10/11/15

Soluzioni dettagliate di esercizi non svolti in aula

⑦ • $p(AA) = 0.4^2$, $p(Aa) = 2 \times 0.4 \times 0.6$, $p(aa) = 0.6^2$

• $p(F=A \mid P=A \text{ e } M=a)$

$$= p(F=AA \text{ o } F=Aa \mid (P=AA \text{ o } P=Aa) \text{ e } M=aa)$$

$$= \cancel{p(F=AA \mid (P=AA \text{ o } P=Aa) \text{ e } M=aa)}^{\,0}$$

$$+ p(F=Aa \mid (P=AA \text{ o } P=Aa) \text{ e } M=aa)$$

$$= \frac{p(F=Aa \text{ e } (P=AA \text{ o } P=Aa) \text{ e } M=aa)}{p((P=AA \text{ o } P=Aa) \text{ e } M=aa)}$$

$$= \frac{p(F=Aa \text{ e } P=AA \text{ e } M=aa) + p(F=Aa \text{ e } P=Aa \text{ e } M=aa)}{p(P=AA \text{ o } P=Aa) \cdot p(M=aa)}$$

$$= \frac{\left(p(F=Aa \mid P=AA \text{ e } M=aa) \cdot p(AA) \cdot \cancel{p(aa)} + p(F=Aa \mid P=Aa \text{ e } M=aa) \cdot p(Aa) \cdot \cancel{p(aa)}\right)}{(1-p(aa)) \cdot \cancel{p(aa)}}$$

$$= \frac{p(AA) + \frac{1}{2} p(Aa)}{1 - p(aa)}$$

• $p(P = A \mid F = a)$

$$= p(P = AA \text{ o } P = Aa \mid F = aa)$$

$$= \cancel{p(P = AA \mid F = aa)}^{\,0} + p(P = Aa \mid F = aa)$$

$$= \cancel{p(P = Aa \text{ e } M = AA \mid F = aa)}^{\,0}$$
$$+ p(P = Aa \text{ e } M = Aa \mid F = aa)$$
$$+ p(P = Aa \text{ e } M = aa \mid F = aa)$$

$$= \frac{p(P = Aa \text{ e } M = Aa \text{ e } F = aa) + p(P = Aa \text{ e } M = aa \text{ e } F = aa)}{p(aa)}$$

$$= \frac{\left(\begin{array}{l} p(F = aa \mid P = Aa \text{ e } M = Aa) \cdot p(Aa)^2 \\ \quad + p(F = aa \mid P = Aa \text{ e } M = aa) \cdot p(Aa) \cdot p(aa) \end{array} \right)}{p(aa)}$$

$$= \frac{\frac{1}{4} p(Aa)^2 + \frac{1}{2} p(Aa) \cdot p(aa)}{p(aa)}$$

• $p(F = A \mid P = a \text{ e } M = a)$

$$= p(F = AA \mid P = aa \text{ e } M = aa) = 0$$

• ~~Se x è il valore di $p(F=A \mid P = A \text{ e } M = a)$ trovato al secondo punto, viene~~

$$\cancel{1 - \left((1-x)^5 + 5 \cdot x \cdot (1-x)^4 \right)}$$

- Lo risolviamo in generale supponendo che A abbia frequenza p e a frequenza $q = 1 - p$.

Dobbiamo calcolare

$$1 - \underbrace{p(F1 = a \text{ e } \dots \text{ e } F5 = A \mid P = A \text{ e } M = a)}_{u}$$

$$- 5 \cdot \underbrace{p(F1 = A \text{ e } F2 = a \text{ e } \dots \text{ e } F5 = A \mid P = A \text{ e } M = a)}_{w}.$$

Per calcolare u e w uso questa formula generale: se X e Y sono incompatibili e Z è indipendente da ciascuno di loro si ha

$$p(E \mid (X \text{ o } Y) \text{ e } Z) = \frac{p(X) \cdot p(E \mid X \text{ e } Z) + p(Y) \cdot p(E \mid Y \text{ e } Z)}{p(X) + p(Y)}.$$

