$$h(x) = \frac{f(x)}{g(x)} \qquad h(x) = f(x) \cdot \frac{1}{g(x)}$$

$$h'(x) = f'(x) \cdot \frac{1}{g(x)} + f(x) \cdot \left(- \frac{g'(x)}{g(x)^2} \right)$$

$$= \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$$

- $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$

$$D(\tan(x)) = \frac{\cos(x)\cdot\cos(x) - (-\sin(x))\cdot\sin(x)}{\cos(x)^2}$$

$$= \frac{1}{\cos(x)^2} = 1 + \tan(x)^2$$

- $a^x = \left(e^{\ln(a)}\right)^x = e^{x\cdot\ln(a)}$

$$D(a^x) = D\left(e^{x\cdot\ln(a)}\right)$$

$$= e^{x \cdot \ln(a)} \cdot \ln(a) = \ln(a) \cdot a^{x}$$

$$D(\operatorname{asin}(x)) = \frac{1}{\cos(y)} \qquad y = \operatorname{asin}(x)$$

$$\cos^2 y + \sin^2 y = 1$$

$$\Rightarrow \cos(y) = \sqrt{1 - \sin^2(y)}$$

$$= \sqrt{1 - x^2}$$

$$D(\operatorname{asin}(x)) = \frac{1}{\sqrt{1-x^2}}$$

• $D(\operatorname{atan}(x)) = \dfrac{1}{1+\tan(y)^2}$ $\quad y = \operatorname{atan}(x)$

$$= \frac{1}{1+x^2}$$

Fatto: se f è derivabile in x_0 la retta tangente al grafico di f nel punto $(x_0, f(x_0))$ è $y = f'(x_0)\cdot(x-x_0)+f(x_0)$

(la retta che passa per $(x_0, f(x_0))$
con coeff. ang. $f'(x_0)$).

Ese 1 : trovare tg a grafico f in $(x_0, f(x_0))$

(a) $f(x) = x + \ln(x) \quad x_0 = 5 \qquad f(5) = 5 + \ln(5)$

$$f'(x) = 1 + \frac{1}{x} \qquad f'(5) = \frac{6}{5}$$

$$\Longrightarrow \quad y = \frac{6}{5}(x-5) + 5 + \ln(5)$$

$$y = \frac{6}{5}x + \ln(5) - 1$$

(b) $f(x) = e^{x} + 5\sin(x) - 3x \quad x_0 = 0 \qquad f(0) = 1 + 0 + 0 = 1$

$f'(x) = e^{x} + 5\cos(x) - 3 \qquad f'(0) = 1 + 5 - 3 = 3$

$\Rightarrow y = 3x + 1$

(c) $f(x) = \cos(x) + \log_{\pi}(x) \quad x_0 = \pi \qquad f(\pi) = -1 + 1 = 0$

$f'(x) = -\sin(x) + \dfrac{1}{x \cdot \ln(\pi)} \qquad f'(\pi) = 0 + \dfrac{1}{\pi \cdot \ln\pi}$

$\Rightarrow y = \dfrac{1}{\pi \cdot \ln(\pi)} \cdot (x - \pi) + 0$

$= \dfrac{x}{\pi \cdot \ln(\pi)} - \dfrac{1}{\ln(\pi)}$

(d) $f(x) = \dfrac{5x + \sin(2x)}{3x - \cos(4x)}$ $\quad x_0 = 0 \quad f(0) = \dfrac{0+0}{0-1} = 0$

$$f'(x) = \frac{\big(5 + 2\cos(2x)\big)\big(3x - \cos(4x)\big) - \big(5x + \sin(2x)\big)\big(3 + 4\sin(4x)\big)}{\big(3x - \cos(4x)\big)^2}$$

$$f'(0) = \frac{7 \cdot (-1) - 0 \cdot (3+0)}{(-1)^2} = -7$$

$$\Longrightarrow \quad y = -7x$$

Ese 3 : troviamo pto su grafico f con tangente ℓ

(a) $f(x) = x^2 + 2x - 1$ $\qquad \ell : y = 6x - 5$

Cerchiamo $x_0 \in \mathbb{R}$ t.c.

