Matematica A – 24.11.15

① $f(x) = mx + q$

• $f(0) = 2 \qquad f(1) = 0$

$$\begin{cases} m \cdot 0 + q = 2 \\ m \cdot 1 + q = 0 \end{cases} \qquad \begin{cases} q = 2 \\ m = -2 \end{cases}$$

$$\Rightarrow f(x) = -2x + 2 = 2(1 - x)$$

- $f^{-1}(y) = 3y - 4$

I modo : In generale $f(x) = mx + q$.

Trovare f^{-1} significa risolvere rispetto a x l'equaz.

$$y = mx + q$$

$$x = \frac{1}{m}(y - q) \implies f^{-1}(y) = \frac{1}{m} \cdot y - \frac{q}{m}$$

Dunque $\begin{cases} 3 = \frac{1}{m} \\ -4 = -\frac{q}{m} \end{cases}$ $\begin{cases} m = \frac{1}{3} \\ q = \frac{4}{3} \end{cases}$.

II modo: $f^{-1}(y) = 3y-4$

Trovare f significa risolvere rispetto a y l'equaz.

$$x = 3y-4$$

$$\Rightarrow \quad 3y = x+4 \quad \Rightarrow y = \frac{x}{3} + \frac{4}{3}$$

$$\Rightarrow f(x) = \frac{x}{3} + \frac{4}{3}$$ cioè $m = \frac{1}{3}$, $q = \frac{4}{3}$ —

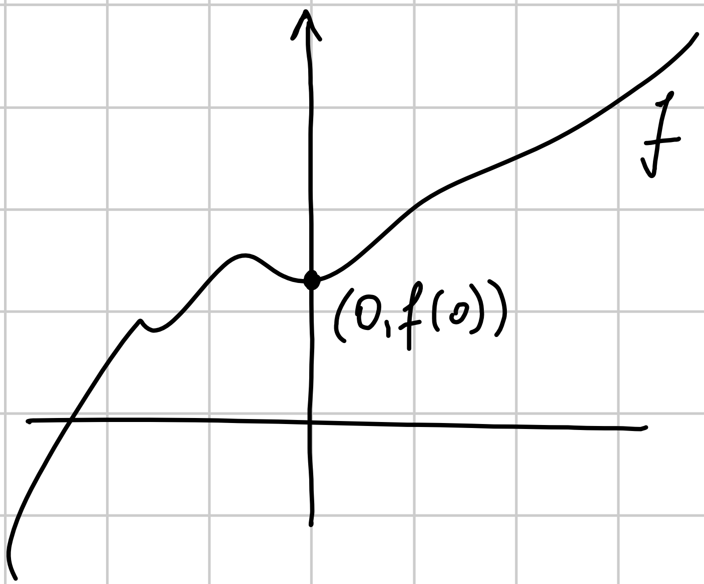
In generale: per trovare l'inversa di f si risolve $y = f(x)$ rispetto a x, trovando $x = f^{-1}(y)$ –

Se conosco $f^{-1}(y)$ per trovare f risolvo
$x = f^{-1}(y)$ rispetto a y, trovando
$y = f(x)$.

- $f(2) = 0$, f^{-1} ha coeff ang $= -2$

$$\begin{cases} m \cdot 2 + q = 0 \\ \frac{1}{m} = -2 \end{cases} \qquad \begin{cases} m = -\frac{1}{2} \\ q = 1 \end{cases} \implies f(x) = -\frac{1}{2}x + 1$$

• $\underbrace{\text{grafico di } f \text{ contiene } (0,0)}_{\substack{f(0)=0 \\ q=0}}$ $\underbrace{\text{ed è parallelo a } 2x+y=10}_{m=-2}$



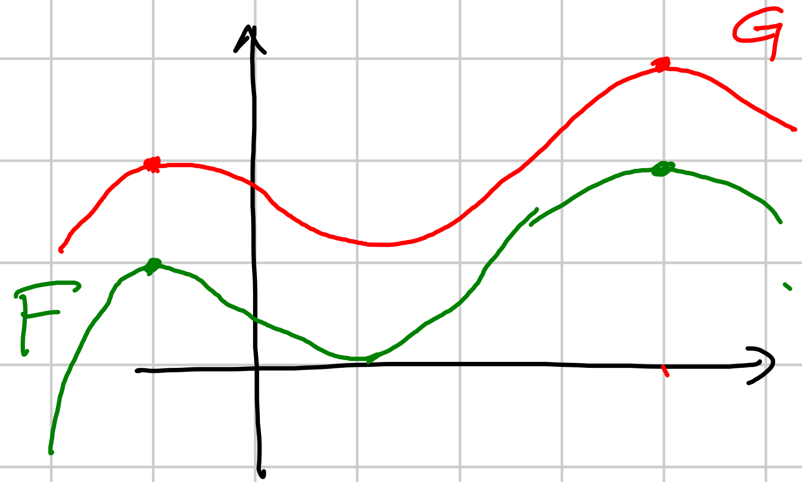
② $f(x) = 6x - x^2$ grafico F

Trovare la g che ha grafico ottenuto da F traslando di 1 a sx e 2 in alto.

In generale: data f con grafico F

$g(x) = f(x) + c$ con grafico G

$c = 1/2$



In generale data f con grafico F

$g(x) = f(x+c)$

$c = +1/2$

$f(0)=1$

$\Downarrow$

$g(-\frac{1}{2})=1$

$f(3/2)=0$

$\Downarrow$

$g(1)=0$

$f(3)=-1/2$

$\Downarrow$

$f(5/2)=-1/2$

Morale: • effetto di $g(x) = f(x) + c$

traslare in alto di $c > 0$

in basso di $c < 0$

• effetto di $g(x) = f(x+c)$

traslare a sinistra di $c > 0$

a destra di $c < 0$

$f(x) = 6x - x^2$ 1 a sx 2 in alto

$$f(x+1)+2 = 6(x+1)-(x+1)^2+2$$

$$= \underline{6x}+6-x^2\underline{-2x}-1+2 =$$

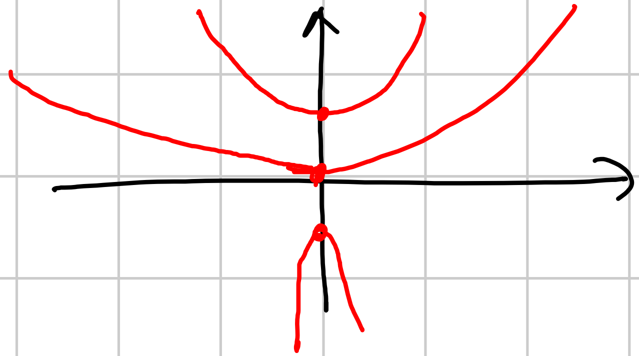
$$= -x^2+4+7$$

③ $f(x) = ax^2+bx+c$ grafico F.

