Geom 19/5/2017

14.3.3 (c)

$$\alpha(s) = \begin{pmatrix} 3s + s^3 \\ s \cdot \log(s) \\ \cos(\pi \cdot s) \end{pmatrix}$$

$$\alpha'(s) = \begin{pmatrix} 3 + 3s^2 \\ \log(s) + 1 \\ -\pi \sin(\pi \cdot s) \end{pmatrix}$$

rif. di Frenet, curv. tors. in $\alpha(s_0)$, $\boxed{s_0 = 1}$

vettore tangente unitario (1° del rif. di Frenet)

$$t(s_0) = \frac{\alpha'(s_0)}{\|\alpha'(s_0)\|} =$$

$$= \begin{pmatrix} 6 \\ 1 \\ 0 \end{pmatrix} \Big/ \sqrt{37}$$

$$\alpha'''(s) = \begin{pmatrix} 6s \\ 1/s \\ -\pi^2 \cos(\pi \cdot s) \end{pmatrix} \qquad \alpha'''(s_0 = 1) = \begin{pmatrix} 6 \\ 1 \\ -\pi^2 \end{pmatrix}$$

$$\alpha'(s_0) \wedge \alpha'''(s_0) = \begin{pmatrix} 6 \\ 1 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 6 \\ 1 \\ -\pi^2 \end{pmatrix} = \begin{pmatrix} -\pi^2 \\ +6\pi^2 \\ 0 \end{pmatrix}$$

$$b(s_0 = 1) = \frac{\alpha'(s_0) \wedge \alpha'''(s_0)}{\|\alpha'(s_0) \wedge \alpha'''(s_0)\|} = \frac{1}{\sqrt{37}} \begin{pmatrix} -1 \\ +6 \\ 0 \end{pmatrix}$$

3° vett. di [illegible] di Frenet

$$n(s_0 = 1) = b(s_0) \wedge t(s_0) = \begin{pmatrix} -1 \\ 6 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 6 \\ 1 \\ 0 \end{pmatrix} \cdot \frac{1}{37} =$$

$$= \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

Curvatura

$$\kappa = \frac{\|\alpha' \wedge \alpha''\|}{\|\alpha'\|^3} = \frac{\sqrt{37}\cdot \pi^2}{\sqrt{37}^3} = \frac{\pi^2}{37}$$

$$\alpha'''(s) = \begin{pmatrix} 6 \\ -1/s^2 \\ \pi^3 \sin(\pi s) \end{pmatrix} \qquad \alpha'''(s_0 = 1) = \begin{pmatrix} 6 \\ -1 \\ 0 \end{pmatrix}$$

$$\tau(s_0) = \frac{\langle \alpha'(s_0) \wedge \alpha''(s_0) \,|\, \alpha'''(s_0) \rangle}{\|\alpha'(s_0) \wedge \alpha''(s_0)\|} = \frac{-12\pi^2}{37\pi^4} = \frac{-12}{37\pi^2}$$

15.2.1

$$d\left(e^{x+\cos(y)} \cdot \log\left(\frac{\cos x}{y}\right)\right) = \frac{\partial}{\partial x}$$

$$= e^{x+\cos y}\left(\log\left(\frac{\cos x}{y}\right) + \frac{y}{\cos y}\left(-\sin x\right)\right)\boxed{dx} +$$

$$+\ e^{x+\cos y}\left(-\sin y \log\left(\frac{\cos x}{y}\right) - \frac{\cos x}{y^2}\ \cancel{\frac{y}{\cos x}}\right) dy$$

$$\frac{\partial}{\partial y}$$

15.2.2

$$d\left(e^{xy}\cos x\, dx + e^{x-y}\sin(xy)\, dy\right) =$$

$$= -\underbrace{x e^{xy}\cos x\, dx\, dy}_{} + \underbrace{e^{x-y}\left(\sin(xy) + y\cos(xy)\right)}_{} dx\, dy$$

$$\frac{\partial}{\partial y}\left(e^{xy}\cos x\right) \qquad \frac{\partial}{\partial x}\left(e^{x-y}\sin(xy)\right)$$

$d(\omega)$ se $\omega = f\,dx + g\,dy$

$$d\omega = -\frac{\partial f}{\partial y}\,dx\,dy + \frac{\partial g}{\partial x}\,dx\,dy =$$

$$= \left(-\frac{\partial f}{\partial y} + \frac{\partial g}{\partial x}\right) dx\,dy$$

15.2.3 Per quali $f : \mathbb{R}^2 \to \mathbb{R}$ $\omega(x,y) = f(x,y)\,dy$ è chiusa?

ω chiusa $\overset{\text{def}}{\iff} d\omega = 0$

$$\omega = f\,dy \qquad d\omega = \frac{\partial}{\partial x} f\,dx\,dy = 0$$

Solo quando $\frac{\partial}{\partial x} f = 0$ cioè f è costante rispetto alla x, cioè

$f(x, y) = g(y)$ —

15.2.4 $\omega = (x + Ky^2)\,dx + (xy - K^2 y^2)\,dy$

Per quali K la 1-forma ω ha potenziale? (cioè è ESATTA) —

Già visto ESATTA $\Rightarrow$ CHIUSA cioè $d\omega = 0$

$$d\omega = -2ky\, dx\, dy + y\, dx\, dy = 0$$

(solo se $k = \frac{1}{2}$

è definita su tutto $\mathbb{R}^2$

quindi chiusa $\Rightarrow$ esatta

15.2.5 $\quad \alpha : [-1,1] \to \mathbb{R}^2 \qquad \alpha(t) = (-\sin(\pi t), 2t^2)$