$$\ell : \; y = f'(x_0) \cdot (x - x_0) + f(x_0)$$

cioè

$$\begin{cases} f'(x_0) = 6 \\ -x_0 \cdot f'(x_0) + f(x_0) = -5 \end{cases}$$

Ora $f'(x) = 2x + 2$ dunque $\begin{cases} 2x_0 + 2 = 6 \\ \dots \end{cases}$

$$\begin{cases} x_0 = 2 \\ -2\cdot 6 + 7 \overset{?}{=} -5 \end{cases} \quad \text{Sì}$$

$\Longrightarrow$ punto $(2, 7)$

(b) $f(x) = x^3 + 2x \quad l: y = 5x + 2$

$f'(x) = 3x^2 + 2$

Trovo i punti in cui $f'(x) = 5$ e poi vedo se in qualcuno di loro la tang. è l:

$3x^2 + 2 = 5 \qquad x = \pm 1$

Tang in -1 : $y = 5(x+1) + (-1-2) = 5x+2$

OK

Tang in $+1$: $y = 5(x-1) + (1+2) = 5x-2$

NO

(c) $f(x) = x^4 - 2x^2$ $\quad l : y = -1$

$f'(x) = 4x^3 - 4x$

Impongo $f'(x) = 0$ $\quad 4x^3 - 4x = 0$

$x = 0, \; x = 1, \; x = -1$

Tangenti rispettive sono $y = 0, \; y = -1, \; y = -1$

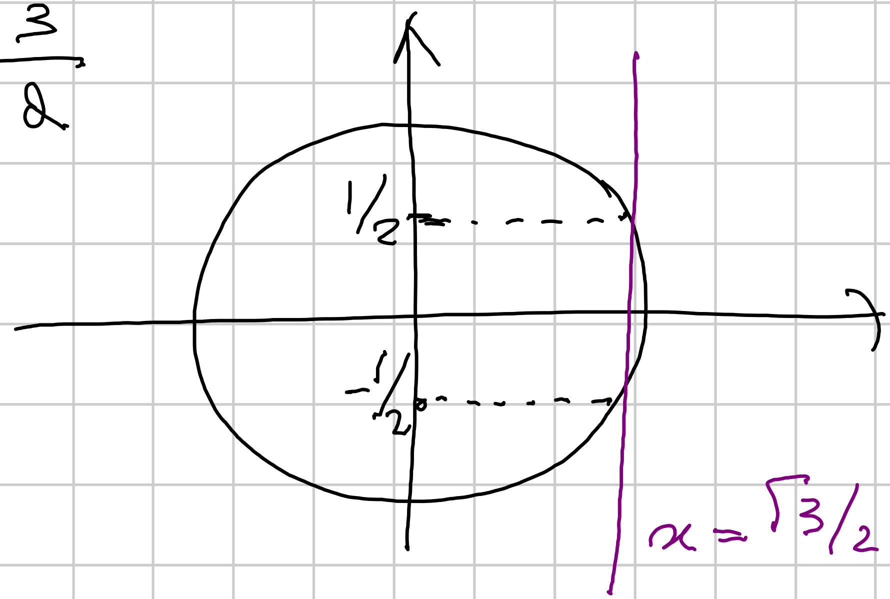
vanno bene i punti $(-1,-1)$ e $(1,-1)$

(d) $f(x) = \sin(x)$ $\quad l: \frac{\sqrt{3}}{2} \cdot x + \frac{1}{2} - \frac{\pi}{4\sqrt{3}}$

Imponiamo $f'(x) = \frac{\sqrt{3}}{2}$

$\cos(x) = \frac{\sqrt{3}}{2}$

$x = \pm \frac{\pi}{6} + 2k\pi$



tangente nel pto di ascissa $\pm \frac{\pi}{6} + 2k\pi$

$$y = \frac{\sqrt{3}}{2}\left(x \mp \frac{\pi}{6} + 2k\pi\right) \pm \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2}\cdot x \pm \frac{1}{2} \mp \frac{\pi}{4\sqrt{3}} + k\pi\sqrt{3}$$

Viene l con segno $+$ e $k=0$

$\Longrightarrow$ l è tangente al grafico del seno nel punto di ascissa $\pi/6$ –

