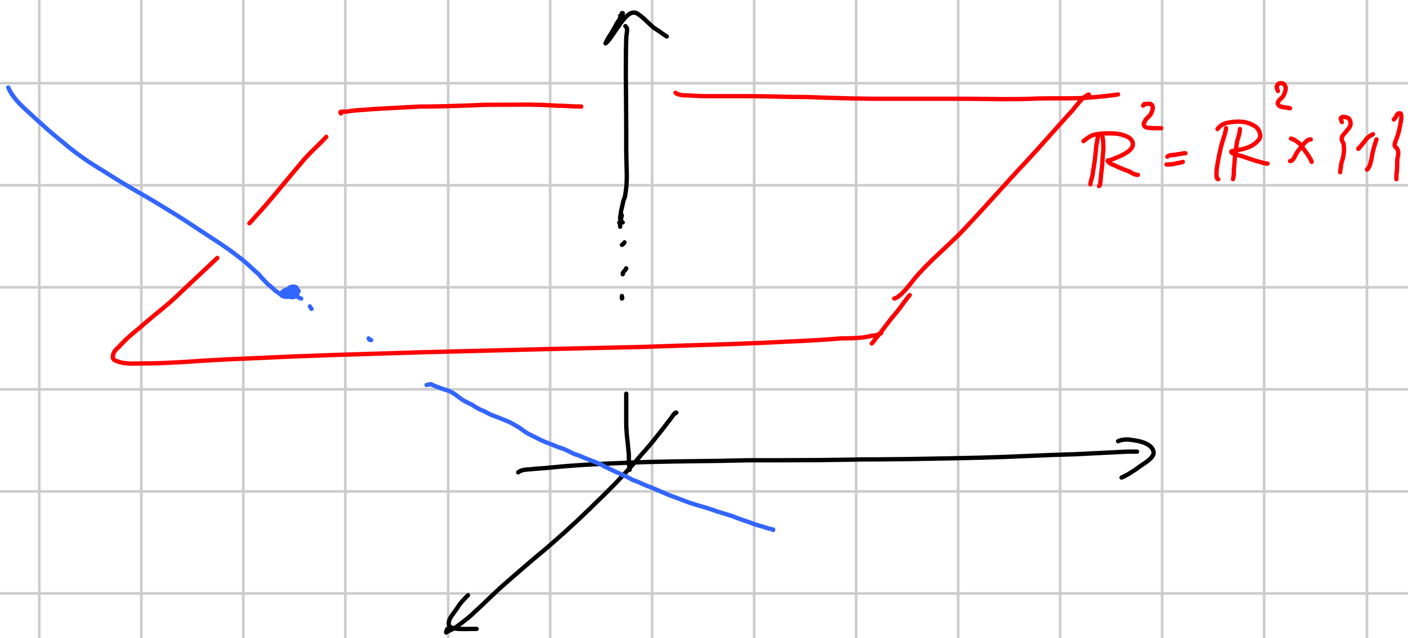
Geometrie 21/4/15

$\mathbb{P}^2(\mathbb{R})$ = le rette in $\mathbb{R}^3$

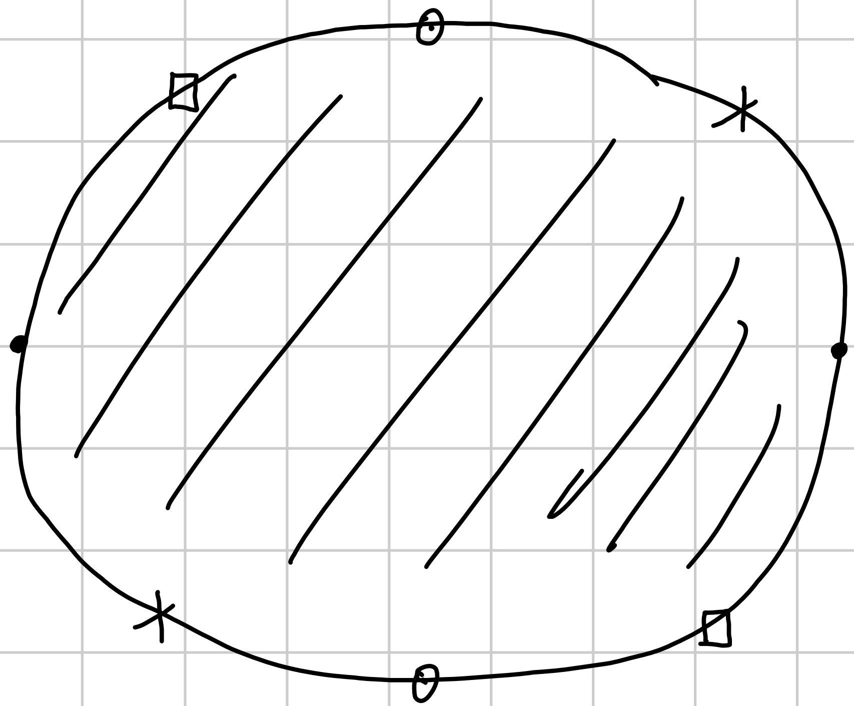


$$\Rightarrow \mathbb{P}^2(\mathbb{R}) = \mathbb{R}^2 \cup \underbrace{\mathbb{P}(\mathbb{R}^2 \times \{0\})}_{\mathbb{P}^1(\mathbb{R})}$$