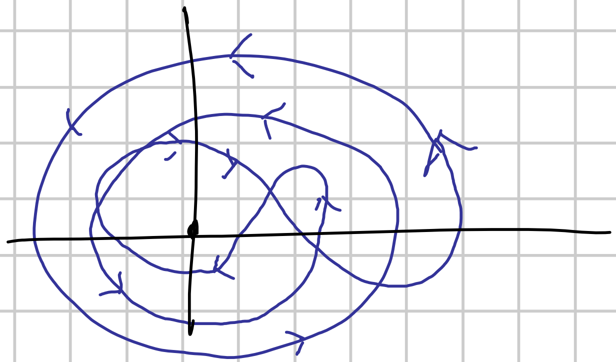
$$\omega(x,y) = \frac{x\, dy - (y-1)\, dx}{x^2 + (y-1)^2}$$

Anello mandando $(0,0)$ in $(0,1)$

già visto $\quad \omega_0(x,y) = \dfrac{-y\, dx + x\, dy}{x^2+y^2} = d\left(\arctan\left(\frac{x}{y}\right)\right)$

$$\int_\beta \omega_0 \quad \text{conta gli avvolgimenti di } \beta \text{ attorno a } (0,0)$$

ES



2 giri in senso orario +

1 in senso antiorario =

Complessivamente 1 giro orario

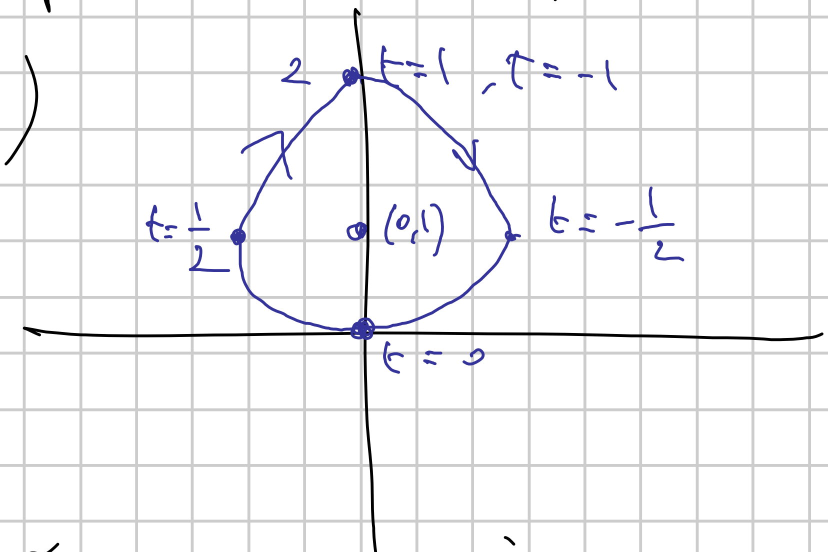
$$\int_\beta \omega_0 = 2\pi \cdot 1$$

in gen

$$= 2\pi \cdot \# \text{ di avvolgimenti}$$

$$\int_\alpha \omega = 2\pi \cdot (\#\text{ di avvolgimenti di } \alpha \text{ attorno al punto } (0,1))$$

$$\alpha(t) = (-\sin(\pi t), 2t^2)$$

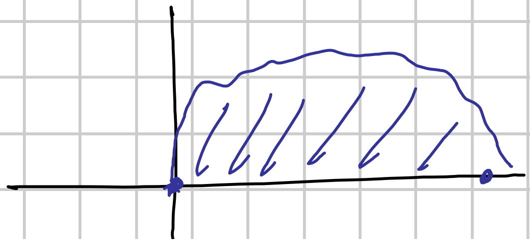


$$\int_\alpha \omega = -2\pi$$

15.3.1 Area delimitata da α e asse ascisse

$$\alpha(t) = \begin{pmatrix} t - \sin(t) \\ 1 - \cos(t) \end{pmatrix}$$

$t \in [0, 2\pi]$



$$\text{Area} = \int_{0} y\,dx = \int_{\alpha} y\,dx = \int_0^{2\pi} (1-\cos t)(1-\cos t)\,dt =$$

$$= \int_0^{2\pi} (1 + \cos^2 t - 2\cos t)\,dt = \int_0^{2\pi} 1\,dt + \int_0^{2\pi} \cos^2 t\,dt +$$

$$+ \int_0^{2\pi} (-2\cos t)\,dt = 2\pi + \pi + 0 = 3\pi$$

15.3.2 $A \subset \mathbb{R}^2$ aperto limitato, bordo è unione di due curve , $f: \mathbb{R}^2 \to \mathbb{R}$ con derivate parziali cont.

$$\int_{\partial A} df = 0 \quad ? \qquad \text{sì} \qquad df \text{ è esatta}$$

$$\Rightarrow \int_{\partial A} df = 0$$

(vale per curve semplici chiuse integrando 1-forme esatte)

13.3.3

$$\int_{\partial Q} \left((x^2+y^2)\,dx + (2xy + e^y)\,dy \right)$$

$$Q = [0,1] \times [0,1]$$

$$\omega = (x^2 + y^2)\,dx + 2xy + e^y\,dy$$

$$d\omega = 0 \implies \int_{\partial Q} \omega = 0$$

15.3.4 ω_0, ω_1 1-forme su $\mathbb{R}^2$ $d\omega_0 = d\omega_1$

$d(\omega_0 - \omega_1) = 0$ su $\mathbb{R}^2$, $\mathbb{R}^2$ è sempl. conn.

$$\implies \omega_0 - \omega_1 = dU \qquad U : \mathbb{R}^2 \to \mathbb{R}$$

$$\omega_0 = \omega_1 + dU$$