Matematica — 13/10/15

$$\sqrt[3]{x^3} = \begin{cases} x & \sqrt[3]{\text{alg.}} \\ \begin{cases} x & x>0 \\ ? & x<0 \end{cases} & \sqrt[3]{\text{init}} \end{cases}$$

① $\left(\sqrt{1000}\right)^3 = \left(\left(10^3\right)^{\frac{1}{2}}\right)^3$

$= 10^{3\cdot\frac{1}{2}\cdot 3} = 10^{9/2}$

10^5 NO ; $100\sqrt{10} = 10^2 \cdot 10^{\frac{1}{2}} = 10^{2+\frac{1}{2}} = 10^{\frac{5}{2}}$ NO

$10\sqrt{1000} = 10^{1} \cdot 10^{\frac{3}{2}} = 10^{\frac{5}{2}}$ NO

$10 \cdot \sqrt{10} = 10^{1} \cdot 10^{\frac{1}{2}} = 10^{1+\frac{1}{2}} = 10^{\frac{3}{2}}$ SÌ

10^{4} NO

② $\log_8 4 = ?$

($\log_b a$ è la soluz. dell'equaz. $b^x = a$)

* $8^x = 4$

$2^{3x} = 2^2$

$3x = 2 \qquad x = 2/3$

$* \log_8 4 =$

$\left(\log_b a = \frac{\log_c a}{\log_c b} \right)$

$\left(\log_b a^n = n \cdot \log_b a \right)$

$= \frac{\log_{10}(4)}{\log_{10}(8)} = \frac{\log_{10}(2^2)}{\log_{10}(2^3)}$

$= \frac{2 \cdot \log_{10}(2)}{3 \cdot \log_{10}(2)} = \frac{2}{3}$

$* \log_8 4 = \frac{\log_2 4}{\log_2 8} = \frac{2}{3}$

(3) $\frac{1}{0.2} = \frac{1}{\frac{2}{10}} = \frac{1}{\frac{1}{5}} = 5$

④ $x > 0$ si ha sempre...

$x \geq \sqrt{x}$: $x = \frac{1}{4}$ $\sqrt{x} = \frac{1}{2}$ NO

$x^2 \geq x$: $x = \frac{1}{2}$ $x^2 = \frac{1}{4}$ NO

$\frac{1}{x} \geq \frac{1}{\sqrt{x}}$ cioè $\sqrt{x} \geq x$: $x = 4$ $\sqrt{x} = 2$ NO

$\frac{1}{x} \geq \frac{1}{x^2}$ cioè $x^2 \geq x$. NO

⑤ $30\% \cdot L = 1.5\ h$

$\frac{30}{100} \cdot L = \frac{3}{2} h \Rightarrow \frac{3}{10} \cdot L = \frac{3}{2} h$

$\Rightarrow L = 5h$

Per Luino: $3.5\,h = 3.5 \times 60\,min = 210\,min$

⑥ Miscela A+Z: 50 kcal / 100 ml

Z: 400 kcal / 100 g = 4 kcal / g

In una bottiglia da 330 ml, quanti cucchiaini da 5 g?

kcal nella bottiglia: $50\,kcal : 100\,ml = x : 330\,ml$

$x = \frac{330}{100} \cdot 50\,kcal = \frac{330}{2}\,kcal =$

$= 165$ kcal

g Zucchero nella lattina $= \frac{165}{4} g = 41.25$

quanti cucchiaini? $\frac{41.25\ g}{5\ g} = \ldots$

(7) 20 oggetti $= 24$ g. Quanti in un peso da 1 kg

oppure $= \frac{24}{20} g = \frac{6}{5} g = 1.2$ g.

Quanti: $\frac{1000\,g}{1.2\,g} = 833,\ldots$

(8) Se V aumenta del 25%, di quanto deve diminuire per tornare V?

V_0 ; $V_1 = \left(1 + \frac{25}{100}\right) \cdot V_0$

$V_2 = \left(1 + \frac{x}{100}\right) \cdot V_1$

Vogliamo: $V_2 = V_0$

$$\left(1+\frac{x}{100}\right)\cdot\left(1+\frac{25}{100}\right)V_0 = V_0$$

$$\left(1+\frac{x}{100}\right)\left(1+\frac{\cancel{25}^{1}}{\cancel{100}_{4}}\right) = 1$$

$$\left(1+\frac{x}{100}\right) = \frac{4}{5} = 0.8$$

$$x = 100\,(0.8-1) = 100\cdot(-0.2) = -20$$

(9) -35% e poi $+35\%$: in tutto?