• $f(1)=1$ $f(2)=4$ f pari

$$\begin{cases} a+b+c=1 \\ 4a+2b+c=4 \\ -b/2a=0 \end{cases}$$

l'ascisse del vertice della parabola



$$\begin{cases} b = 0 \\ a + c = 1 \\ 4a + c = 4 \end{cases} \qquad \begin{cases} a = 1 \\ b = 0 \\ c = 0 \end{cases} \qquad f(x) = x^2$$

• F passa per $(0,2)$ $(-2,0)$ $(1,0)$

cioè $f(0) = 2$ $f(-2) = 0$ $f(1) = 0$

$$\begin{cases} c = 2 \\ 4a - 2b + c = 0 \\ a + b + c = 0 \end{cases} \qquad \begin{cases} c = 2 \\ 2a - b = -1 \\ a + b = -2 \end{cases} \qquad \begin{cases} a = -1 \\ b = -1 \\ c = 2 \end{cases}$$

- F ha vertice in $(1,-1)$ e passa per $(0,0)$.

$$\begin{cases} -b/2a = 1 \\ a\cdot\left(-\frac{b}{2a}\right)^2 + b\cdot\left(-\frac{b}{2a}\right) + c = -1 \\ c = 0 \end{cases}$$

$$\begin{cases} c = 0 \\ b = -2a \\ a + b + c = -1 \end{cases} \qquad \begin{cases} c = 0 \\ b = -2a \\ a - 2a = -1 \end{cases} \qquad \begin{cases} a = 1 \\ b = -2 \\ c = 0 \end{cases}$$

④ $f(x) = -2x^2+3x-1$ F grafico di f

- $\mathrm{Im}(f)$ + disegnare F

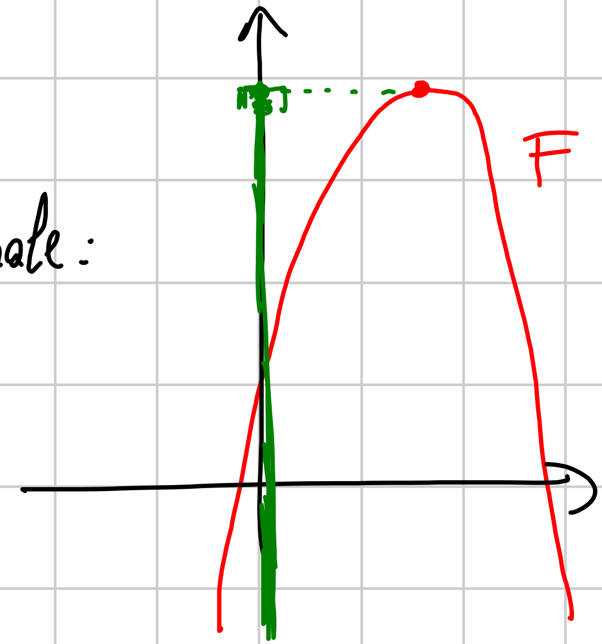
$x_v = 3/4$

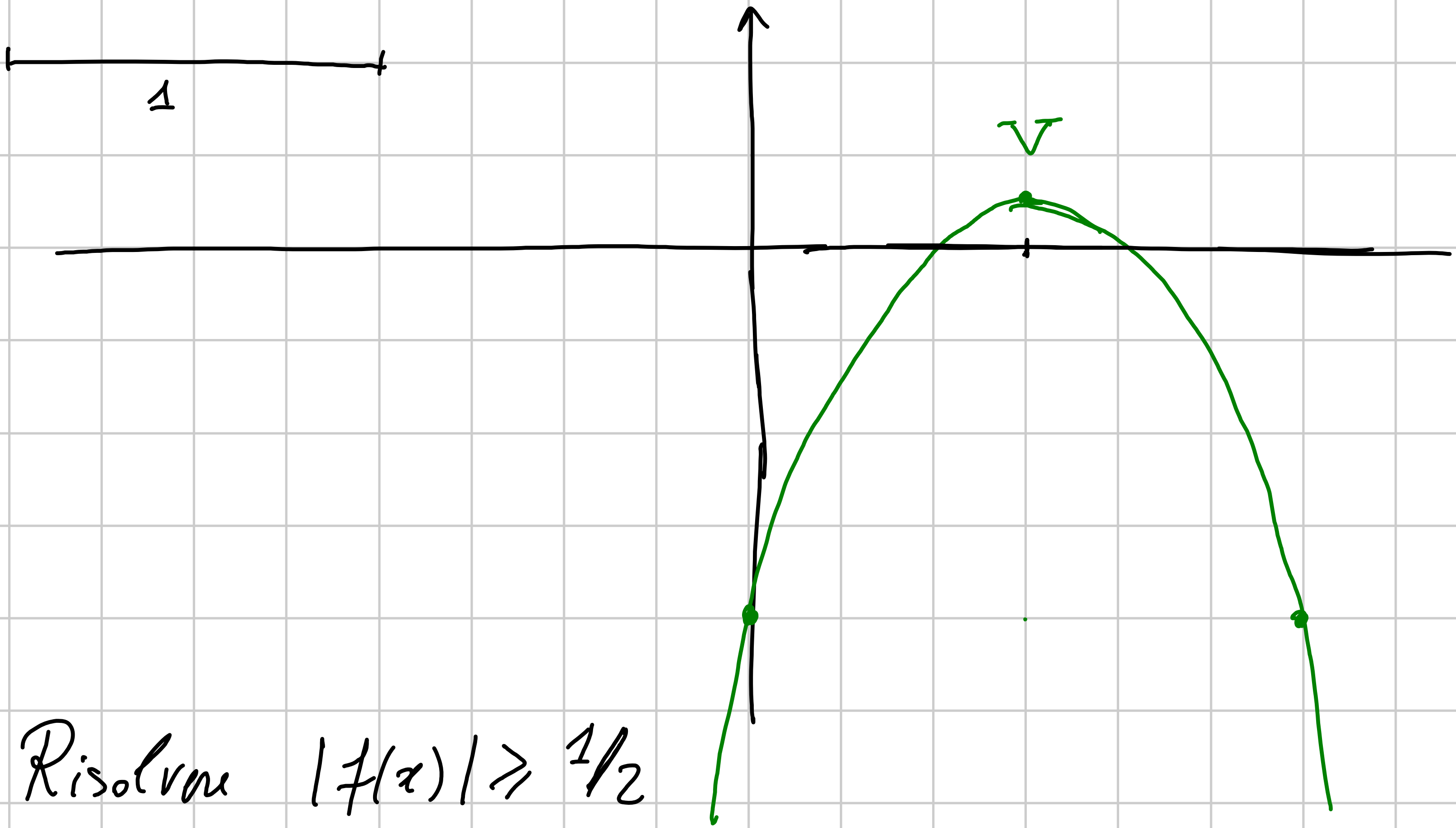
$y_v = -2 \cdot \frac{9}{16} + \frac{9}{4} - 1$

