Geometria 4/5/2017

11.2.1

10) $4x^2+6y^2-5xy-2x-y+22=0$

$$\begin{pmatrix} 4 & -5/2 & -1 \\ -5/2 & 6 & -1/2 \\ -1 & -1/2 & 22 \end{pmatrix}$$

$d_1 > 0$

$d_2 = \frac{71}{4} > 0$

$$d_3 = \frac{71}{4} \cdot 22 + \frac{5}{2} \cdot \left(-\frac{111}{2}\right) - \left(\frac{29}{4}\right) > 0$$

ϕ (insieme vuoto $\sim x^2+y^2+1=0$)

11) $x^2+y^2+2xy+x+y=0$

$$\begin{pmatrix} 1 & 1 & 1/2 \\ 1 & 1 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

$d_1 > 0$

$d_2 = 0$

$d_3 = 0$

$\Rightarrow$ due rette e il pol. si fattorizza

$$(x+ay+b)\left(x+\frac{y}{a}\right) = x^2+2xy+y^2+x+y$$

una delle due costanti deve essere nulla, suppongo la seconda.

$$\begin{array}{ll} (xy) & \left\{ \begin{array}{ll} a + \dfrac{1}{a} = 2 & \checkmark \\[2ex] \end{array} \right. \\ (x) & \quad b = 1 \qquad \Rightarrow \ b = 1 \\ (y) & \quad \dfrac{b}{a} = 1 \qquad \Rightarrow \ a = 1 \end{array}$$

$$(x+y+1)(x+y) = 0$$

due rette parallele

38) $\begin{pmatrix} k & \sqrt{k} & \sqrt{k} \\ \sqrt{k} & 3 & 0 \\ \sqrt{k} & 0 & k \end{pmatrix}$ $k \geq 0$

$k = 0$ è la retta

$y = 0$ contata due volte

Supponiamo $\underline{\underline{k > 0}}$

$d_1 > 0$

$d_2 = 3k - k = 2k > 0$

$d_3 = -3k + 2k^2 = k(2k-3)$

$0 < k < 3/2$ $\quad d_3 < 0 \quad \Rightarrow$ <u>ellisse</u>

$k = 3/2$ un punto $(\sim x^2 + y^2 = 0)$

$k > 3/2$ $\quad d_3 > 0 \quad \Rightarrow \emptyset$

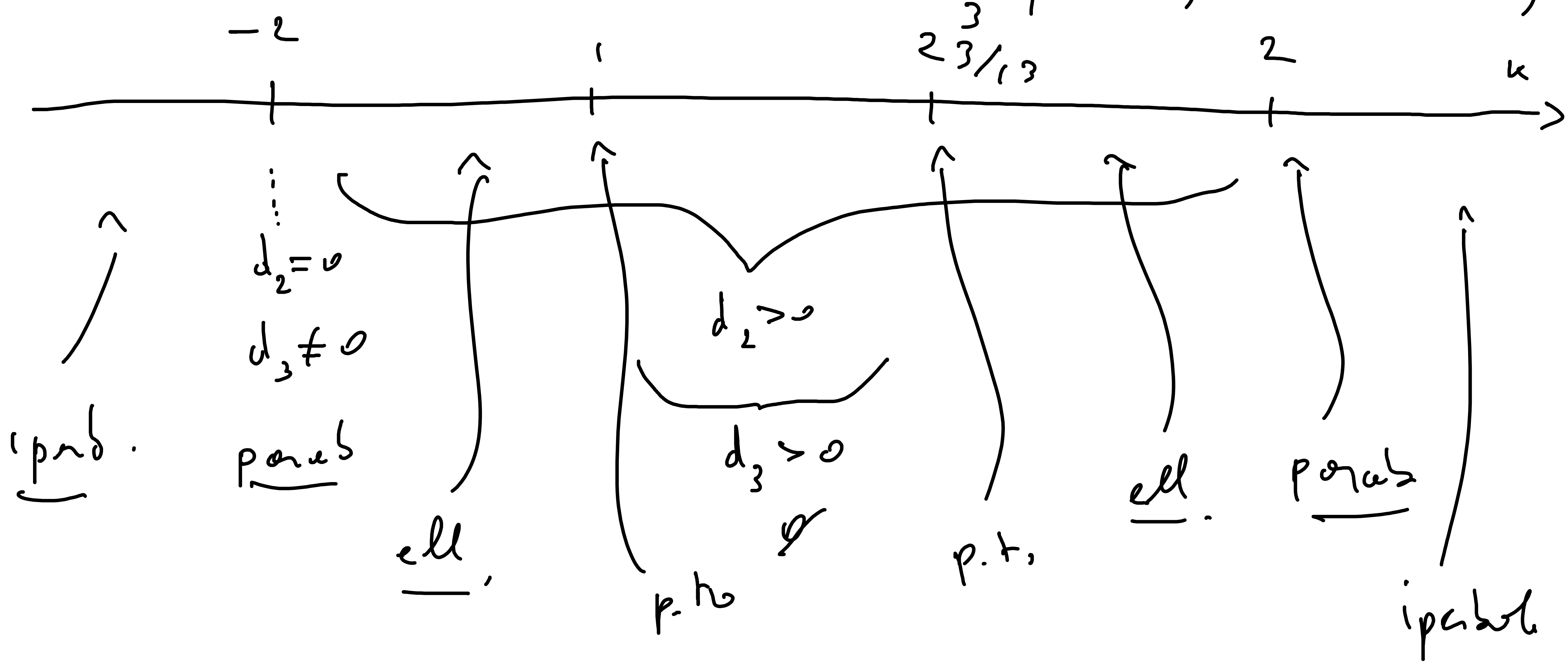
39) $\begin{pmatrix} 1 & -k & -2 \\ -k & 4 & 3 \\ -2 & 3 & 13/3 \end{pmatrix}$