$$V_0 \longrightarrow V_1 = \left(1 - \frac{35}{100}\right) V_0 \longrightarrow V_2 = \left(1 + \frac{35}{100}\right) V_1$$

$$= \left(1 - \frac{35}{100}\right)\left(1 + \frac{35}{100}\right) \cdot V_0$$

$$= \left(1 + \frac{x}{100}\right) V_0$$

$$1 + \frac{x}{100} = \frac{65}{100} \cdot \frac{135}{100} \quad \dots \quad x = -12.25$$

(10) $C = \dfrac{S}{S + a}$

S = massa sale

a = massa acqua

- $a = 100\ ml = 100g$ $\qquad$ $S = 25g$

$$C = \frac{s}{s+a} = \frac{25}{25+100} = \frac{1}{5} = 20\ \%$$

- C_0 concentraz. iniz. $= 10\% = \frac{10}{100} = \frac{1}{10}$

  C_1 Concentraz. fin $= 6\% = \frac{6}{100} = \frac{3}{50}$

$$C_0 = \frac{s_0}{s_0 + a_0} \qquad C_1 = \frac{s_1}{s_1 + a_1} = \frac{s_0}{s_0 + a_0 + 100g}$$

$$\begin{cases} \dfrac{s_0}{s_0 + a_0} = \dfrac{1}{10} \\ \dfrac{s_0}{s_0 + a_0 + 100g} = \dfrac{3}{50} \end{cases} \qquad \begin{cases} 10 s_0 = s_0 + a_0 \\ \text{---} \end{cases}$$

$$\begin{cases} a_0 = 9 s_0 \\ \dfrac{9 s_0}{10 s_0 + 100\,g} = \dfrac{3}{50} \end{cases} \qquad \begin{cases} 450 s_0 = 30 s_0 + 300\,g \\ a_0 = 9 s_0 \end{cases}$$

Calcoli...

Oppure: $C_0 = \dfrac{s_0}{m_0}$ $\qquad C_1 = \dfrac{s_1}{m_1} = \dfrac{s_0}{m_0 + 100\,g}$

$$\begin{cases} \dfrac{s_0}{m_0} = \dfrac{1}{10} \\ \dfrac{s_0}{m_0 + 100\,g} = \dfrac{6}{100} \end{cases} \qquad \begin{cases} s_0 = \dfrac{1}{10} m_0 \\ \dfrac{\frac{1}{10} m_0}{m_0 + 100\,g} = \dfrac{3}{50} \end{cases}$$

$5m_0 = 3m_0 + 300g \Rightarrow 2m_0 = 300g$

$m_0 = 150g$

(11) Aumento del 10% le dimensioni di un parallelepipedo. Come deve cambiare il prezzo totale del liquido contenuto perché il prezzo per unità di volume cali del 10%?

V_0 = vol iniziale = alt. × largh. × prof.

V_1 = vol finale = alt × 1.1 × largh × 1.1 × prof × 1.1

$$= dt \times larg \times prof \times (1.1)^3$$

$$= V_0 \cdot (1+0.1)^3$$

$$= V_0 \cdot (1 + 0.3 + 0.03 + 0.001)$$

$$\left((a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \right)$$

$$\Longrightarrow \quad V_1 = V_0 \cdot 1.331$$

P_0 = vecchio prezzo

P_1 = nuovo prezzo

Voglio: $\frac{P_1}{V_1} = \frac{P_0}{V_0} \cdot 0.9$

$\Rightarrow P_1 = \frac{V_1}{V_0} \cdot 0.9 \cdot P_0$

$\Rightarrow \frac{P_1}{P_0} = 1.331 \times 0.9 = 1.1979 \cong 1.20$

$\Rightarrow$ aumento: 20 %

(12) — non lo faccio

(13) $21.4\ \mu m \pm 0.3\ \mu m$

val. atteso — err. ass.

posso aver osservato 21.1 μm ? Sì

21.1 μm ? No

(14) $\sqrt{5} = 2.230679\ldots \cong 2.23068$

↑ ↑ ↑ ↑

(15) $a = 188 \pm 18$ $\quad b = 176 \pm 16$

facendo operaz. con misure approssimate:

- il val. atteso è quello ovvio
- per somme e diff, l'errore assoluto è la somma degli errori assoluti
- per prodotto e quoziente, l'errore relativo è la somma degli errori relativi

$$\frac{188 \pm 18}{176 \pm 16}$$

val. atteso: $\frac{188}{176} = 1.06818$

err. rel. $= \frac{18}{188} + \frac{16}{176} = 0.18665$

err. ass. $= 0.18665 \times 1.06818 \simeq 0.19938$

(16) $5.3 \leq a \leq 7.2$

$6.4 \leq b \leq 8.1$ $\quad (-8.1 \leq -b \leq -6.4)$

$1/8.1 \leq 1/b \leq 1/6.4$

$\Rightarrow 5.3+6.4 \leq a+b \leq 7.2+8.1$

$5.3-8.1 \leq a-b \leq 7.2-6.4$

$5.3 \times 6.4 \leq a \cdot b \leq 7.2 \times 8.1$

$5.3 / 8.1 \leq a/b \leq 7.2/6.4$

(17) 5.3 m 6.2 m 5.1 m 6.4 m 5.9 m

$$v = \frac{6.4+5.1}{2}\,m = 5.75\,m$$

$$\text{Durchm.} = \frac{6.4-5.1}{2}\,m = 0.65\,m$$

$$\text{err. rel} = \frac{0.65}{5.75} = 0.113\ldots = 11.3\ \%$$

(18) Auto percorre 170 ± 5 km in 50 ± 5 min.

$$\text{v.a. (vel. media)} = \frac{170\ \text{km}}{50\ \text{min}} = \frac{170\ \text{km}}{50 \cdot \frac{1}{60}\ \text{h}} = \frac{6}{5} \cdot 170\ \text{km/h}$$

$(1\text{h} = 60\text{min})$

$$= 204\ \text{km/h}$$

$$\text{Imp. rel.} = \frac{5}{170} + \frac{5}{50} = 0.1294$$

$$\text{err. ass} = \text{v.a.} \times \text{err. rel} = 204 \times 0.1294 \text{ km/h}$$
$$= 26.4 \text{ km/h}$$

(19) Stima vol. cubo $= 27 \pm 0.3 \text{ cm}^3$ _
Stima lato?

$$\text{Lato} = l \pm d$$

Sappiamo: $(l \pm d)^3 = 27 \pm 03 \text{ cm}^3$

$$l^3 \pm 3l^2 d + \underbrace{3ld^2 \pm d^3}$$

trascurabile se $d/l \ll 1$

Quindi $l^3 = 27\,cm^3 \Longrightarrow l = 3\,cm$

err. ass. $3l^2 d = 0.3\,cm^3$

$$3 \cdot 9\,cm^2 \cdot d = 0.3\,cm^3$$

$$d = \frac{3}{10} \cdot \frac{1}{3} \cdot \frac{1}{9}\,cm = \frac{1}{90}\,cm$$

$$= 0.011\ldots cm$$

$$\text{err. rel} = \frac{0.011}{3} = \dots$$

(20) diam. sfera $= 2.3 \pm 0.2$ mm ; volume?

raggio $= 1.15 \pm 0.1$ mm

$$\text{vol} = \frac{4}{3}\pi (1.15 \pm 0.1)^3 \ \text{mm}^3$$

$$\cong \frac{4}{3}\pi (1.15^3 \pm 3 \cdot 1.15^2 \cdot 0.1) \ \text{mm}^3$$

$$\cong 6.37 \pm 1.66 \ \text{mm}^3$$

$\varepsilon_{rel.} \quad \frac{1.66}{6.37} = 0.26$

(21) Sciolgo 50 ± 2 g sale in 150 ± 2 g acqua
Concentrazione ?

$$C = \frac{S}{S+a} = \frac{50 \pm 2}{(50 \pm 2) + (150 \pm 2)} = \frac{50 \pm 2}{200 \pm 4}$$

$$= \frac{50}{200} \pm \varepsilon_{rel.\,ass.} = 25\% \pm \varepsilon_{rel.\,ass}$$

$$\varepsilon_{rel.} = \frac{2}{50} + \frac{4}{200} = \frac{3}{50}$$

$$\text{err. ass.} = \frac{3}{50} \cdot 25\,\% = 1.5\,\%$$

$$\Longrightarrow (25 \pm 1.5)\,\%$$

(22) soluz. 30 g sol. h_2o $c = 25 \pm 5\,\%$

acqua?

$$c = \frac{s}{s + a}$$

$$\frac{25 \pm 5}{100} = \frac{30}{30 + a}$$

$$\frac{30+a}{30} = \frac{20}{5\pm1}$$

$$a = \frac{600}{5\pm1} - 30$$

err. rel $\frac{1}{5}$

$$\frac{600}{5\pm1} = 120 \pm \frac{120}{5} = 120 \pm 24$$

$$\Rightarrow a = 90 \pm 24$$ err. ass.

em. rel. $\frac{24}{90} \cong 26\%$