$= \frac{-9+18-8}{8} = \frac{1}{8}$

$\mathrm{Im}(f) = (-\infty, 1/8]$

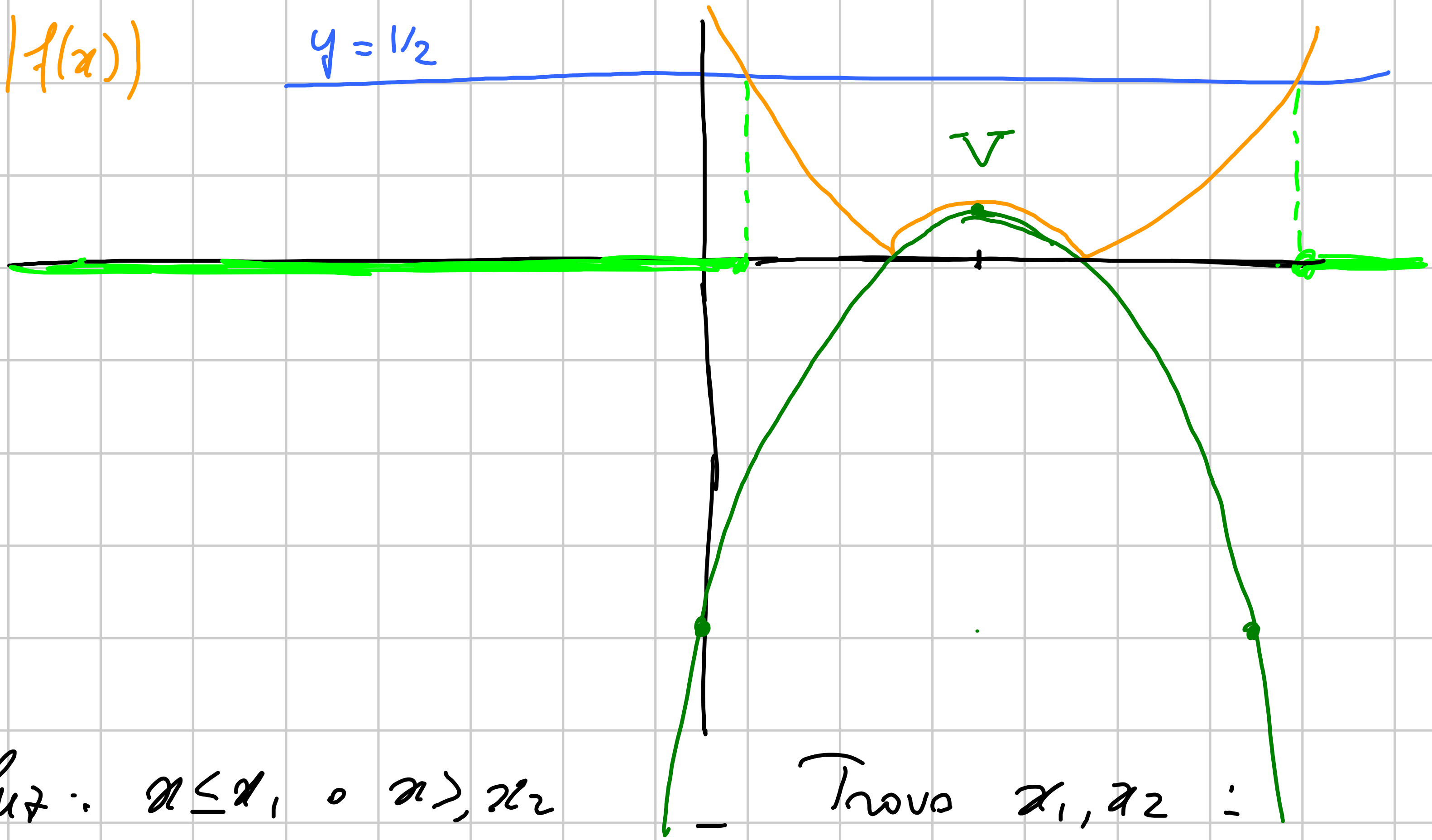
In generale:





- Risolvere $|f(x)| \geq 1/2$

$g(x) = |f(x)|$



Soluz: $x \le x_1$ o $x \geq x_2$ — Trovo x_1, x_2:

$-f(x) = 1/2$

$$2x^2 - 3x + 1 = \frac{1}{2}$$

$$4x^2 - 6x + 1 = 0$$

$$x_{1,2} = \frac{3 \pm \sqrt{9-4}}{4} = \frac{3 \pm \sqrt{5}}{4} .$$

- $f(x) = -2x^2 + 3x + 1$

  Traslare di 2 a dx e 3 in alto:

$$f(x-2) + 3 = 2(x-2)^2 - 3(x-2) + 1 + 3 = \dots$$

6 $f(x) = mx + q$ grafico F

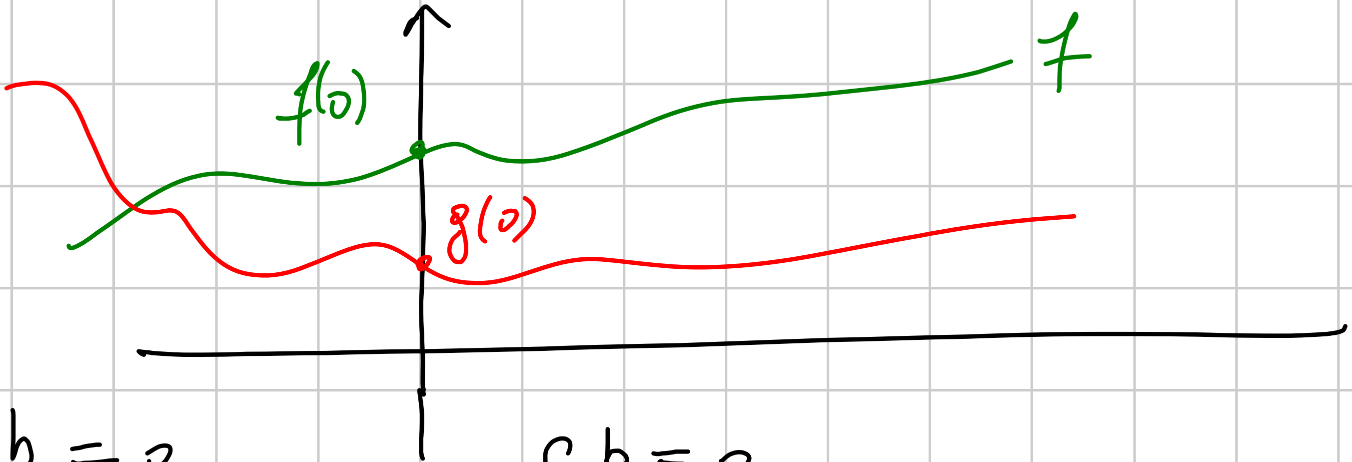
$g(x) = ax^2 + bx + c$ grafico G

- F contiene $(1,3)$ e è $\perp$ $y = \frac{x}{2} + 5$

$$\begin{cases} m + q = 3 \\ m = -2 \end{cases} \qquad f(x) = -2x + 5$$

- $\underbrace{g \text{ pari}}_{b=0}$ $\qquad F \cap G = \{(0, ?), (2, ?)\}$