Ese 4

(b) $f(x) = -3x^2 + 5x + 1$ $\qquad x_1 = 0 \qquad x_2 = -1$

$f'(x) = -6x + 5$

$$\begin{cases} y = 5 \cdot (x - 0) + 1 \\ y = 11 \cdot (x + 1) - 7 \end{cases} \qquad \begin{cases} y = 5x + 1 \\ y = 11x + 4 \end{cases}$$

$$\begin{cases} x = -\frac{1}{2} \\ y = -\frac{3}{2} \end{cases}$$

(c) $f(x) = 3\sin(x) - x \qquad x_1 = \frac{\pi}{2} \qquad x_2 = \pi$

$$f'(x) = 3\cos(x) - 1$$

$$\begin{cases} y = (3 \cdot 0 - 1) \cdot \left(x - \frac{\pi}{2}\right) + \left(3 \cdot 1 - \frac{\pi}{2}\right) \\ y = (3 \cdot (-1) - 1) \cdot (x - \pi) + (3 \cdot 0 - \pi) \end{cases}$$

$$\begin{cases} y = -x + 3 \\ y = -4x + 3\pi \end{cases}$$

...

Ese 5: derivare

(a) $D\left(x^2+\cos(x)^3\right) = 2x + 3\cos(x)^2 \cdot (-\sin(x))$

(b) $D\left(x \cdot \ln(x^2-3)\right) =$

$= \ln(x^2-3) + x \cdot \dfrac{1}{x^2-3} \cdot (2x)$

(c) $D\left((e^x+\sin(x))^2 \right)$

$= 2\left(e^x+\sin(x)\right) \cdot \left(e^x+\cos(x)\right)$

(d) $D\left(e^{-x} \cdot (x^3+5x^2-1)\right)$

$$= e^{-x}\cdot(-1)\cdot(x^3+5x^2-1)+e^{-x}\cdot(3x^2+10x)$$

$$= e^{-x}\left(\begin{aligned}-x^3-5x^2&\qquad +1\\ +3x^2+10x&\end{aligned}\right)$$

$$= e^{-x}\left(-x^3-2x^2+10x+1\right)$$

(e) $D\left(x\cdot e^x-\arctan(e^x)\right)$

$$= D\left(e^x+x\cdot e^x-\frac{1}{1+(e^x)^2}\cdot e^x\right)$$

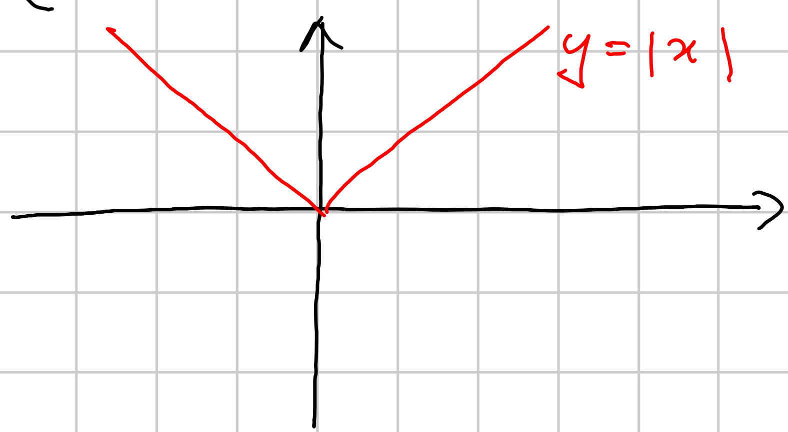
$$= e^x\left(1+x-\frac{1}{1+e^{2x}}\right)$$

