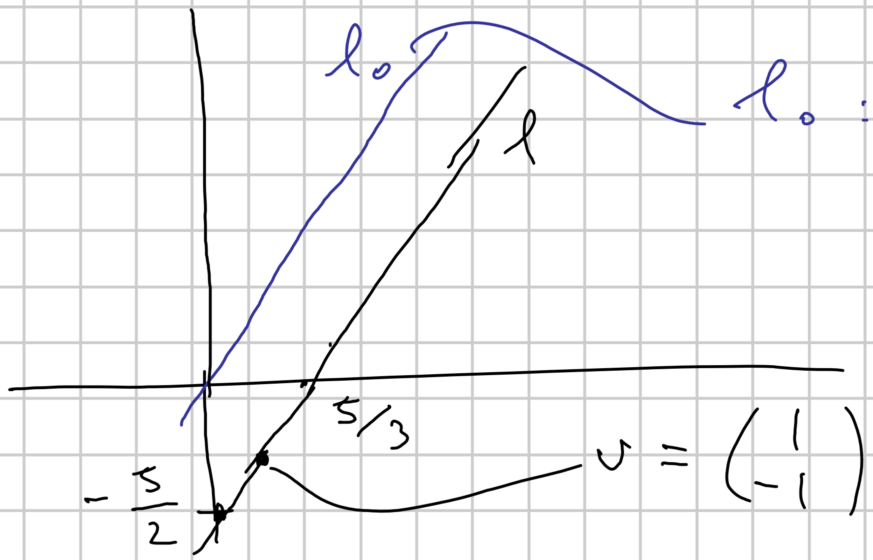
$\lambda_1 + \lambda_2 = i = \mathrm{tr}\, A \qquad \lambda_1 = \dfrac{i+\sqrt{-25}}{2} = 3i$

$\lambda_1 \cdot \lambda_2 = 2+4 = 6 \qquad \lambda_2 = \dfrac{i-\sqrt{-25}}{2} = -2i$

11. 1. 2 Trovare l'espressione delle seguenti isometrie di $\mathbb{R}^2$

$\hookrightarrow$ Riflessione rispetto alla retta

$\ell : \quad 3x - 2y = 5$



$\ell_0 : 3x - 2y = 0$

$v = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \in \ell$

cerco riflessione rispetto a l

$v = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \in l$

posso traslare di $-v$

faccio riflessione rispetto a l_0,

infine trasla di v

$$\sigma_0 \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} - 2 \frac{\left\langle \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \end{pmatrix} \right\rangle}{\left\| \begin{pmatrix} 3 \\ -2 \end{pmatrix} \right\|^2} \begin{pmatrix} 3 \\ -2 \end{pmatrix} =$$

$$= \begin{pmatrix} x \\ y \end{pmatrix} - 2 \frac{3x - 2y}{13} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = \frac{1}{13} \begin{pmatrix} -5 & 12 \\ 12 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\sigma\begin{pmatrix}x\\y\end{pmatrix} = v + \sigma_0\left(\begin{pmatrix}x\\y\end{pmatrix} - v\right) = \begin{pmatrix}-5/13 & 12/13\\ 12/13 & 5/13\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \frac{10}{13}\begin{pmatrix}3\\-2\end{pmatrix}$$

<u>11.1.2</u> rotazione ρ di angolo $-\frac{\pi}{6}$ attorno a $\begin{pmatrix}\sqrt{3}\\-2\end{pmatrix}$

rotaz. di $-\frac{\pi}{6}$ attorno a O: $R_\theta = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} =$

$\overset{\displaystyle -\frac{\pi}{6}}{\underset{\displaystyle \theta}{\shortparallel}}$

$$= \begin{pmatrix}\sqrt{3}/2 & 1/2\\ -1/2 & \frac{\sqrt{3}}{2}\end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \xmapsto{-v} \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} \sqrt{3} \\ -2 \end{pmatrix} \xmapsto{R_\theta} R_\theta\left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} \sqrt{3} \\ -2 \end{pmatrix} \right) \longmapsto$$

$$\xrightarrow{+v} v + R_\theta\left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} \sqrt{3} \\ -2 \end{pmatrix} \right) =$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \sqrt{3} - 5/4 \\ \frac{3\sqrt{3}}{2} - 2 \end{pmatrix}$$