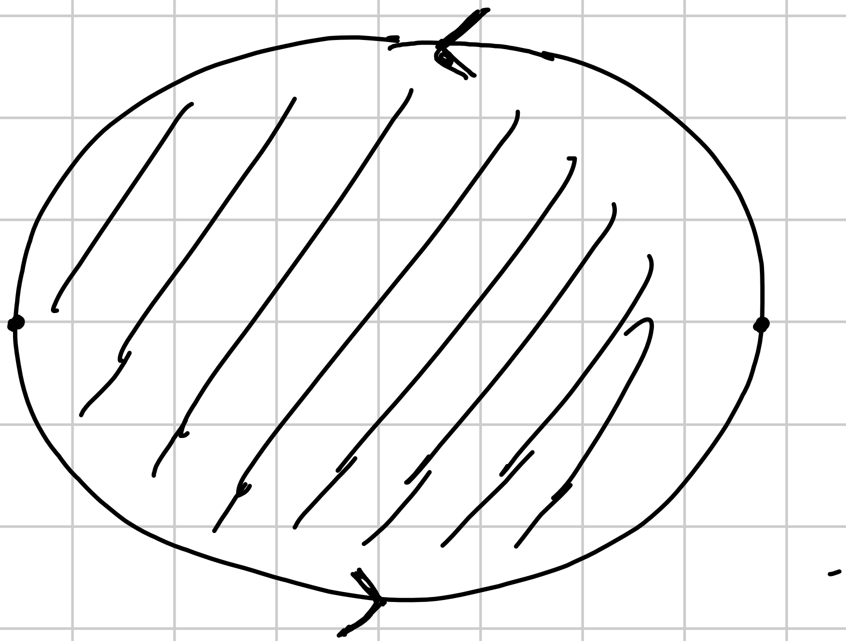
- i pti all'∞ di $\mathbb{R}^2$
- le direzioni delle rette in $\mathbb{R}^2$

$\Rightarrow \mathbb{P}^2(\mathbb{R})$ si ottiene da $\mathbb{R}^2$ aggiungendo i punti all'orizzonte MA: quelli opposti sono uguali.

