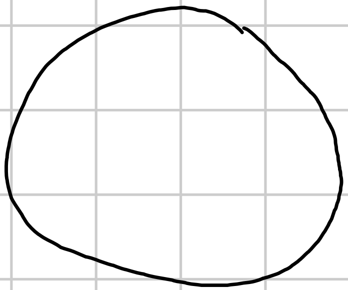
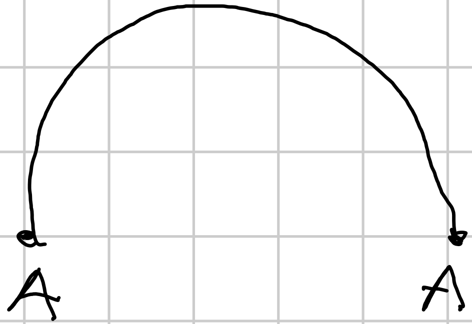
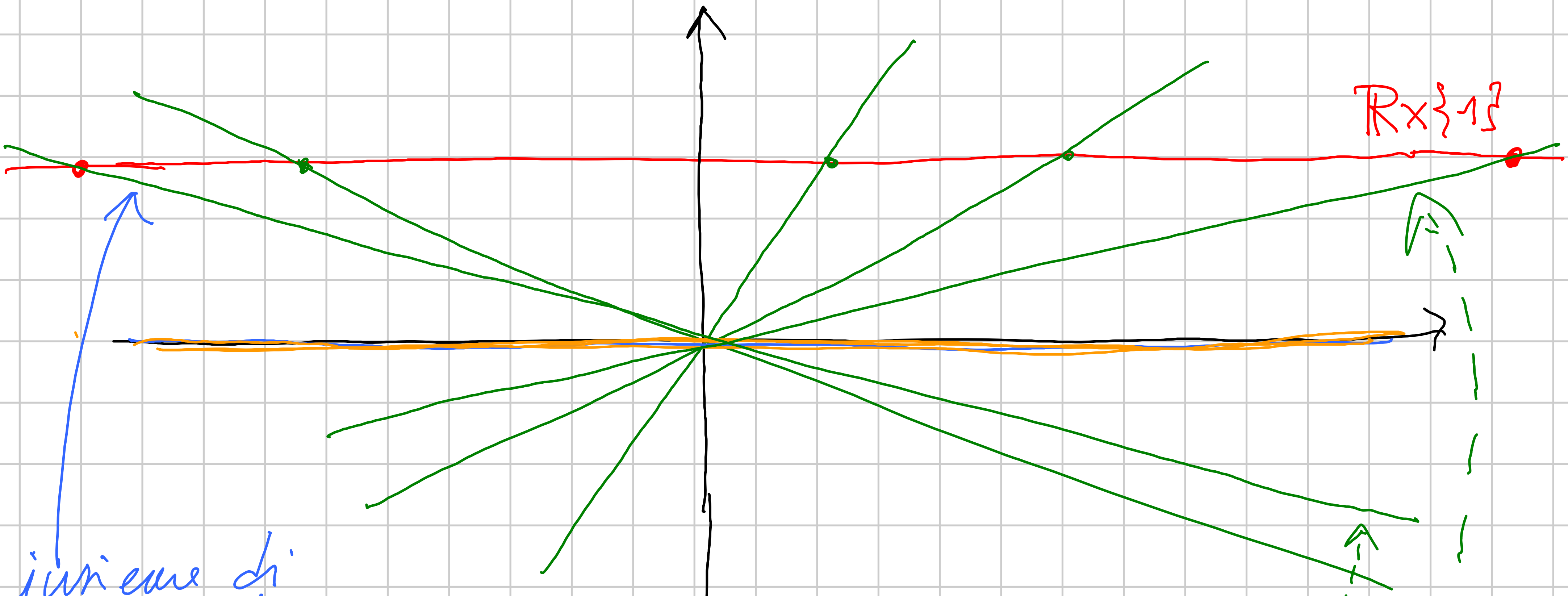
Geom 22/4/16

$$\mathbb{P}^n(\mathbb{K}) = \mathbb{K}^{n+1} \setminus \{0\} \Big/ \begin{array}{l} xRy \text{ se} \\ \exists \lambda \neq 0 \text{ t.c.} \\ y = \lambda \cdot x \end{array} = \text{l'insieme delle rette in } \mathbb{K}^{n+1}$$

