Geom 17/5/2017

13. 4.1   ERRATA   c) $(+ - +) -$

cioè $x^2 - y^2 + z^2 - 1 = 0$

$$x^2 + z^2 = y^2 + 1$$

<u>NON</u> iperboloide a 2 falde

<u>MA</u> " a <u>1</u> <u>falde</u>

d) $2x^2 + y^2 + z^2 + 2xy - 4yz + 6z = 0$

$$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 1 & -2 & 0 \\ 0 & -2 & 1 & 3 \\ 0 & 0 & 3 & 0 \end{pmatrix}$$

$d_1 > 0$

$d_2 > 0$

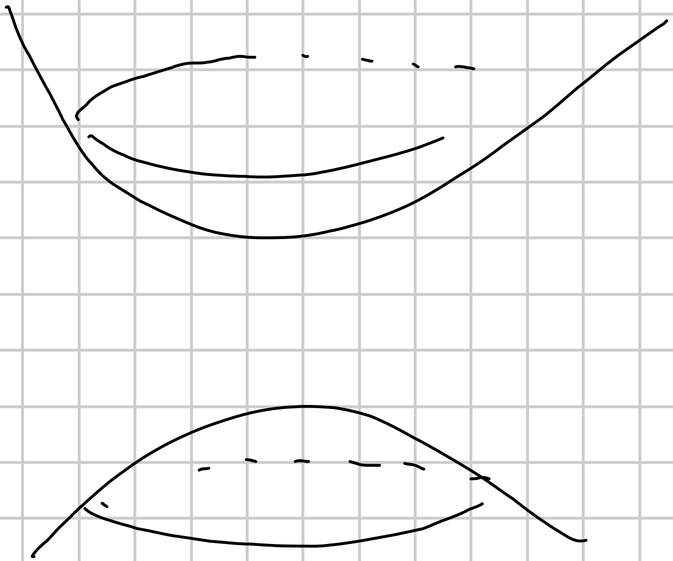
$d_3 = 2(-3) - 1 \cdot 3 = -9 < 0$

$d_4 = -3 \cdot \left[2 \cdot 3 - 1 \cdot 3 \right] = -9 < 0$

Ant. $(+\ +\ -)\ +$

$x^2 + y^2 - z^2 + 1 = 0$

cioè $z^2 = x^2 + y^2 + 1$



Iperboloide a 2 falde

(y) $\begin{pmatrix} 1 & 1 & 0 & -2 \\ 1 & 2 & -2 & 1 \\ 0 & -2 & 5 & 0 \\ -2 & 1 & 0 & 0 \end{pmatrix}$

$d_1 > 0$

$d_2 > 0$

$d_3 = 10 - 4 - 5 > 0$

$$d_4 = \det \begin{pmatrix} 3 & 1 & 0 & -2 \\ 5 & 2 & -2 & 1 \\ -4 & -2 & 5 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} =$$

$$= \det \begin{pmatrix} 3 & 0 & -2 \\ 5 & -2 & 1 \\ -4 & 5 & 0 \end{pmatrix} = -2(17) - 15 < 0$$

<u>Autoval</u> $(+\ +\ +)\ -$

$x^2 + y^2 + z^2 - 1 = 0$

elissoide

$$(m)\quad \begin{pmatrix} 1 & 1 & -k & 0 \\ 1 & 5 & k & 0 \\ -k & k & k^2+9 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$d_1 > 0$

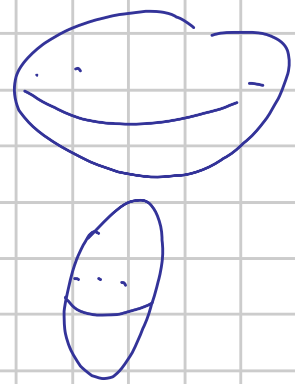
$\lambda_2 > 0$

$$d_3 = 5k^2 + 45 - k^2 - k^2 + \\ - 5k^2 - k^2 - 9 - k^2$$

$$= 36 - 4k^2 = 4(9 - k^2)$$

Ultimo autovalore è sempre -1

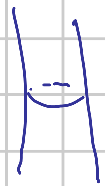
se $9 - k^2 > 0$, ovvero $|K| < 3$



$+ \; + \; + \; -$ elissoide

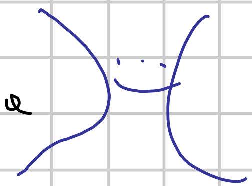
se $9 - k^2 = 0$ $\quad |K| = 3$

$+ \; + \; 0 \; -$ cilindro



se $9 - k^2 < 0$ $\quad |K| > 3$

$+ \; + \; - \; -$ iperboloide a 2 falde



(f)