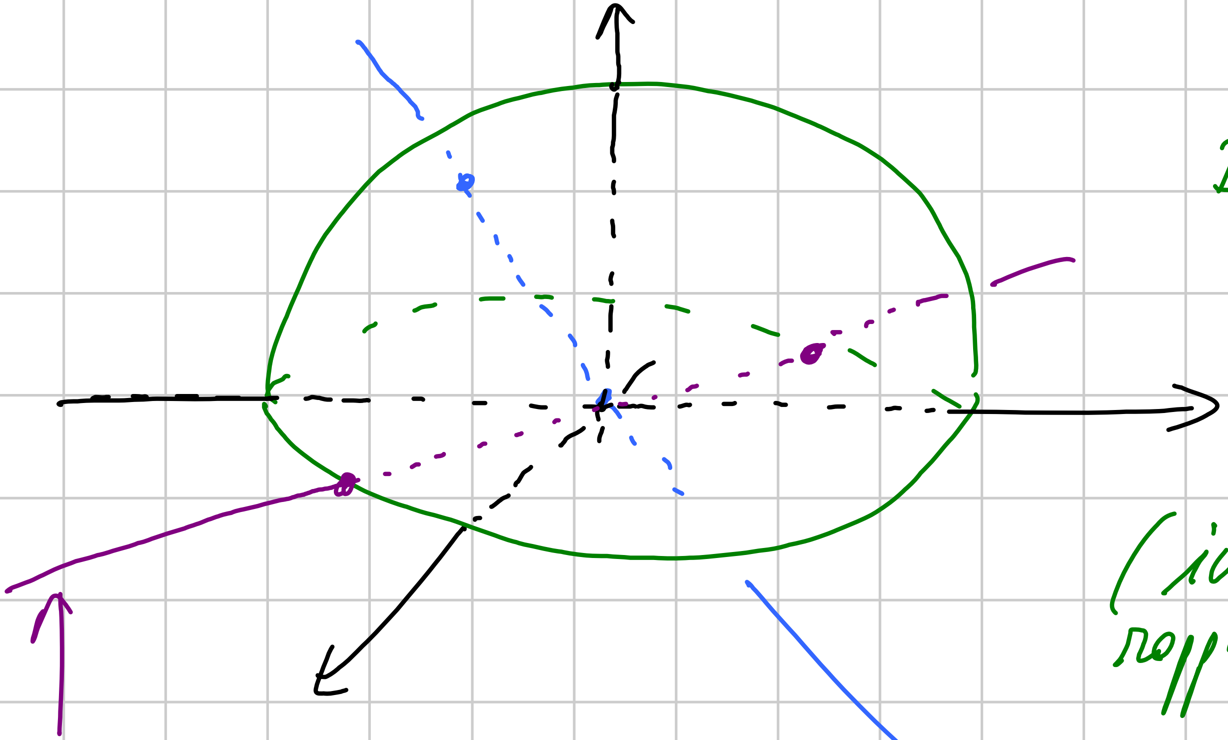
$\Rightarrow \mathbb{P}^2(\mathbb{R})$ si ottiene da un disco chiuso (cerchio con l'interno e il bordo) identificando fra loro i punti antipodali del bordo:



sintetizzo così:



Oppure.



$$D^2_+ = \{x \in \mathbb{R}^3 : \|x\| = 1,\ x_3 \geq 0\}$$

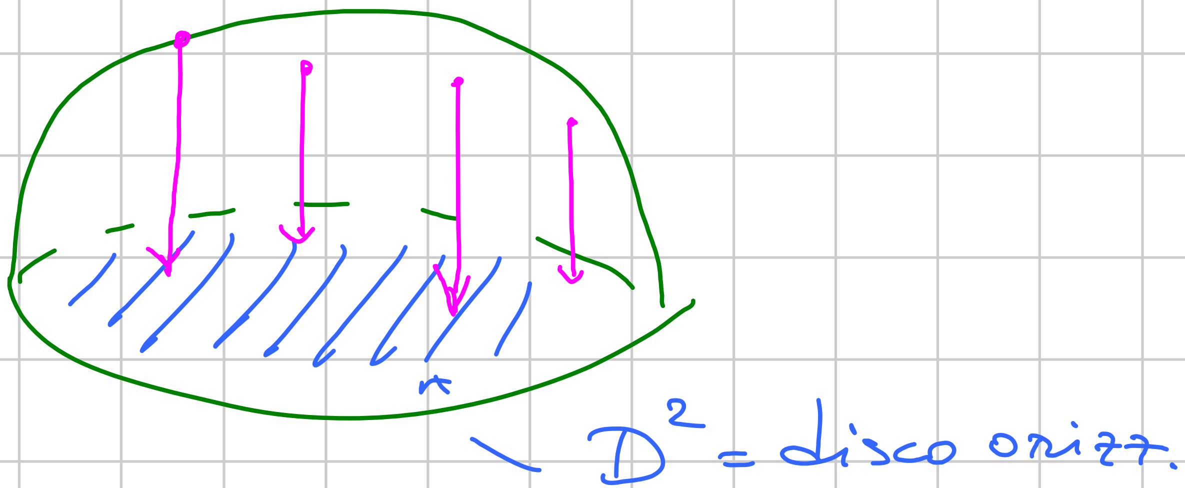
(insieme di rappresentanti per $\mathbb{P}^2(\mathbb{R})$ sovrabbondante)

retta orizz :
due rappresentanti
(sul bordo di D^2_+ : antipodali sull'equatore)

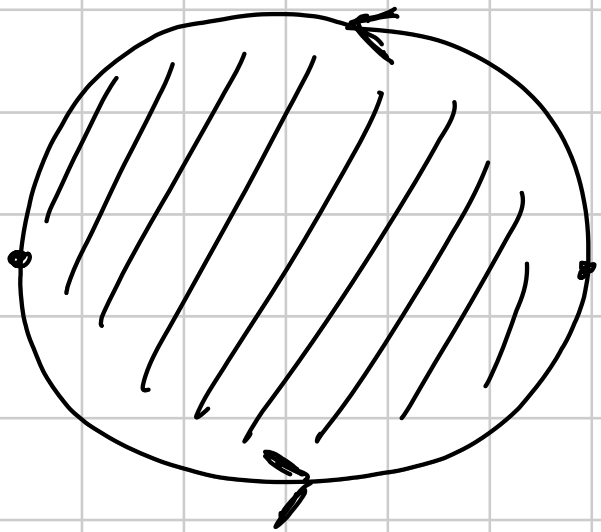
non orizz : unico rappr ;