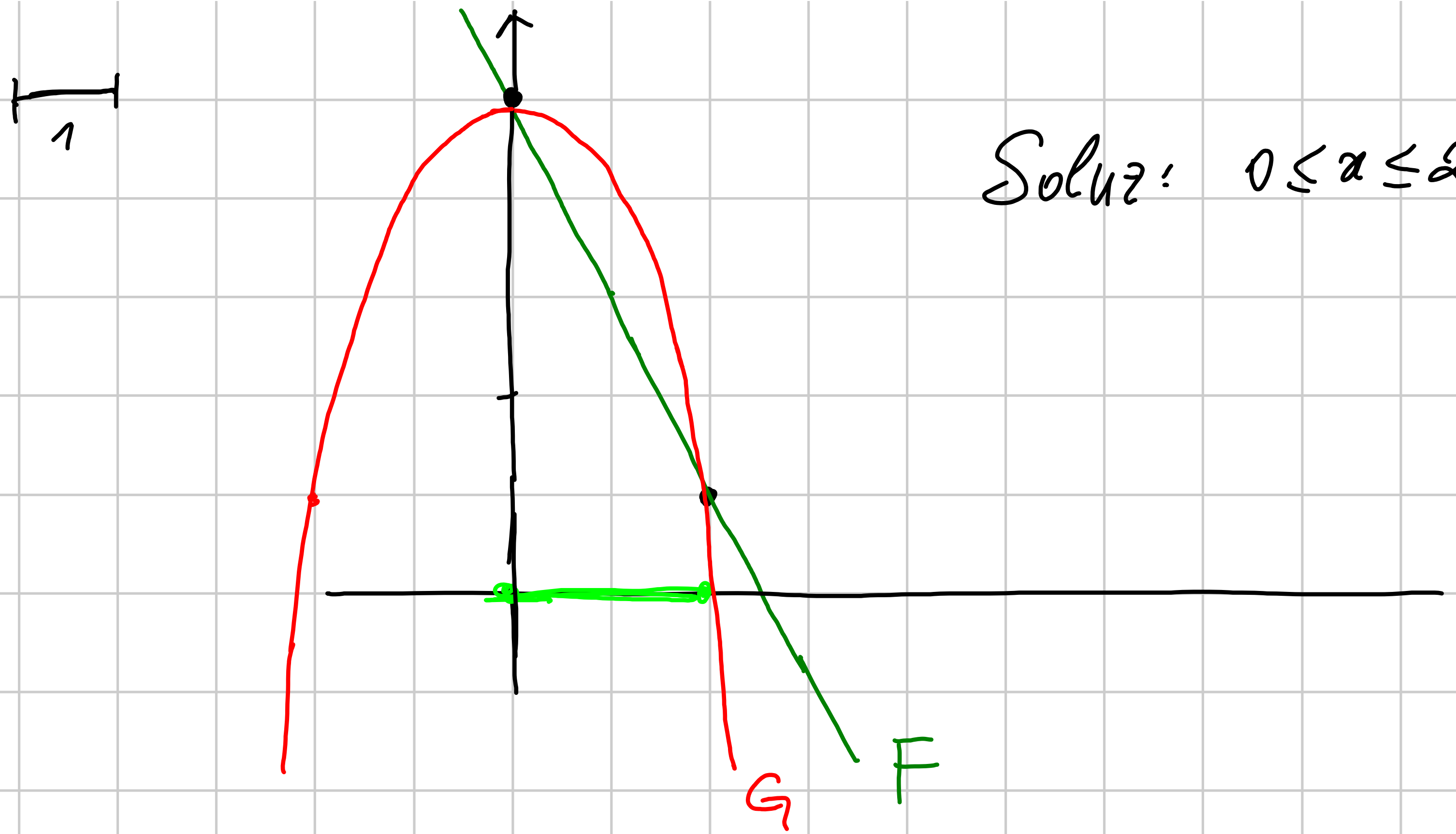
$f(0) = g(0) \qquad f(2) = g(2)$



$$\begin{cases} b = 0 \\ c = 5 \\ 4a + 2b + c = 1 \end{cases} \qquad \begin{cases} b = 0 \\ c = 5 \\ a = -1 \end{cases}$$

$$\Longrightarrow g(x) = -x^2 + 5$$

- Risolvere $f(x) \leq g(x)$



Soluz: $0 \leq x \leq 2$

10 $f(x) = 3x^2$ F

$g(x) = 3x^2 - 24x + 11$ G

- Posizione di G rispetto a F. Voglio scrivere

$$g(x) = f(x+k) + h$$

$$3x^2 - 24x + 11 = 3(x+k)^2 + h$$

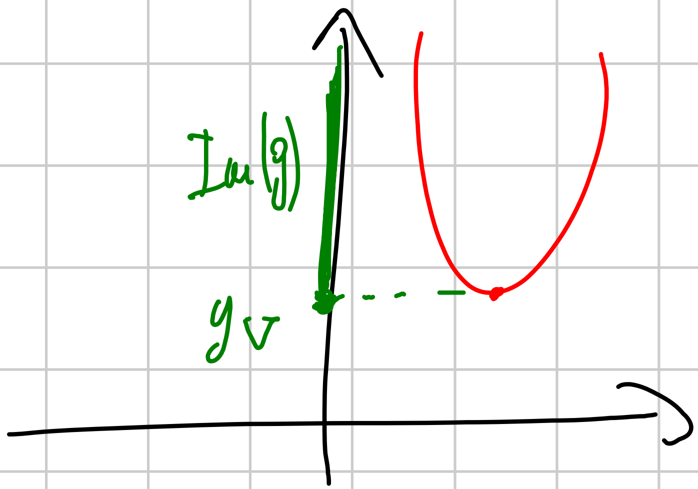
$$= 3x^2 + 6kx + 3k^2 + h$$

$$\Rightarrow k = -4$$

$$h = 11 - 3k^2 = 11 - 48 = -37$$

$\Rightarrow$ G ottenuta da F traslando di 4 verso dx e di 37 verso il basso.