$$\mathbb{P}^0(\mathbb{K}) = \text{un punto}$$

$$\mathbb{P}^1(\mathbb{R}) =$$  $=$ 

insieme di rappresentanti sovrabbondanti



insieme di rappresentanti a cui manca un punto

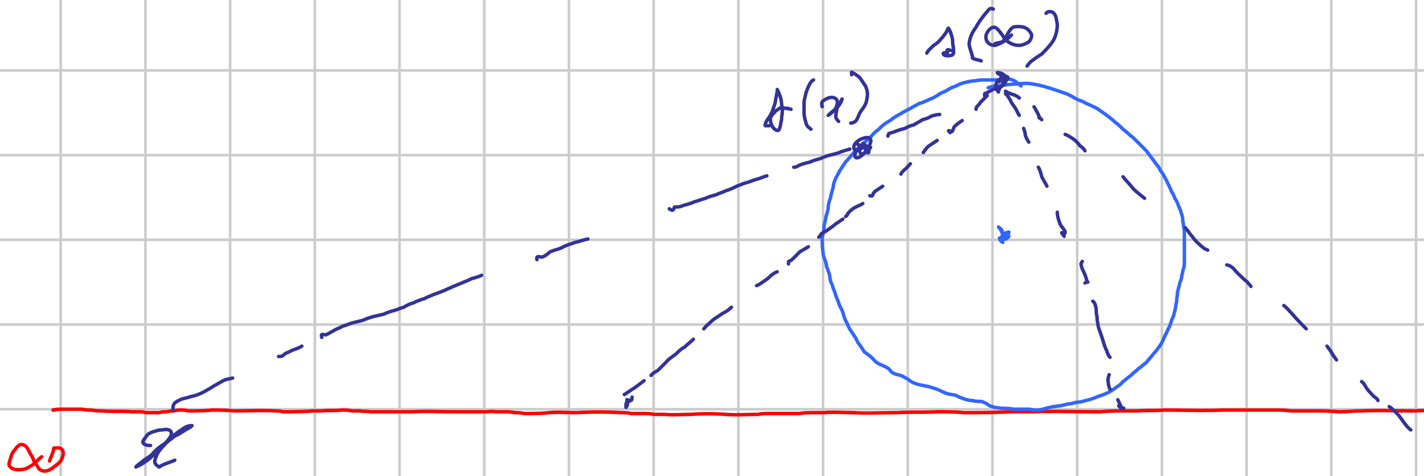
$\Rightarrow \mathbb{P}^1(\mathbb{R})$ "=" $\underbrace{(\mathbb{R}\times\{1\})}_{\mathbb{R}} \cup$ un punto

pto vicino a • è una retta poco inclinata dunque su $\mathbb{R}$ è un pto molto lontano

è naturale interpretarlo
come pto all'∞

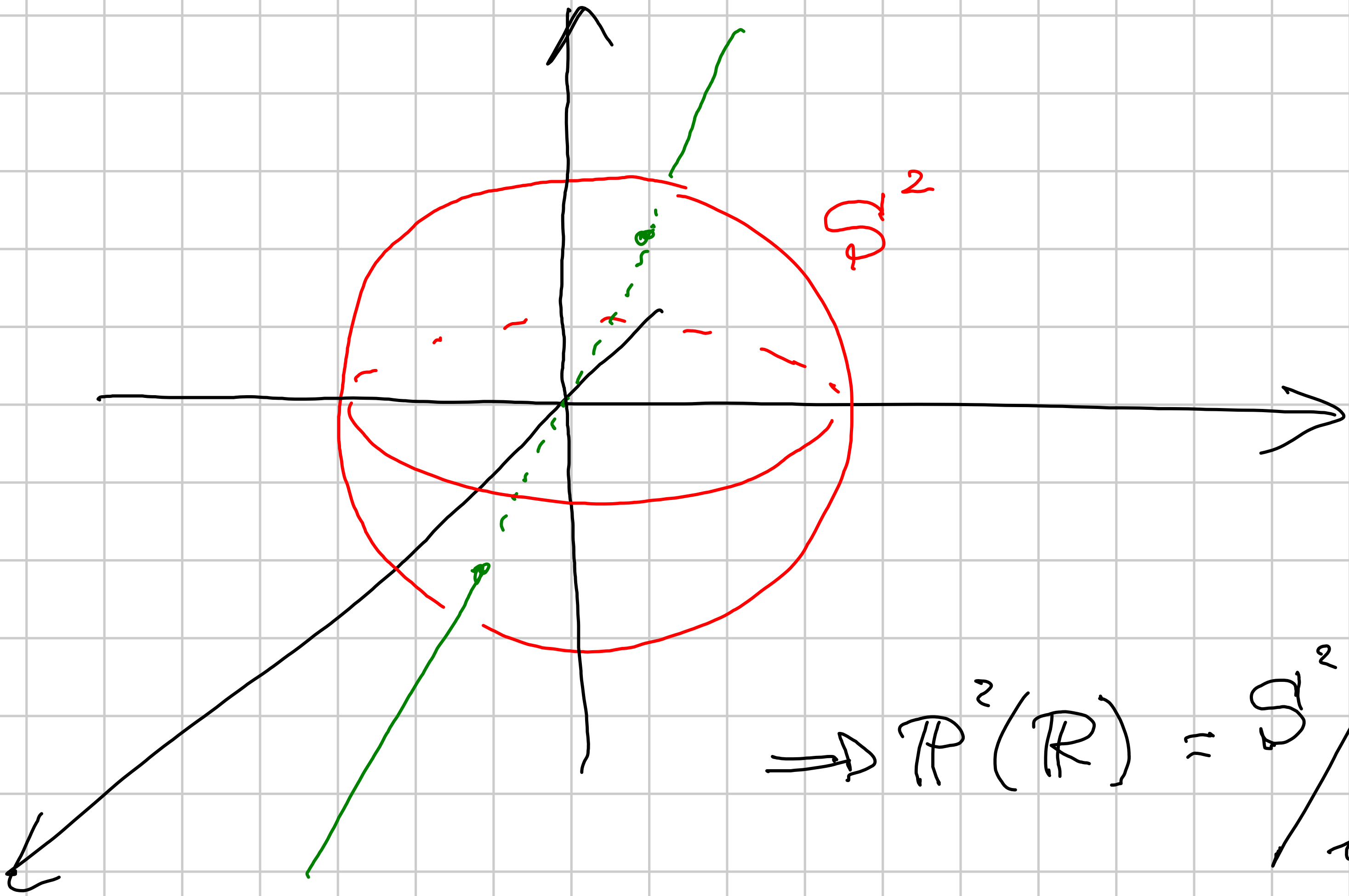
$$\longrightarrow \mathbb{P}^1(\mathbb{R}) = \underbrace{\mathbb{R} \cup \{\infty\}}$$