$\Rightarrow \mathbb{P}^2(\mathbb{R})$ si ottiene da D^2_+ identificando i punti antipodali sul bordo.

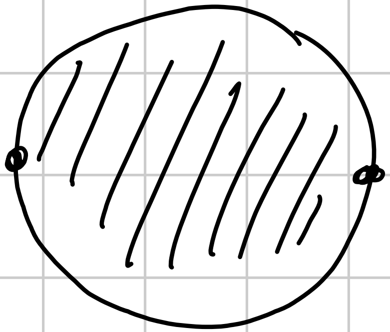
Con la proiez. verticale identifico D^2_+ e D^2:



$\Longrightarrow \mathbb{P}^2(\mathbb{R})$ si ottiene dal disco D^2 identificando pti antipodali sul bordo

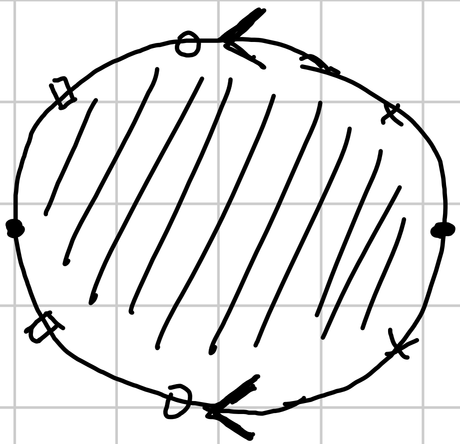


Oggetti (superfici) ottenibili attaccando fra loro lati di un poligono (inteso in modo flessibile:



= poligono con due lati).

1.