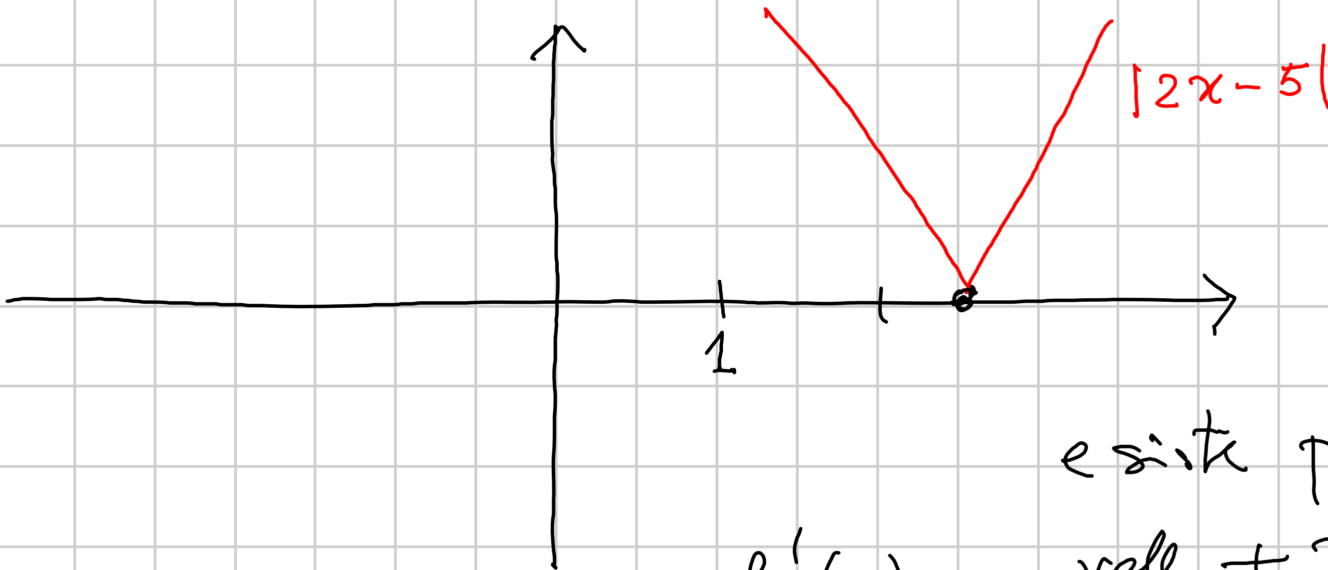
(f) $D\left(\operatorname{acos}\left(\log_3(7x-1)\right)\right)$

$$= -\frac{1}{\sqrt{1-\left(\log_3(7x-1)\right)^2}} \cdot \frac{1}{\ln(3)\cdot(7x-1)} \cdot 7$$

<u>Ese 6</u> : dire dove esiste $f'(x)$ e calcolarla.

(a) $f(x) = |2x-5|$





$f'(x)$ esiste per $x \neq 5/2$

vale $+2$ per $x > 5/2$

vale -2 per $x < 5/2$

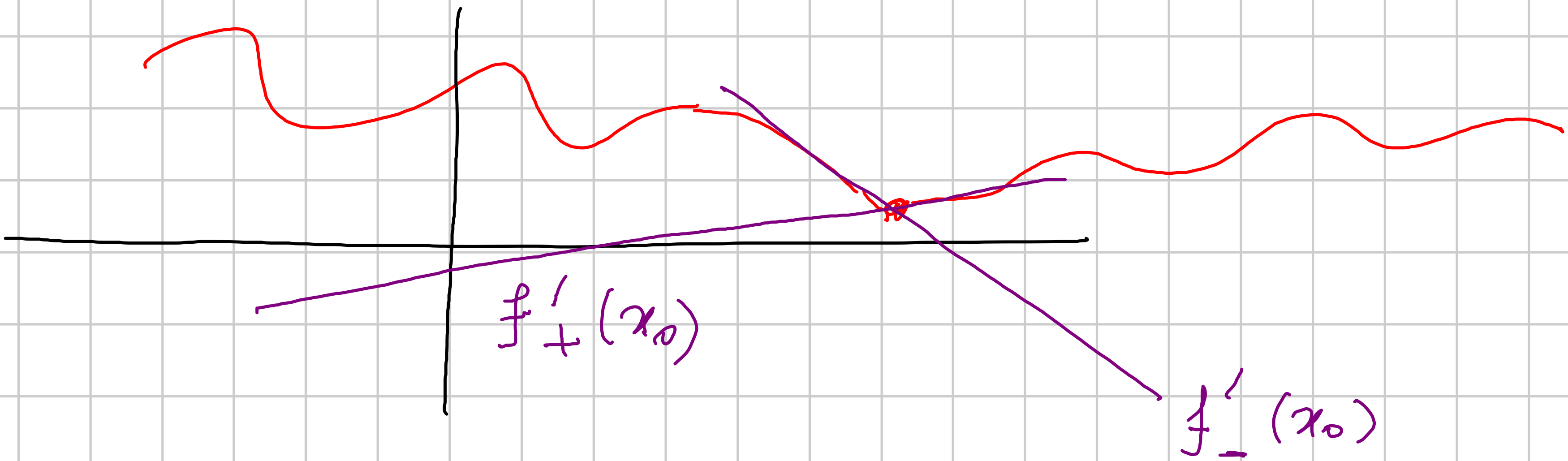
(b) $f(x) = \dfrac{1}{x-1}$ definita per $x \neq 1$.