è di nuovo naturale
interpretato come una
circonferenza.



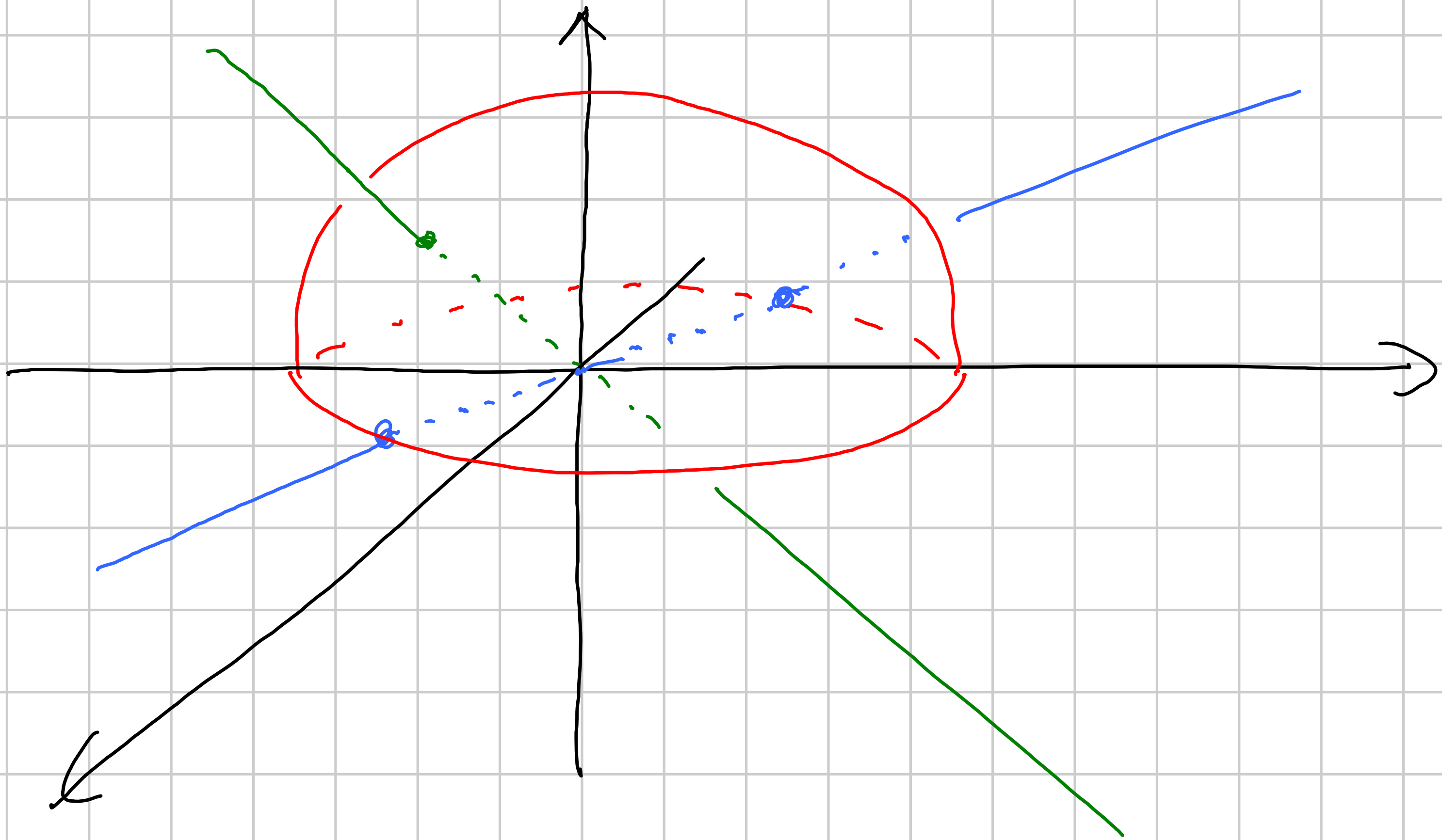
$s : \mathbb{R} \longrightarrow$ circonf escluso
il pto in alto
si estende a
$s : \mathbb{P}^1(\mathbb{R}) \longrightarrow$ circonf