La formula è intuitiva, comunque possiamo verificarla:

$$p(E \mid (X \text{ o } Y) \text{ e } Z) = \frac{p(E \text{ e } (X \text{ o } Y) \text{ e } Z)}{p((X \text{ o } Y) \text{ e } Z)}$$

$$= \frac{p((E \text{ e } X \text{ e } Z) \text{ o } (E \text{ e } Y \text{ e } Z))}{p(X \text{ o } Y) \cdot p(Z)} = \frac{p(E \text{ e } X \text{ e } Z) + p(E \text{ e } Y \text{ e } Z)}{(p(X) + p(Y)) \cdot p(Z)}$$

$$= \frac{p(E \mid X \text{ e } Z) p(X) p(Z) + p(E \text{ e } Y \text{ e } Z) p(Y) p(Z)}{(p(X) + p(Y)) \cdot p(Z)} = \underline{\underline{ok}}$$

ORA $u = p(F1=aa \text{ e } \ldots \text{ e } F5=aa \mid (P=AA \text{ o } P=Aa) \text{ e } M=aa)$

$$= \frac{\begin{pmatrix} p(F1=aa \text{ e} \ldots \text{e } F5=aa \mid P=AA \text{ e } M=aa) \cdot p(P=AA) \\ + p(F1=aa \text{ e} \ldots \text{e } F5=aa \mid P=Aa \text{ e } M=aa) \cdot p(P=Aa) \end{pmatrix}}{p(P=AA) + p(P=Aa)}$$

$$= \frac{0 \cdot p^2 + \left(\frac{1}{2}\right)^5 \cdot 2pq}{p^2+2pq} = \frac{q}{16(p+2q)}$$

$w = p(F1=Aa \text{ e } F2=aa \text{ e} \ldots \text{e } F5=aa \mid (P=AA \text{ o } P=Aa) \text{ e } M=aa)$

$$= \frac{\begin{pmatrix} p(F1=Aa \text{ e } F2=aa \text{ e} \ldots \text{e } F5=aa \mid P=AA \text{ e } M=aa) \cdot p^2 \\ + p(F1=Aa \text{ e } F2=aa \text{ e} \ldots \text{e } F5=aa \mid P=Aa \text{ e } M=aa) \cdot 2pq \end{pmatrix}}{p^2+2pq}$$

$$= \frac{0 \cdot p^2 + \left(\frac{1}{2}\right)^5 \cdot 2pq}{p^2+2pq} = \frac{q}{16(p+2q)}$$

(8) $f(m) = f(MM) = f(M)^2 \Rightarrow f(M) = \sqrt{0.49} = 0.7$

$f(a) = f(AA) + f(AM) = f(A)^2 + 2\, f(A) \cdot f(M)$

$\Rightarrow f(A)^2 + 2 f(A) f(M) - f(a) = 0$

$\Rightarrow f(A) = -f(M) + \sqrt{f(M)^2 + f(a)}$

$= -0.7 + \sqrt{0.49 + 0.15}$

$= -0.7 + \sqrt{0.64} = -0.7 + 0.8 = 0.1$

$f(R) = 1 - f(a) - f(m) = 0.2$

$f(x) = f(RR) + f(RM) = f(R)^2 + 2 f(R) \cdot f(M)$

$= 0.04 + 2 \times 0.2 \times 0.7 = 0.32$ ✓

$f(v) = f(AR) = 2 f(A) \cdot f(R) = 2 \times 0.1 \times 0.2$

$= 0.04$ ✓

(9) •

$\left.\begin{array}{l} p(AA) = 0.3^2 = 0.09 \\ p(AO) = 2 \times 0.3 \times 0.6 = 0.36 \end{array}\right\} p(A) = 0.45$

$\left.\begin{array}{l} p(BB) = 0.1^2 = 0.01 \\ p(BO) = 2 \times 0.1 \times 0.6 = 0.12 \end{array}\right\} p(B) = 0.13$