Dove f è definita lo è anche f' e vale

$$f'(x) = -\frac{1}{(x-1)^2}$$

(c) $f(x) = x \cdot |x-1| = \begin{cases} x^2 - x & \text{per } x \geq 1 \\ x - x^2 & \text{per } x \leq 1 \end{cases}$

Certamente f ha derivata per $x \neq 1$.
Inoltre in $x=1$ ha derivata destra e sinistra: la f risulterà derivabile in $x=1$ se derivata destra e sinistra coincidono.



Per $x > 1$, $f'(x) = 2x - 1 \Longrightarrow f'_+(1) = 2 - 1 = 1$

Per $x < 1$, $f'(x) = 1 - 2x \Longrightarrow f'_-(1) = 1 - 2 = -1$

$\Longrightarrow f'(1)$ non esiste

d) $\cos(|x|) = \begin{cases} \cos(x) & x \geq 0 \\ \cos(-x) & x \leq 0 \end{cases} = \begin{cases} \cos(x) & x \geq 0 \\ \cos(x) & x \leq 0 \end{cases}$

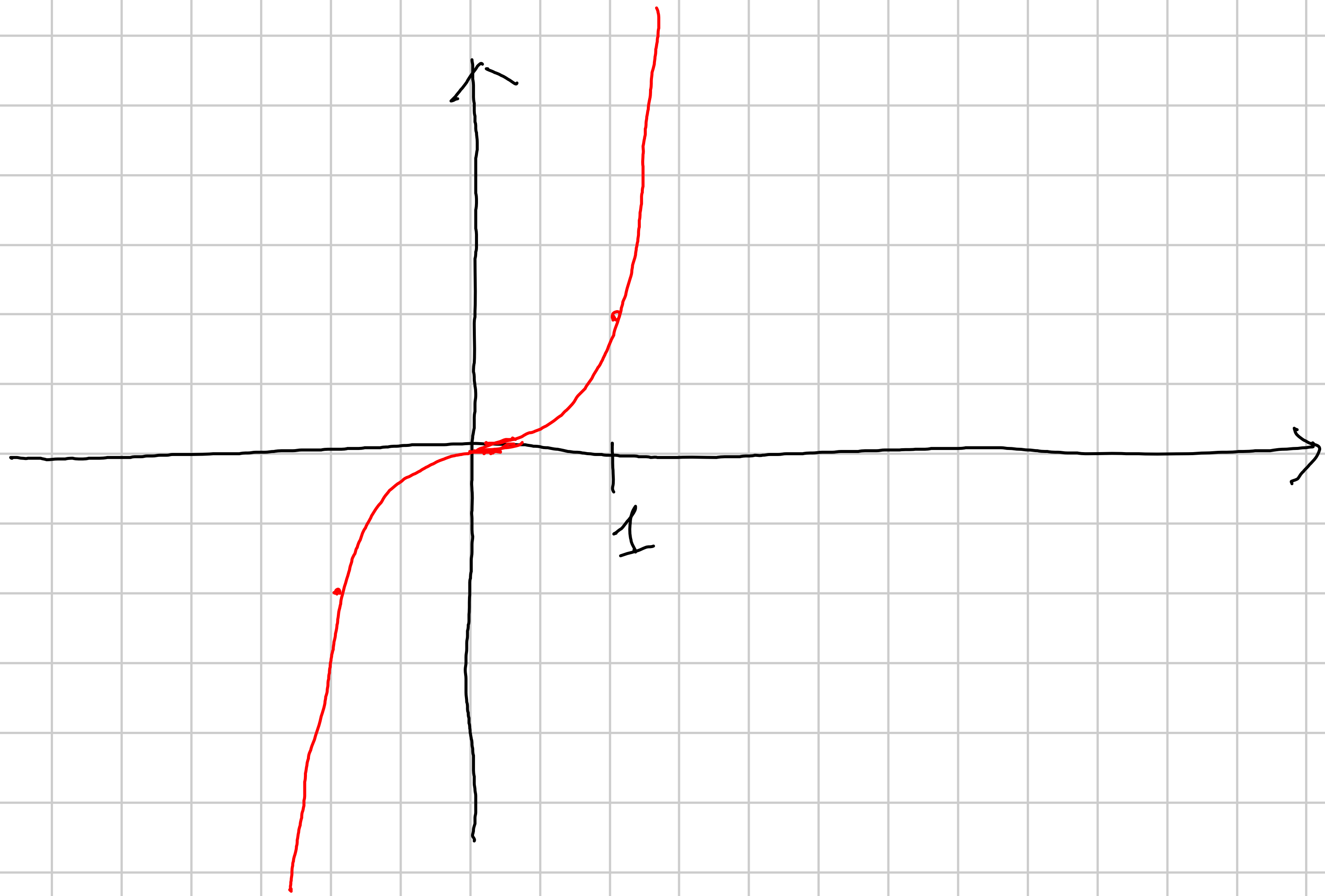
$= \cos(x) \Longrightarrow$ derivabile ovunque.