$\mathbb{P}^2(\mathbb{R})$ = l'insieme delle rette in $\mathbb{R}^3$



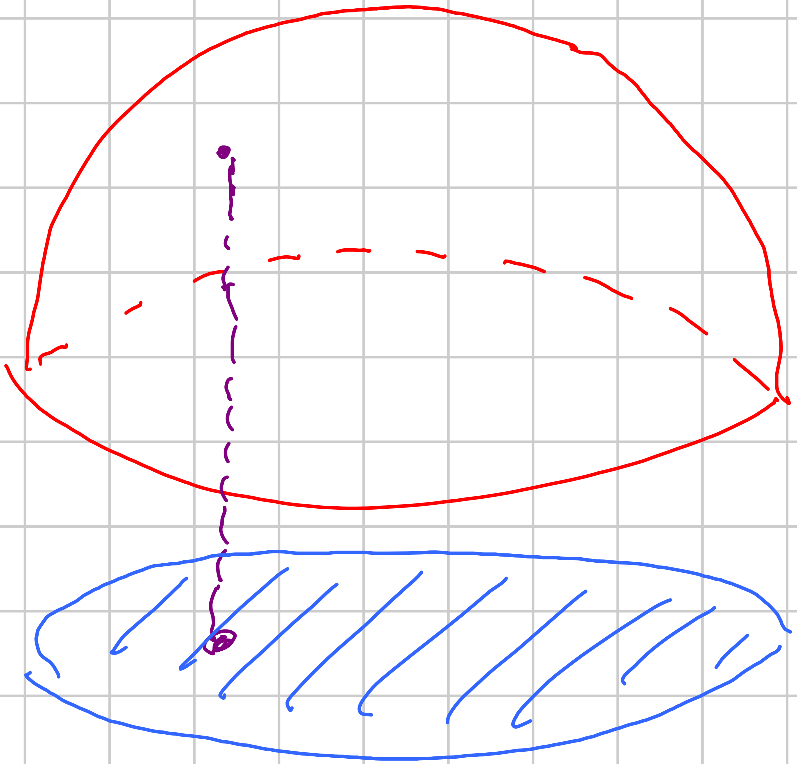
$\Longrightarrow \mathbb{P}^2(\mathbb{R}) = S^2 /$ identificare i pti antipodali tra loro

Oppure prendo $E = \{ x \in \mathbb{R}^3 : \|x\| = 1 \, , \, x_3 \geq 0 \}$



$\Rightarrow$ $\mathbb{P}^2(\mathbb{R})$ si ottiene da E identificando tra loro i punti antipodali sull'equatore (bordo di E).

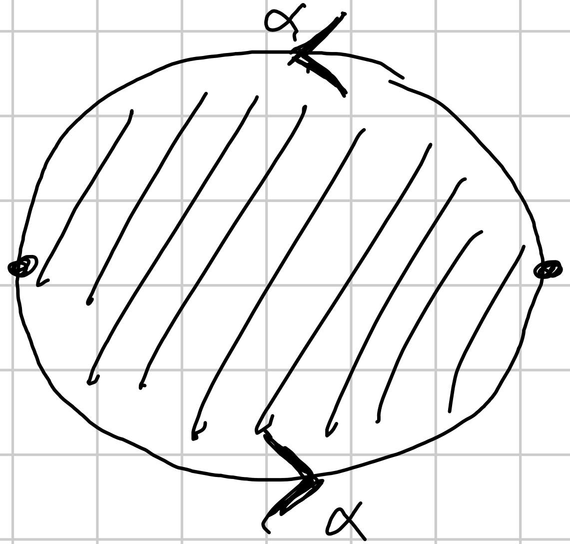
Notiamo che c'è una corrispondenza biunivoca naturale fra E e il disco $D = \{x \in \mathbb{R}^2 : \|x\| \leq 1\}$



$$(x_1, x_2, x_3) \longmapsto (x_1, x_2)$$

rispetto a cui il bordo resta fermo.