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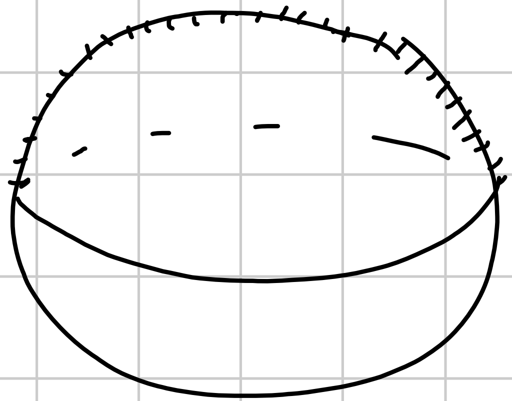
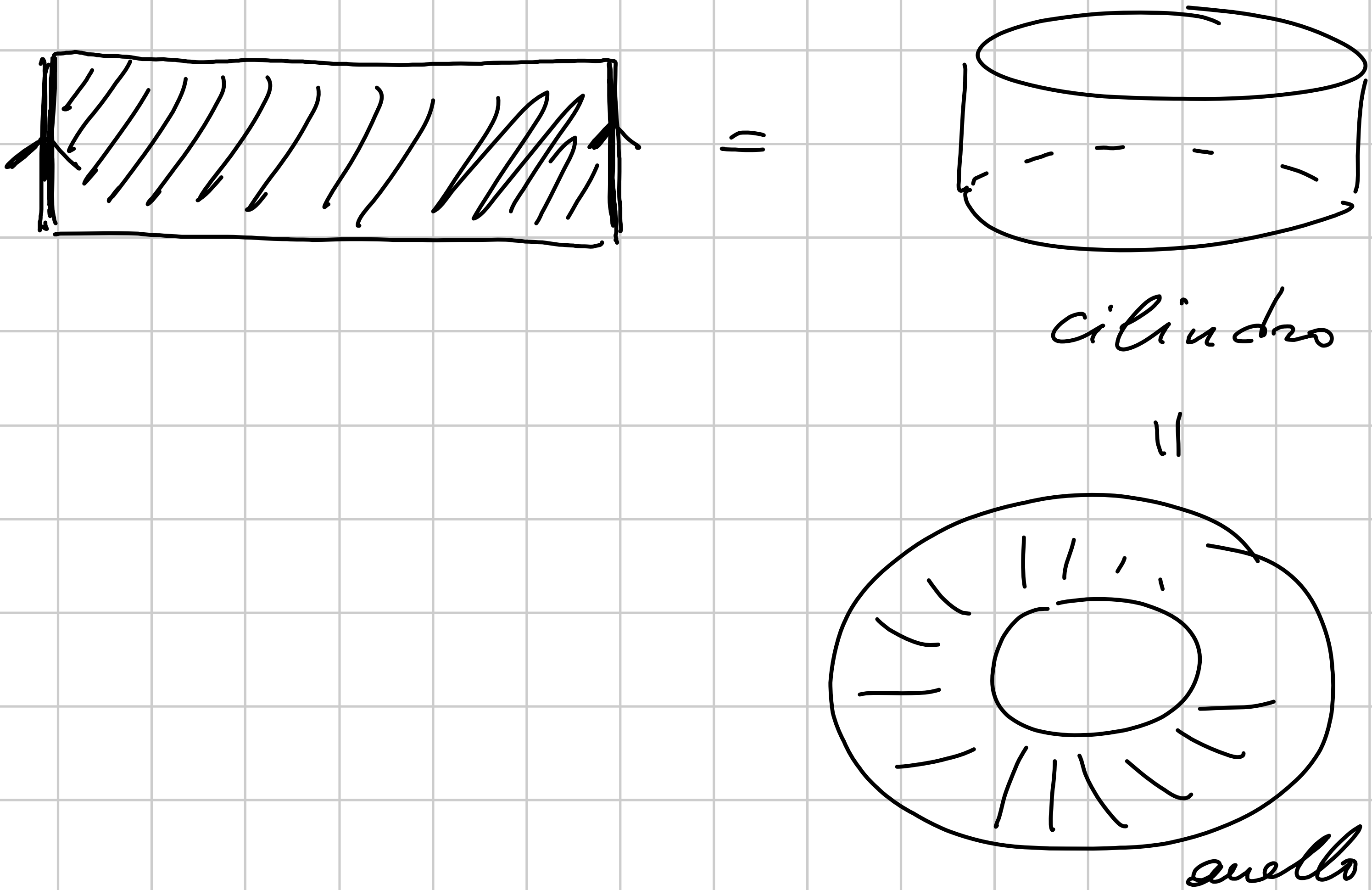
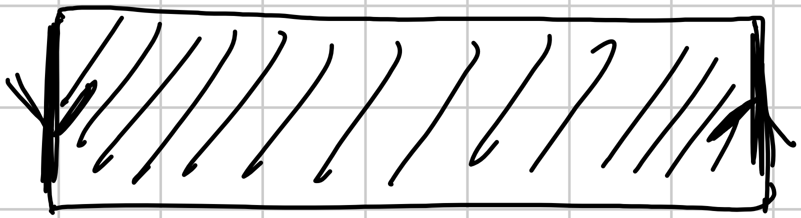
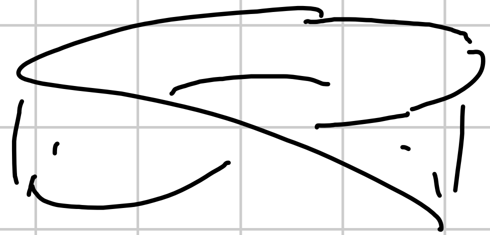


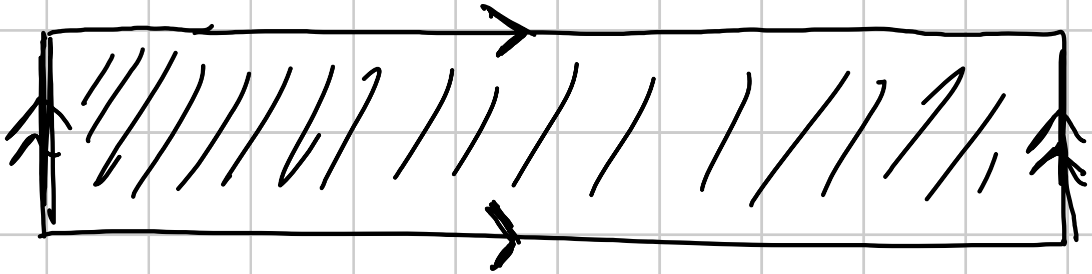
1.