$$\begin{pmatrix} 1 & 1 & 0 & -1 \\ 1 & 2 & k & -1 \\ 0 & k & k^2-1 & 0 \\ -1 & -1 & 0 & 1-k^2 \end{pmatrix}$$

$d_1 > 0$

$d_2 > 0$

$d_3 = 2k^2 - 2 - k^2 + 1 - k^2 =$

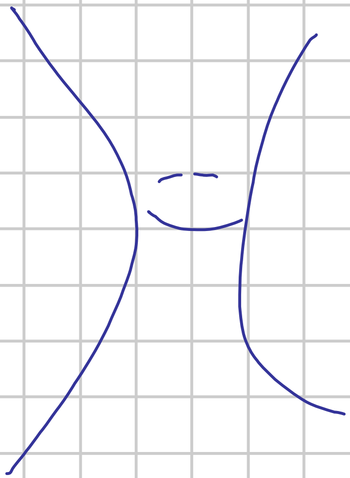
$= -1 < 0$

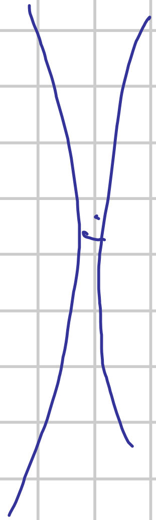
$d_4 = \dots = k$

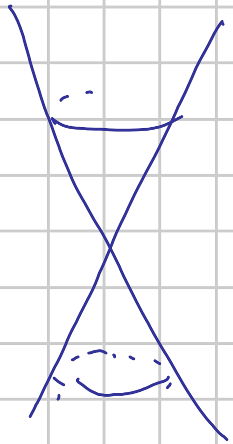
$k > 0$ Autov. $+ \; + \; - \; -$ iperb. a 1 falda

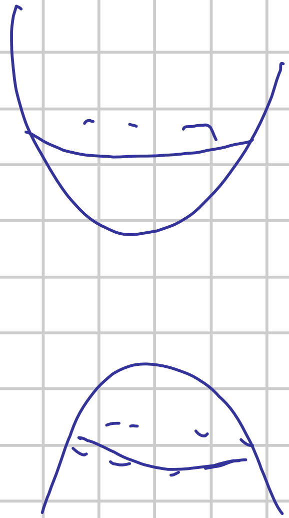
$k = 0$ Autov. $+ \; + \; - \; 0$ cono

$k < 0$ Autov. $+ \; + \; - \; +$ iperb. a 2 falde









(4) $\begin{pmatrix} 1 & -k & 0 & -2 \\ -k & k^2+k & k & 2k \\ 0 & k & k+1 & k \\ -2 & 2k & k & k^2+3 \end{pmatrix}$

$d_1 > 0$

$d_2 = K$

$d_3 = \dots = K$

$d_4 = \dots = -K$

$K > 0$ Anton $(+ + +) -$ elissoide

$K < 0$ Ant $(+ - +) -$ iperb a 1 falda

14.1.3 Dire se curve param. α, β definiscono la stessa curva orientata.

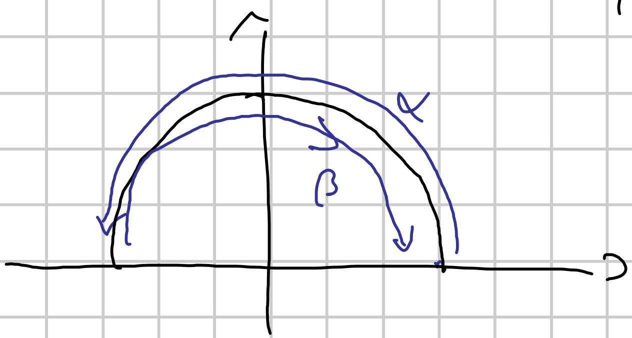
$$\alpha : [0,1] \to \mathbb{R}^2 \qquad \alpha(t) = (\cos(\pi t), \sin(\pi t))$$

$$\beta : [-1,1] \to \mathbb{R}^2 \qquad \beta(t) = (t, \sqrt{1-t^2})$$

β' non è continua in quanto $\lim_{t \to -1} \beta'(t) = (1, +\infty)$

In $(-1,1)$ aperto β è regolare.