• Img $\qquad g(x) = 3x^2 - 24x + 11$



$x_v = 4$

$y_v = 3 \cdot 16 - 24 \cdot 4 + 11$

$= 59 \quad \Rightarrow \mathrm{Img} = [59, +\infty)$

- $g(x) \geq 11$ $\quad$ $3x^2 - 24x \geq 0$ $\quad$ $x^2 - 8x \geq 0$

$x(x-8) \geq 0$

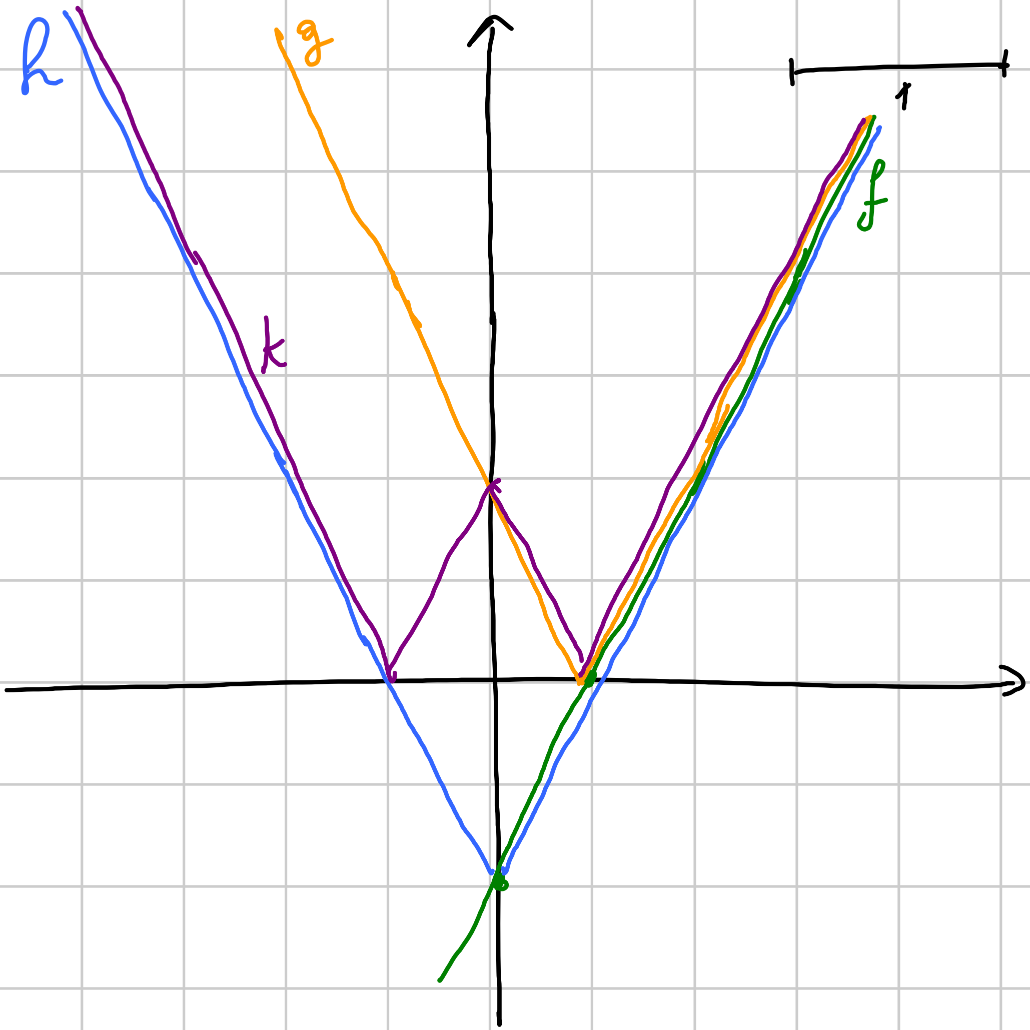
$x \leq 0$ ou $x \geq 8$

(11) $f(x) = 2x - 1$

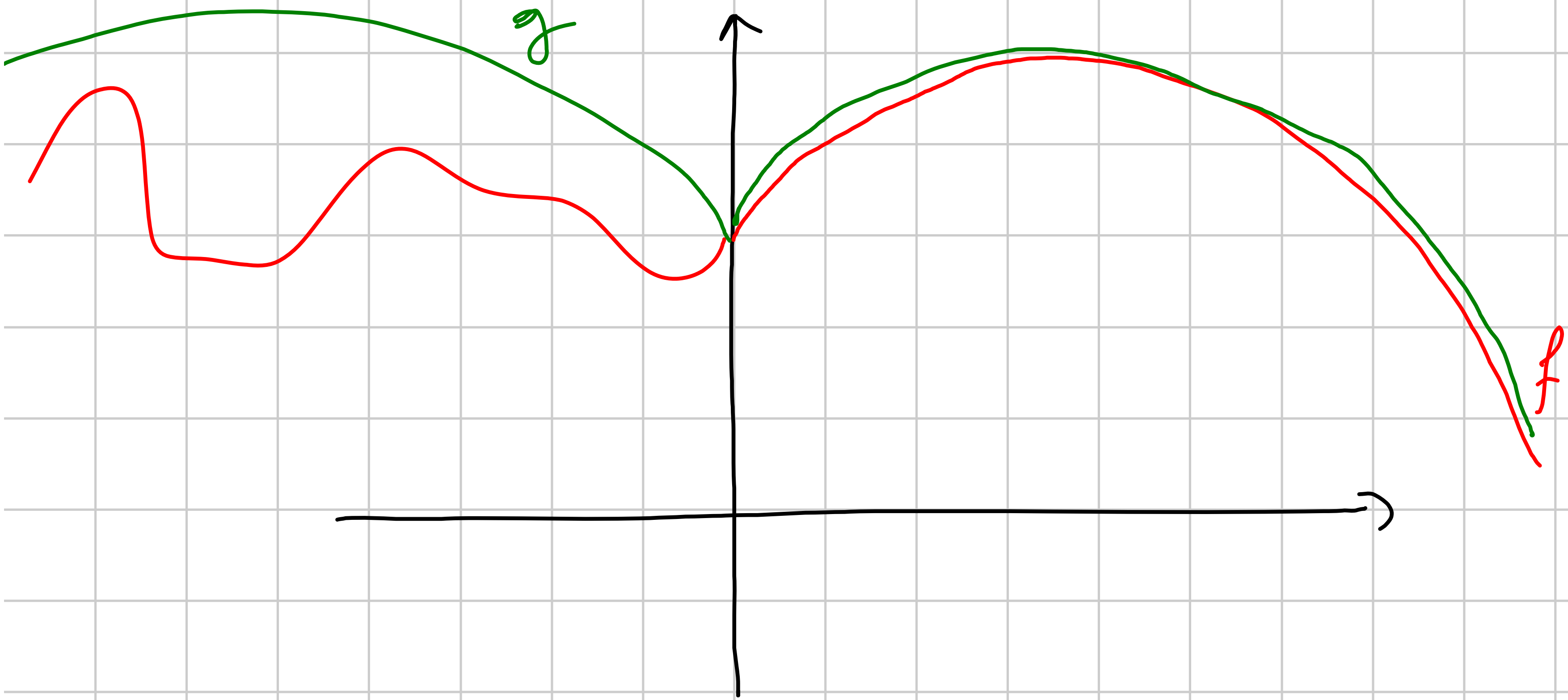
$g(x) = |f(x)|$

$h(x) = f(|x|)$

$k(x) = |f(|x|)|$

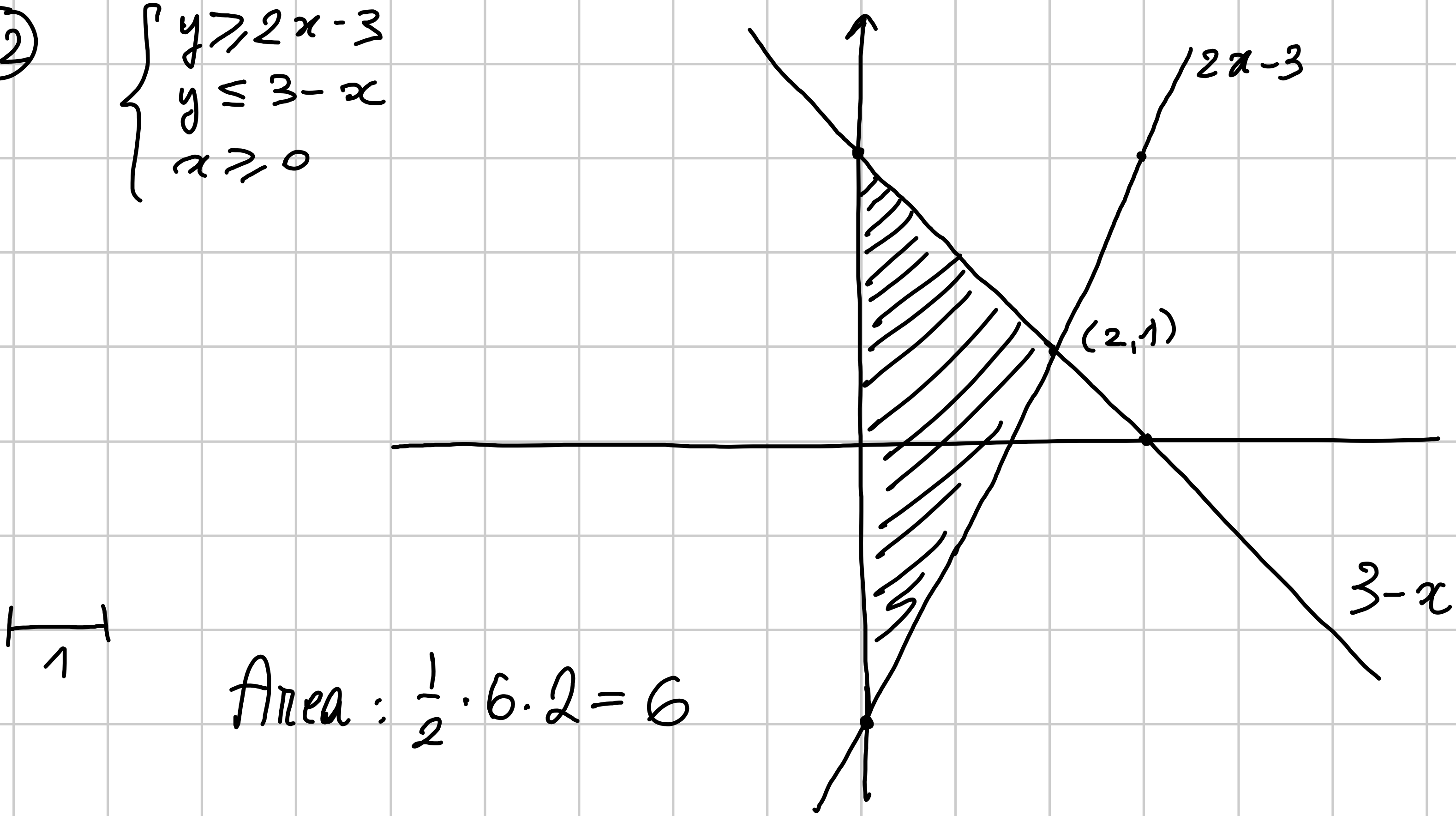


Es $f(x) = \dots$ $\quad g(x) = f(|x|)$



(12)