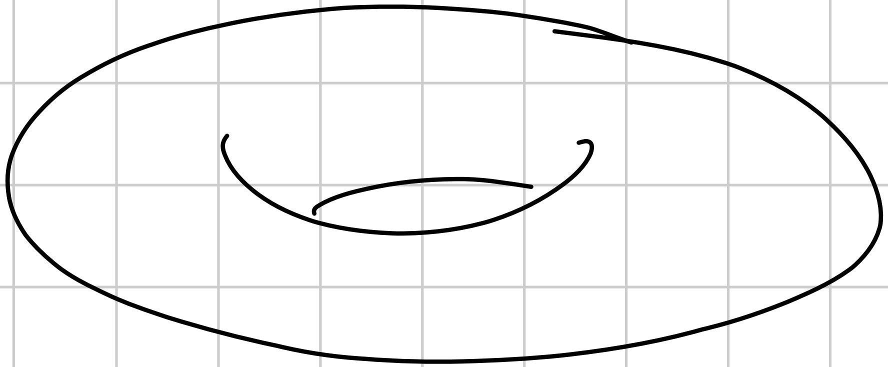


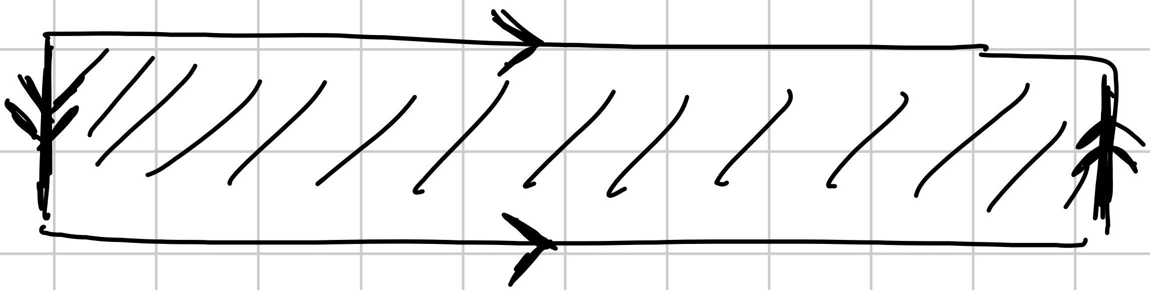
3.  = 

nastro di Möbius

superficie monolatera (non si può colorare con due colori ai lati opposti)

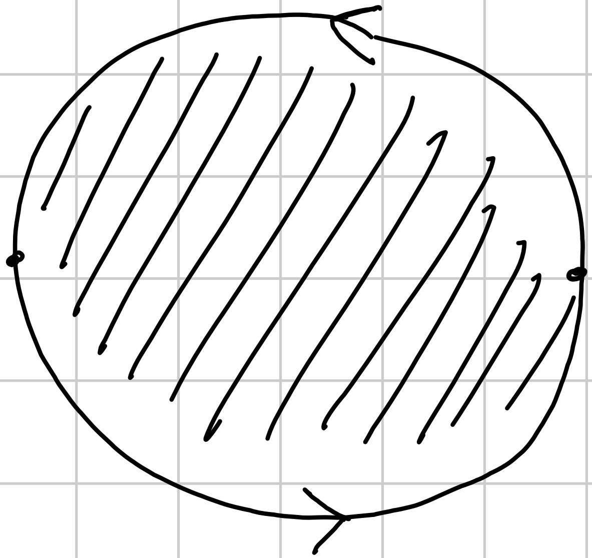
4. 

=  toro

5. 