$$\operatorname{Im} \alpha = \operatorname{Im} \beta = \{ (x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1 , \ y \geq 0 \}$$



α, β non definiscono la stessa curva <u>orientata</u>

$$\alpha\left(\tfrac{1}{2}\right) = \beta(0) = (0,1)$$

$$\beta'(0) = (1,0)$$

$$\alpha'(0) = \left(-\frac{\pi}{2}, 0\right)$$

derivate di segno opposto

quindi posso riparametrizzare α in β, ma il cambio di parametro ha derivata < 0.

Ovvero se $\beta(s) = \alpha(\tau(s))$

$$\beta'(0) = \alpha'(\tau(0)) \cdot \tau'(0) =$$

$$= \alpha\left(\frac{1}{2}\right) \tau'(0) \qquad \tau'(0) = -\frac{\pi}{2} < 0$$

14.1.5 $\quad \alpha : [0, 2\pi] \to \mathbb{R}^3$

$$\alpha(t) = \begin{pmatrix} \sin t \\ \cos t \\ t^2 \end{pmatrix}$$

$$\beta : [0, \pi] \to \mathbb{R}^3$$

$$\beta(t) = \begin{pmatrix} \sin 2t \\ \cos 2t \\ 4t^2 \end{pmatrix}$$

ovvero $\beta(t) = \alpha(2 \cdot t)$, cioè dato

$$\tau : [0, \pi] \to [0, 2\pi] \qquad \tau(t) = 2t$$

$$\beta(t) = \alpha(\tau(t)) \quad \Rightarrow \quad L(\alpha) = L(\beta)$$

perché β è ottenuta da α per cambio di parametro

$$\alpha'(t) = \begin{pmatrix} \cos t \\ -\sin t \\ 2t \end{pmatrix} \qquad L(\alpha) = \int_0^{4\pi} \|\alpha'(t)\| \, dt =$$

$$= \int_0^{2\pi} \sqrt{1 + 4t^2} \, dt$$

14.1.6 Calcolar $\int_\alpha \sqrt{1 + x^2 + y^2}$ $\qquad \alpha : [0,1] \to \mathbb{R}^2$

$$\alpha(t) = \begin{pmatrix} t \cos t \\ t \sin t \end{pmatrix}$$

$$\alpha'(t) = \begin{pmatrix} \cos t - t \sin t \\ \sin t + t \cos t \end{pmatrix} \qquad \|\alpha'(t)\| = \sqrt{1+t^2}$$

$$\int_\alpha \sqrt{1+x^2+y^2} = \int_0^1 \sqrt{1+t^2\cos^2 t + t^2 \sin^2 t} \cdot \|\alpha'(t)\| \, dt =$$

$$= \int_0^1 \sqrt{1+t^2} \cdot \sqrt{1+t^2} \, dt = \int_0^1 (1+t^2) \, dt =$$

$$= \left(t + \frac{t^3}{3} \right) \Big|_0^1 = \frac{4}{3}$$

14.1.3 $\quad \alpha : [0,1] \to \mathbb{R}^2$

$$\alpha(t) = \begin{pmatrix} t \\ e^t \end{pmatrix} \qquad \alpha'(t) = \begin{pmatrix} 1 \\ e^t \end{pmatrix}$$

$$\int_\alpha \sqrt{1+y^2} = \int_0^1 \sqrt{1+e^{2t}} \cdot \sqrt{1+e^{2t}}\, dt =$$

$$= \int_0^1 (1+e^{2t})\, dt = \left(t + \frac{e^{2t}}{2} \right)\Bigg|_0^1 = 1 + \frac{1}{2} e^2 - \frac{1}{2} =$$

$$= \frac{1}{2}\left(e^2 - 1\right)$$