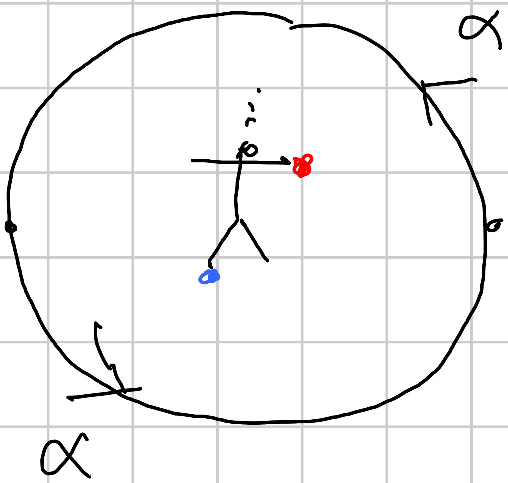
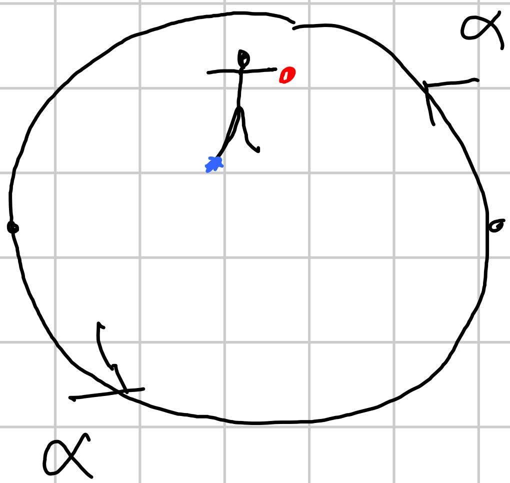
$\Longrightarrow \mathbb{P}^2(\mathbb{R}) = D \Big/$ identificazione dei pti antipodali sul bordo

$=$ 

i punti dei due archi α vengono ordinatamente identificati fra loro

Fatto: è impossibile disegnare questo oggetto in $\mathbb{R}^3$.

Passeggiate in $\mathbb{P}^2(\mathbb{R})$:


α
α

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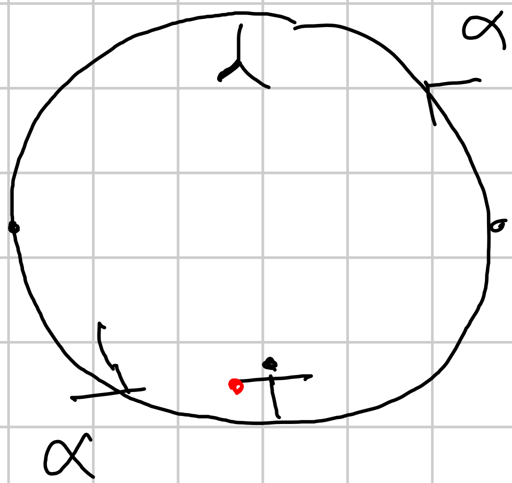
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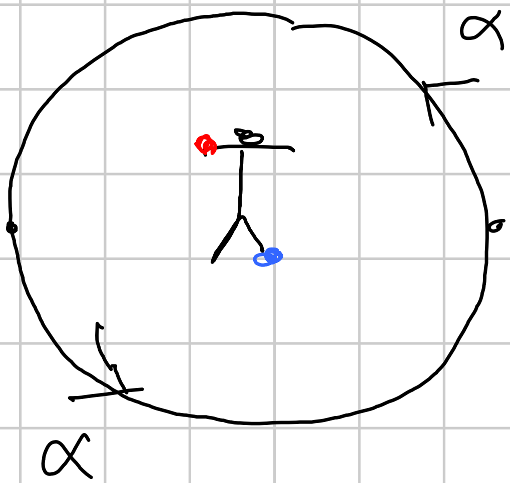
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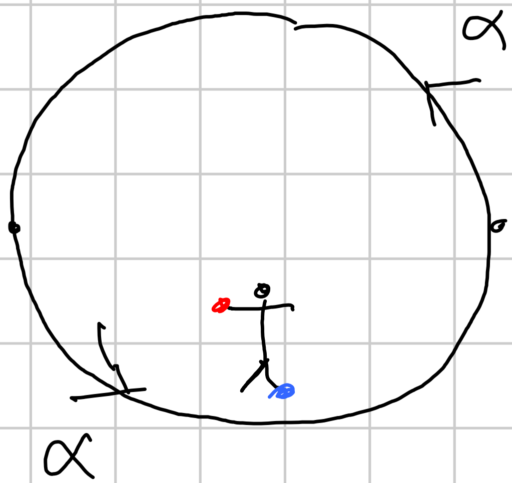
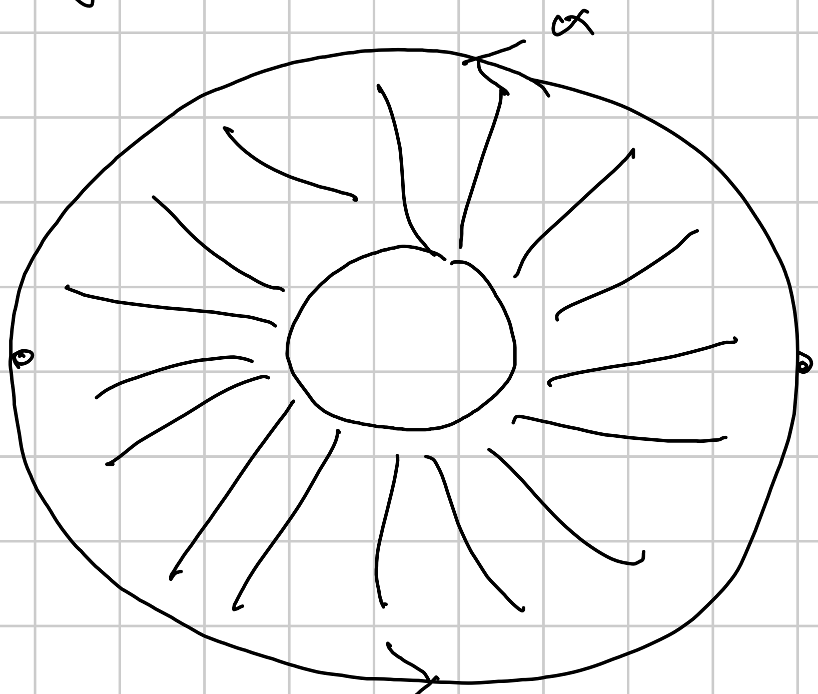
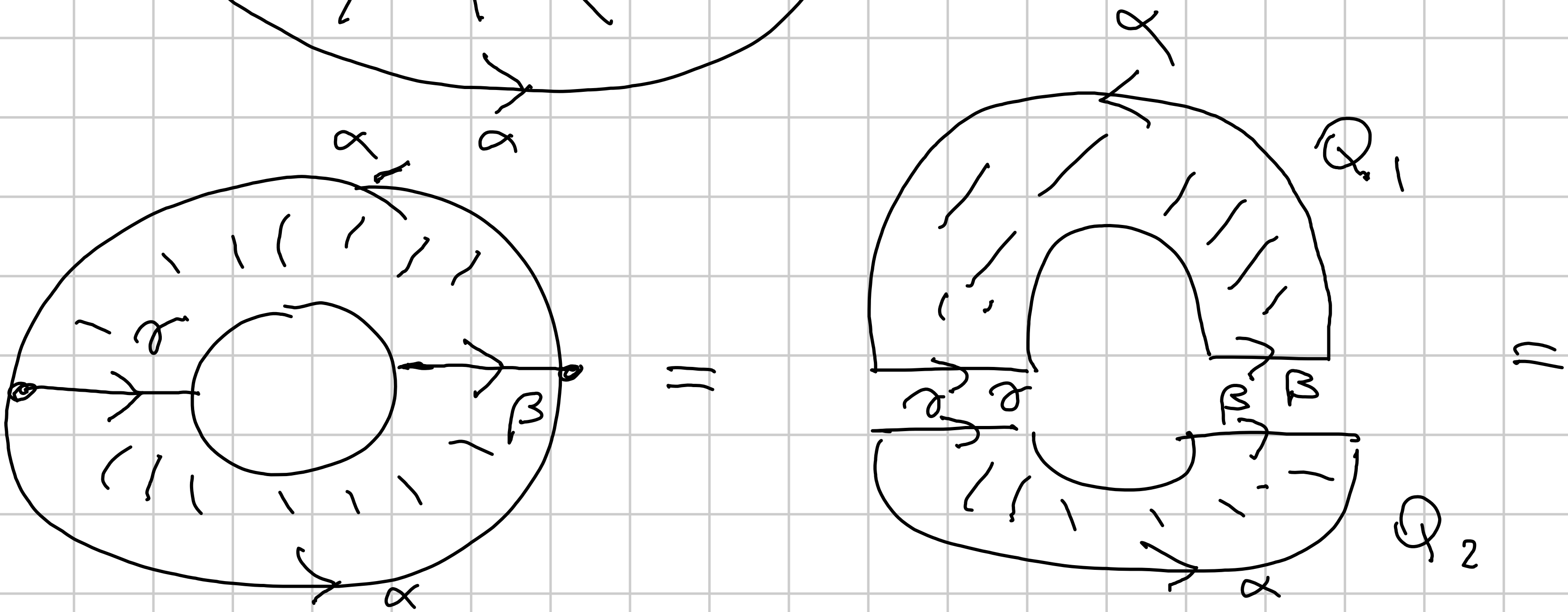