$$\begin{cases} y \geq 2x - 3 \\ y \leq 3 - x \\ x \geq 0 \end{cases}$$



Area: $\frac{1}{2} \cdot 6 \cdot 2 = 6$

(14) $\begin{cases} y = (2+k)x - (2k+3) \\ y = x^2 - 3x + 4 \end{cases}$

I modo : algebricamente

$$(2+k)x - (2k+3) = x^2 - 3x + 4$$

$$x^2 - (5+k)x + 2k + 7 = 0$$

$$\Delta = (5+k)^2 - 4(2k+7)$$

$$= k^2 + 10k + 25 - 8k - 28$$

$$= k^2 + 2k - 3 = (k+3)(k-1)$$

Dunque: due soluzioni per $k < -3$ e $k > 1$

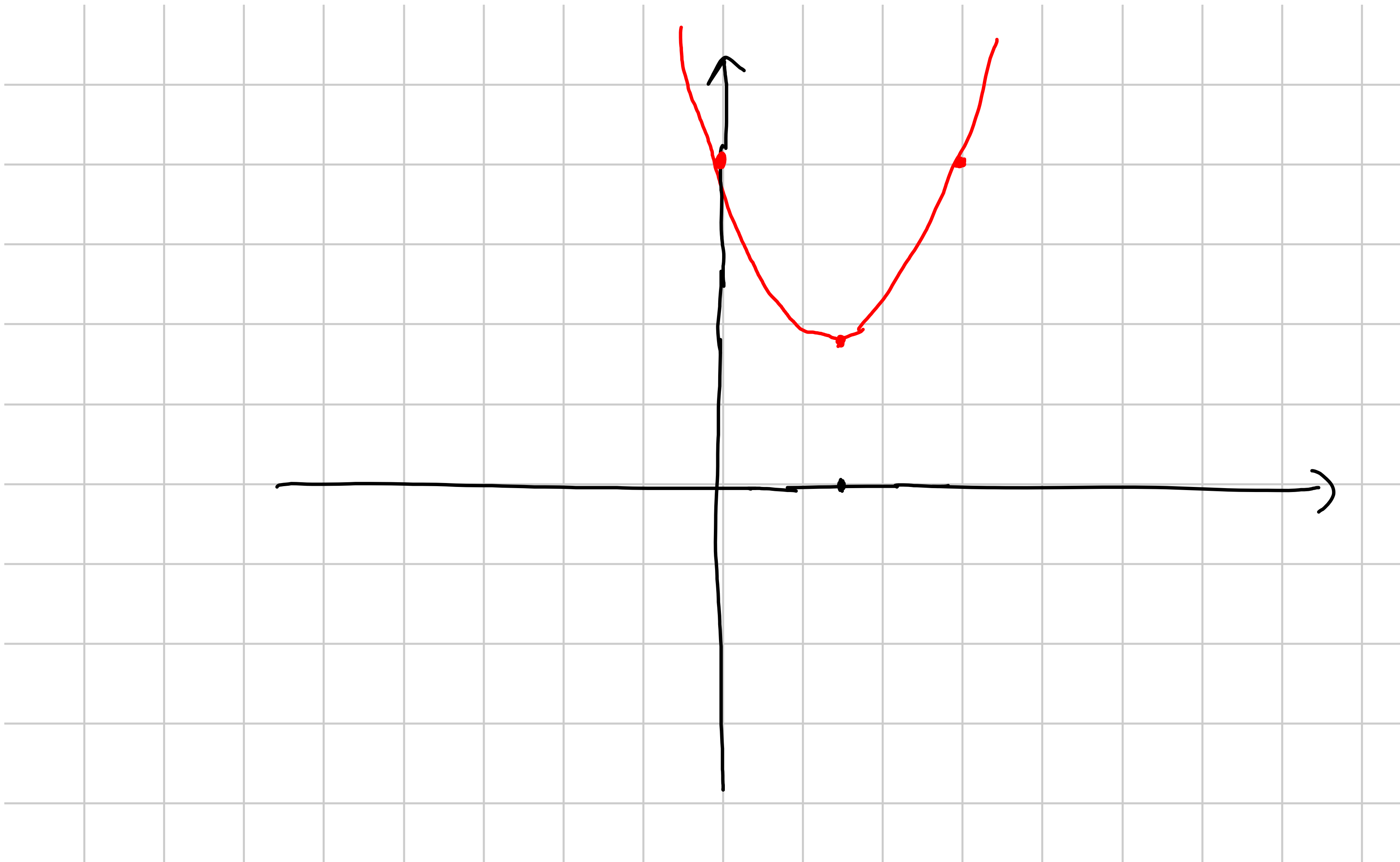
una sola per $k = -3$ e $k = 1$

nessuna per $-3 < k < 1$

II modo: $\begin{cases} y = (2+k)x - (2k+3) \\ y = x^2 - 3x + 4 \end{cases}$

$x_v = \frac{3}{2}$

$y_v = \frac{9}{4} - \frac{9}{2} + 4 = -\frac{9}{4} + 4 = \frac{7}{4}$

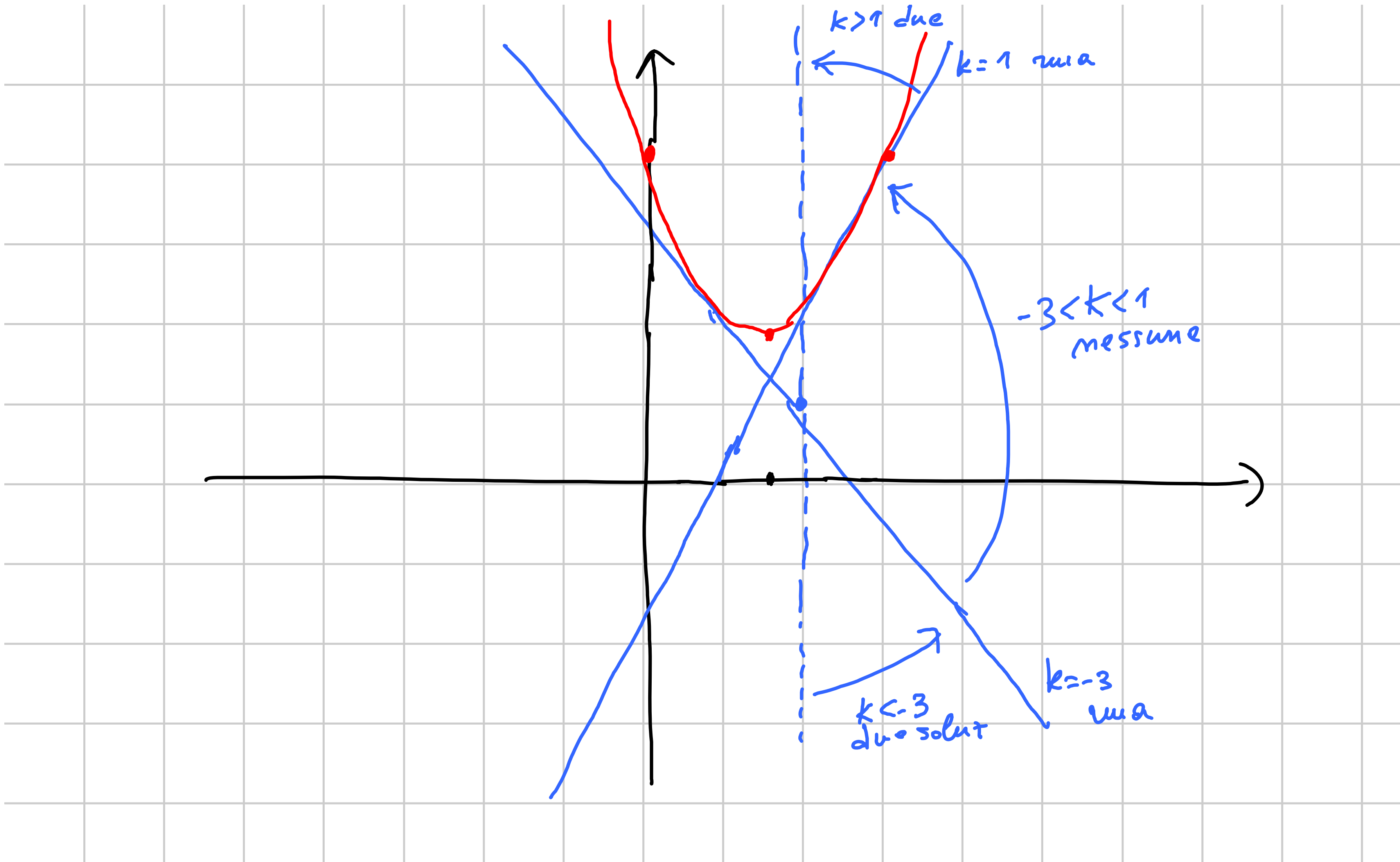


$y = (2+k)x - (2k+3)$ fascio di rette di centro...

$k = -2$ $\quad y = 1$

$x = 2$

$k = 0$ $\quad y = 2x - 3$ $\quad y = 1$


k>1 due
k=1 una
-3<k<1 nessuna
k=-3 una
k<-3 due soluz