$d_1 > 0$

$d_2 = 4 - k^2$

$d_3 = -\frac{13}{3}k^2 + 12k - \frac{23}{3} =$

$= -\frac{1}{3}(k-1)(13k-23)$



(12.2.2) $\mathcal{C}: \quad 8x^2 - 12xy + 17y^2 + 4x + 12y + 4 = 0$

$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 17 & 6 \\ 2 & 6 & 4 \end{pmatrix}$

$d_1 > 0$

$d_2 > 0$

$d_3 = -540 + 490 < 0$

$\Rightarrow$ ellisse

Cerco di scriverla come

$(2x + ay + b)^2 + (2x + cy + d)^2 = e^2$

(a meno di scambiarli)

$$\begin{cases} a^2+c^2=17 & (y^2) \\ a+c=-3 & (xy) \\ b+d=1 & (x) \\ ab+cd=6 & (y) \\ b^2+d^2-e^2=4 & (\text{cost}) \end{cases}$$

$\left.\begin{matrix} a^2+c^2=17 \\ a+c=-3 \end{matrix}\right\} \Rightarrow a=1,\ c=-4$

$\left.\begin{matrix} b+d=1 \\ ab+cd=6 \end{matrix}\right\} \Rightarrow b=2,\ d=-1$

$b^2+d^2-e^2=4 \Rightarrow e=1$

$$(2x+y^2+2)^2+(2x-4y-1)^2=1$$

Trasf. affine:

$$\begin{cases} X = 2x+y+2 \\ Y = 2x-4y-1 \end{cases} \qquad \begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 2 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

Trasf. isometrica:

$$\begin{cases} Z = \alpha(2x+y+2) \\ Y = \beta(2x-4y-1) \end{cases} \qquad \begin{pmatrix} 2\alpha & \alpha \\ 2\beta & -4\beta \end{pmatrix} \text{ ortogonale}$$

$$\Rightarrow \begin{cases} \alpha\beta = \pm 1/10 & (\det = \pm 1) \\ 2\alpha^2 - 4\alpha\beta = 0 \Rightarrow 2\alpha = \beta \\ \dots \\ \dots \end{cases} \leftarrow \text{relazioni da } A^TA = I$$

$$\Rightarrow \alpha = 1/\sqrt{5} \quad \beta = \frac{1}{2\sqrt{5}} \qquad \begin{pmatrix} Z \\ W \end{pmatrix} = \overbrace{\frac{1}{\sqrt{5}}\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}}^{\text{ORTOG.}}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2/\sqrt{5} \\ -1/(2\sqrt{5}) \end{pmatrix}$$