α
α

Disegno in $\mathbb{R}^3$ di $\mathbb{P}^2(\mathbb{R})$ tolto un piccolo disco.





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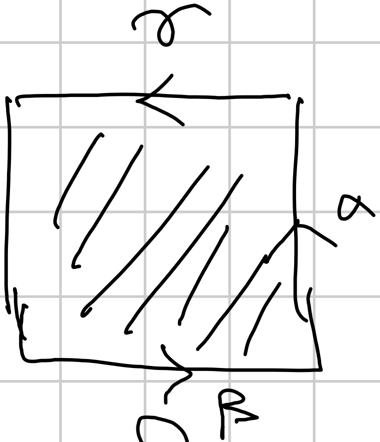
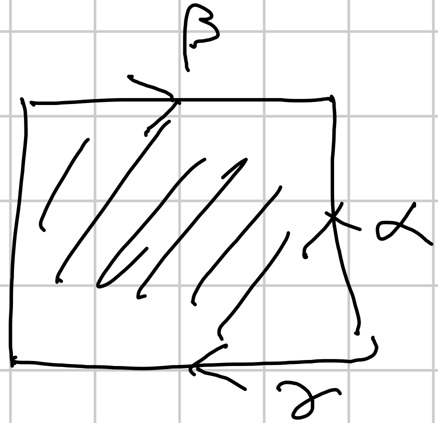
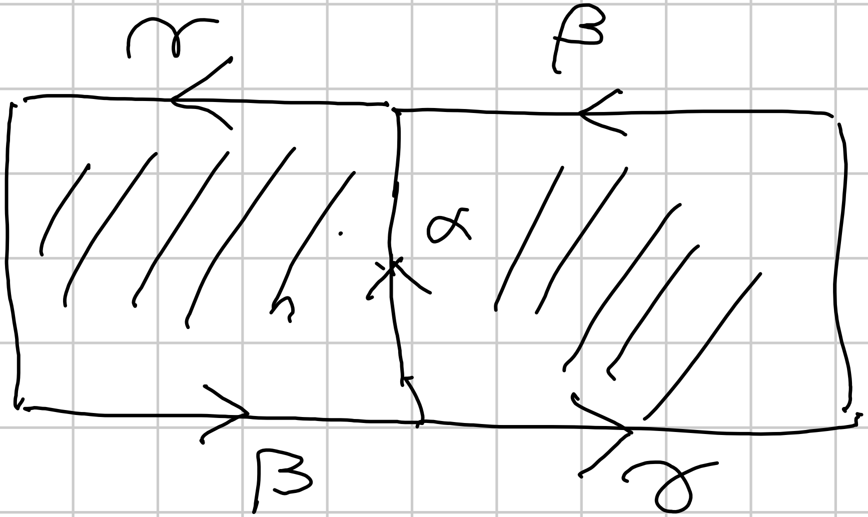


Q_1



Q_2

=



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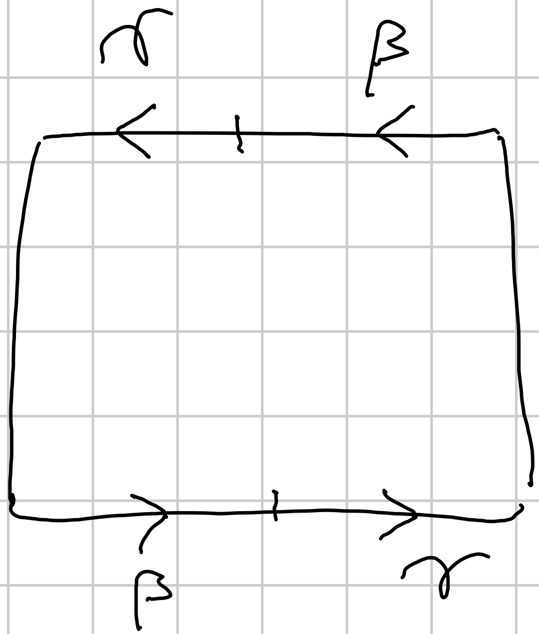