$p(AB) = 2 \times 0.3 \times 0.1 = 0.06$

$p(O) = p(OO) = 0.6^2 = 0.36$

- $p(F=0 \mid P=0 \text{ e } M=\text{non } 0)$

$$= \frac{p(F=0 \text{ e } M=\text{non } 0 \mid P=0)}{p(M=\text{non } 0)}$$

$$= \frac{p(F=0 \text{ e } (M=AO \text{ o } M=BO) \mid P=0)}{1-p(0)}$$

$$= \frac{p(F=0 \text{ e } M=AO \mid P=0) + p(F=0 \text{ e } M=BO \mid P=0)}{1-p(0)}$$

$$= \frac{p(F=0 \mid P=0 \text{ e } M=AO)\cdot p(M=AO) + p(F=0 \mid P=0 \text{ e } M=BO)\cdot p(BO)}{1-p(0)}$$

$$= \frac{\frac{1}{2}\, p(AO) + \frac{1}{2}\, p(BO)}{1-p(0)}$$

- $p(P=0 \mid F=\text{non } 0)$

$$= \frac{p(P=0 \text{ e } F=\text{non } 0)}{p(F=\text{non } 0)}$$

$$= \Big(p(P=0 \text{ e } F=\text{non } 0 \text{ e } M=AA) + p(P=0 \text{ e } F=\text{non } 0 \text{ e } M=AO)$$
$$+ \ \ldots \text{ gli altri casi per } M \Big) \Big/ (1-p(0))$$

$$= \Big(p(F = \text{non } O \mid P = O \text{ e } M = AA) \cdot p(O) \cdot p(AA)$$
$$+ p(\;\text{"}\; \mid \;\text{"}\; \text{ e } M = AO) \cdot p(O) \cdot p(AO)$$
$$\ldots \Big) \Big/ (1 - p(O))$$

$$= \frac{\left(1 \cdot p(AA) + \frac{1}{2} p(AO) + 1 \cdot p(BB) + \frac{1}{2} p(BO) + p(AB)\right) \cdot p(O)}{1 - p(O)}$$

- $p(F = O \mid P = \text{non } O \text{ e } M = \text{non } O)$

$$= \frac{p(F = O \text{ e } P = \text{non } O \text{ e } M = \text{non } O)}{(1 - p(O))^2}$$

$$= \Big(p(F = O \text{ e } P = AO \text{ e } M = AO)$$
$$+ p(F = O \text{ e } P = AO \text{ e } M = BO)$$
$$+ p(F = O \text{ e } P = BO \text{ e } M = AO)$$
$$+ p(F = O \text{ e } P = BO \text{ e } M = BO) \Big) \Big/ (1 - p(O))^2$$

$$= \Big(p(F = O \mid P = AO \text{ e } M = AO) \cdot p(AO)^2$$
$$+ p(F = O \mid P = AO \text{ e } M = BO) \cdot p(AO) \cdot p(BO)$$
$$+ p(F = O \mid P = BO \text{ e } M = AO) \cdot p(BO) \cdot p(AO)$$
$$+ p(F = O \mid P = BO \text{ e } M = BO) \cdot p(BO)^2 \Big) \Big/ (1 - p(O))^2$$

$$= \frac{1}{4}\left(\frac{p(AO)+p(BO)}{1-p(O)}\right)^2$$

• Vale $1 - \underbrace{p(F1 = \text{non } O \text{ e} \dots \text{e } F4 = \text{non } O \mid P = O \text{ e } M = \text{non } O)}_{u}$

$$u = \frac{\begin{aligned} & p(F1 = \text{non } O \text{ e} \dots \text{e } F4 = \text{non } O \mid P = OO \text{ e } M = AA)\cdot a^2 \\ & + p(\_\_\_\_ \mid P = OO \text{ e } M = BB)\cdot b^2 \\ & + p(\_\_\_\_ \mid P = OO \text{ e } M = AB)\cdot 2ab \\ & + p(\_\_\_\_ \mid P = OO \text{ e } M = AO)\cdot 2ao \\ & + p(\_\_\_\_ \mid P = OO \text{ e } M = BO)\cdot 2bo \\ & \cancel{+ p(\_\_\_\_ \mid P = OO \text{ e } M = OO)\cdot o^2} \end{aligned}}{1-o^2}$$

$$= \frac{a^2+b^2+2ab+\left(\frac{1}{2}\right)^4 2ao+\left(\frac{1}{2}\right)^4\cdot 2bo}{1-o^2} = \dots$$