(e) $f(x) = x \cdot |x| = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x \leq 0 \end{cases}$

Per $x > 0$ $\quad f'(x) = 2x \quad f'_+(0) = 0$

Per $x < 0$ $\quad f'(x) = -2x \quad f'_-(0) = 0$

$\Longrightarrow$ $f'(0)$ esiste e vale 0.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f) $f(x) = x \cdot \sin(|x|) = \begin{cases} x \cdot \sin(x) & x \geq 0 \\ x \cdot \sin(-x) & x \leq 0 \end{cases} = \begin{cases} x \cdot \sin(x) & x \geq 0 \\ -x \cdot \sin(x) & x \leq 0 \end{cases}$

Per $x > 0$ $\quad f'(x) = \sin(x) - x \cdot \cos(x)$; $f'_+(0) = 0$

Per $x < 0$ $\quad f'(x) = x \cdot \cos(x) - \sin(x)$; $f'_-(0) = 0$

$\Longrightarrow$ $f'(0)$ esiste e vale 0

<u>Ese 7</u>: dire dove $f'(x)$ soddisfa...

(a) $f(x) = \sin(e^x)$ $\qquad f'(x) = 0$

$$f'(x) = \cos(e^x) \cdot e^x$$

$$f'(x) = 0 \iff \cos(e^x) = 0$$

$$\iff e^x = \frac{\pi}{2} + k\pi \qquad k \in \mathbb{Z}$$

$$\iff x = \log\left(\frac{\pi}{2} + k\pi\right) \qquad k \in \mathbb{N}$$

(b) $f(x) = \ln(x^2+3x+6)$ $\qquad f'(x) = 0$

$$\frac{2x+3}{x^2+3x+6} = 0 \qquad x = -3/2$$

(c) $f(x) = (2+x^2)\cdot e^x$ $\qquad f'(x) > 0$

$$f'(x) = 2x\cdot e^x + (2+x^2)\cdot e^x$$

$$= e^x(x^2+2x+2)$$

$$= e^x\left((x+1)^2+1\right)$$ positivo $\forall x \in \mathbb{R}$

(d) $f(x) = (x^2-3)\cdot e^x$ $\qquad f'(x) > 0$

$$f'(x) = 2x\cdot e^x + (x^2-3)e^x = \underbrace{e^x}_{\text{sempre} > 0}(x^2+2x-3)$$

$x^2+2x-3>0$ $(x+3)(x-1)>0$

$x<-3$ oppure $x>1$