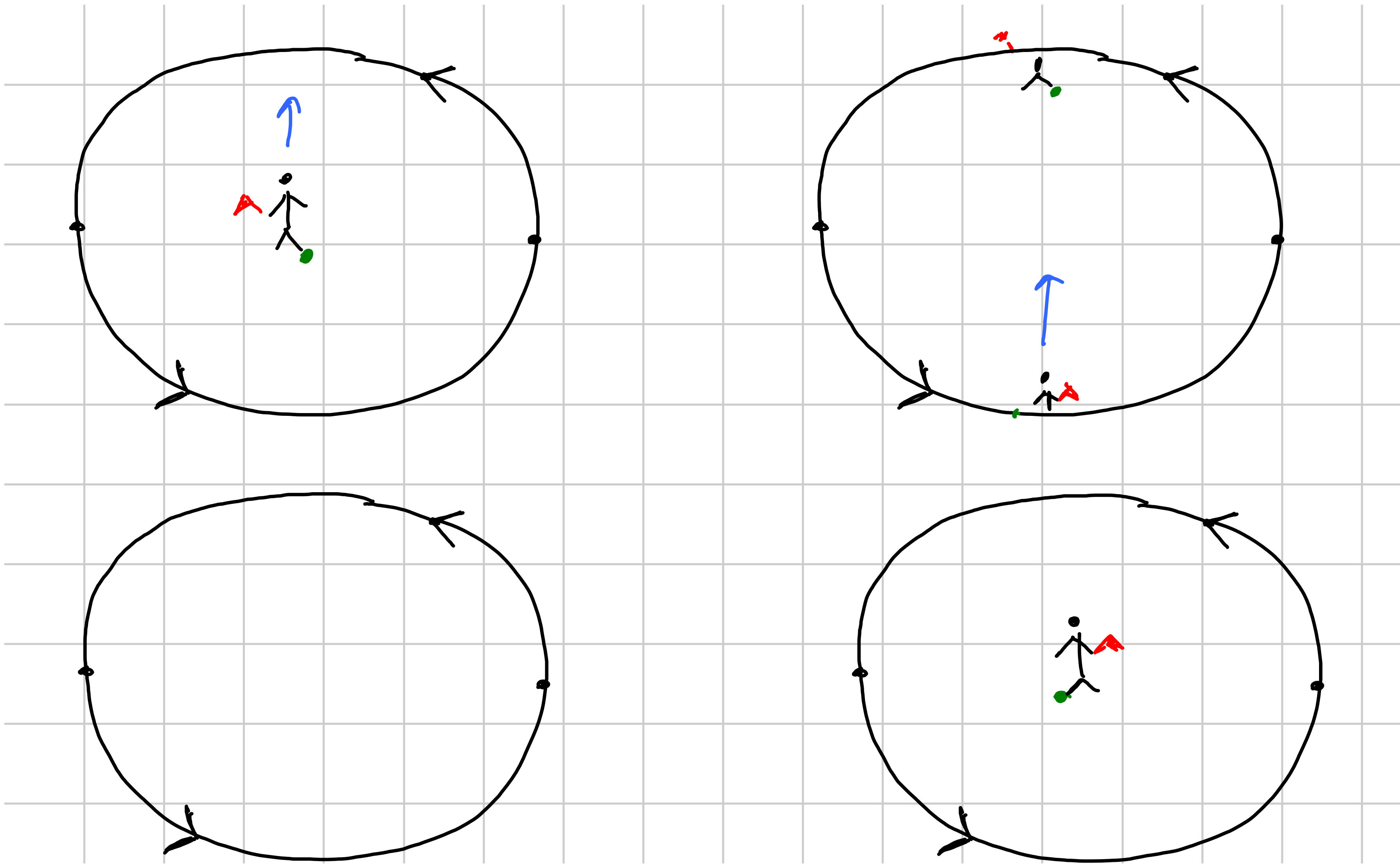
E' una superficie che non si può disegnare in $\mathbb{R}^3$ (in $\mathbb{R}^4$ sì):
"bottiglia di Klein".