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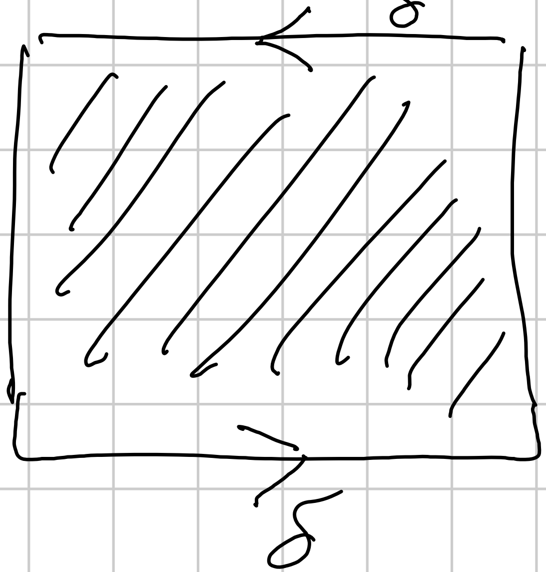

= nastro di Möbius

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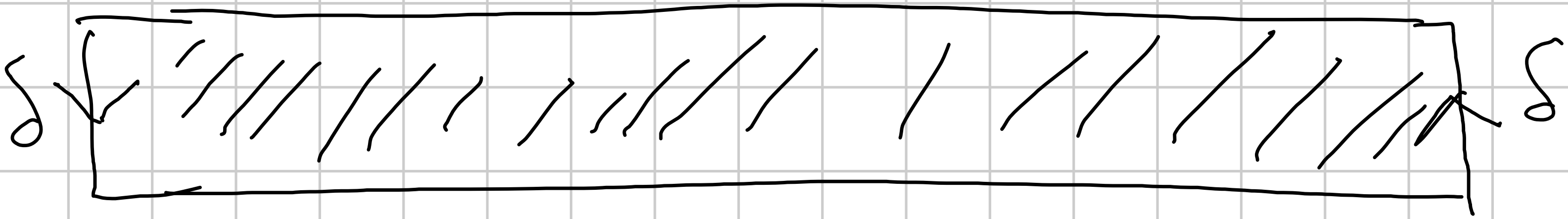

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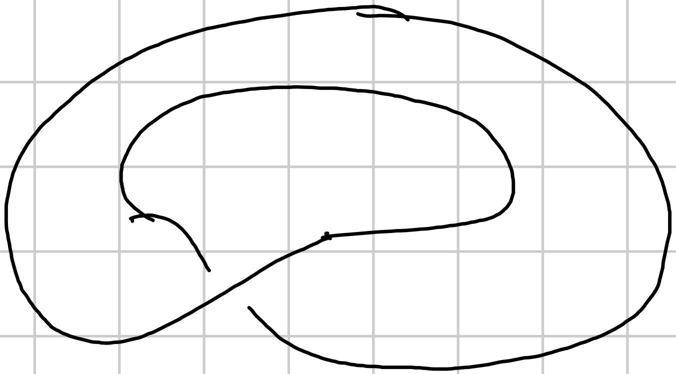

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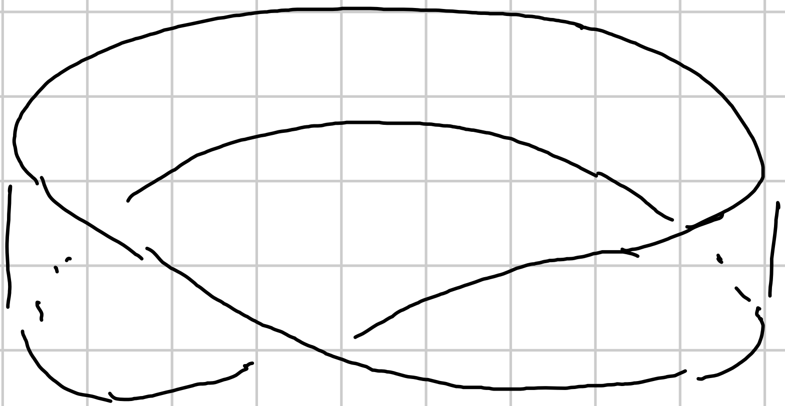

E' una superficie monolaterale (è impossibile colorarne una parte di rosso e l'altra di blu) ___