6. $\mathbb{P}^2(\mathbb{R})$

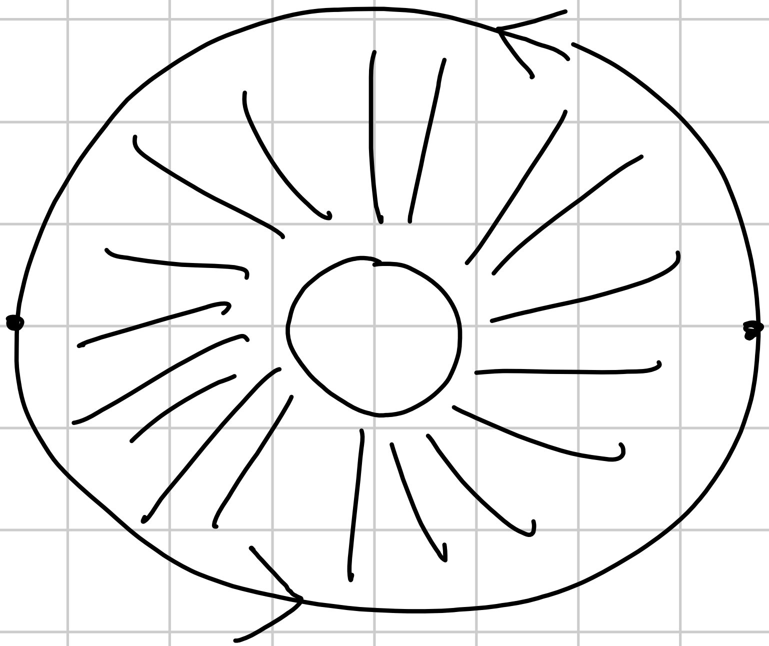


<u>Fatto</u>: non si può disegnare in $\mathbb{R}^3$ –

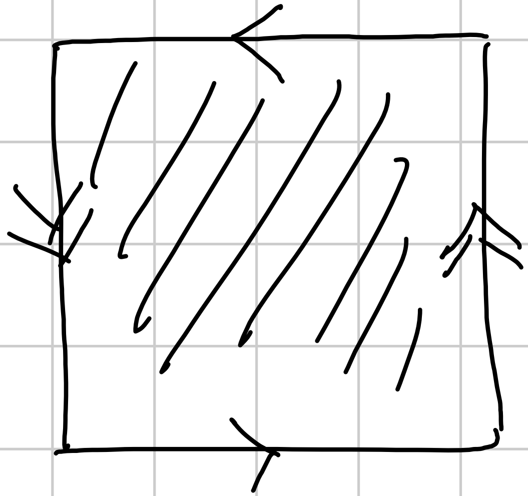
Però esiste. Ed è "monolatera" :

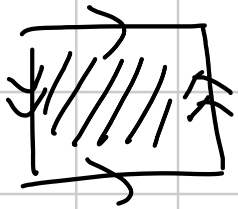


Esercizi: (1) Dire cosa si ottiene togliendo un dischetto a $\mathbb{P}^2(\mathbb{R})$:



(2) Così

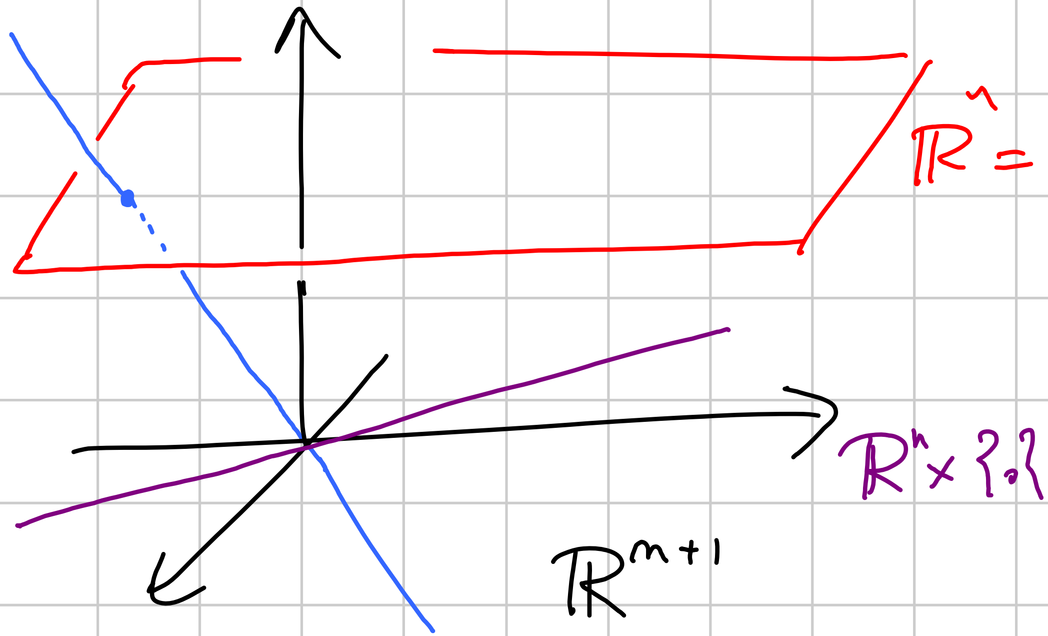


( = toro ;  = Klein)

(3) Cosa succede tagliando a metà un nastro di Möbius?

(4) Cosa succede tagliando in tre un nastro di Möbius?

In generale: $\mathbb{P}^n(\mathbb{R}) = \left(\mathbb{R}^n \times \{1\}\right)$
$\cup\ \mathbb{P}(\mathbb{R}^n \times \{0\})$



$$\|$$
$$\mathbb{R}^n \cup \underbrace{\mathbb{P}^{n-1}(\mathbb{R})}_{\text{i pti all'}\infty \text{ di } \mathbb{R}^n}$$

$\Longrightarrow \mathbb{P}^n(\mathbb{R})$ si ottiene da $\mathbb{R}^n$ aggiungendo i suoi pti all'∞, cioè le direzioni (no verso) delle sue rette affini, che costituiscono un $\mathbb{P}^{n-1}(\mathbb{R})$ –