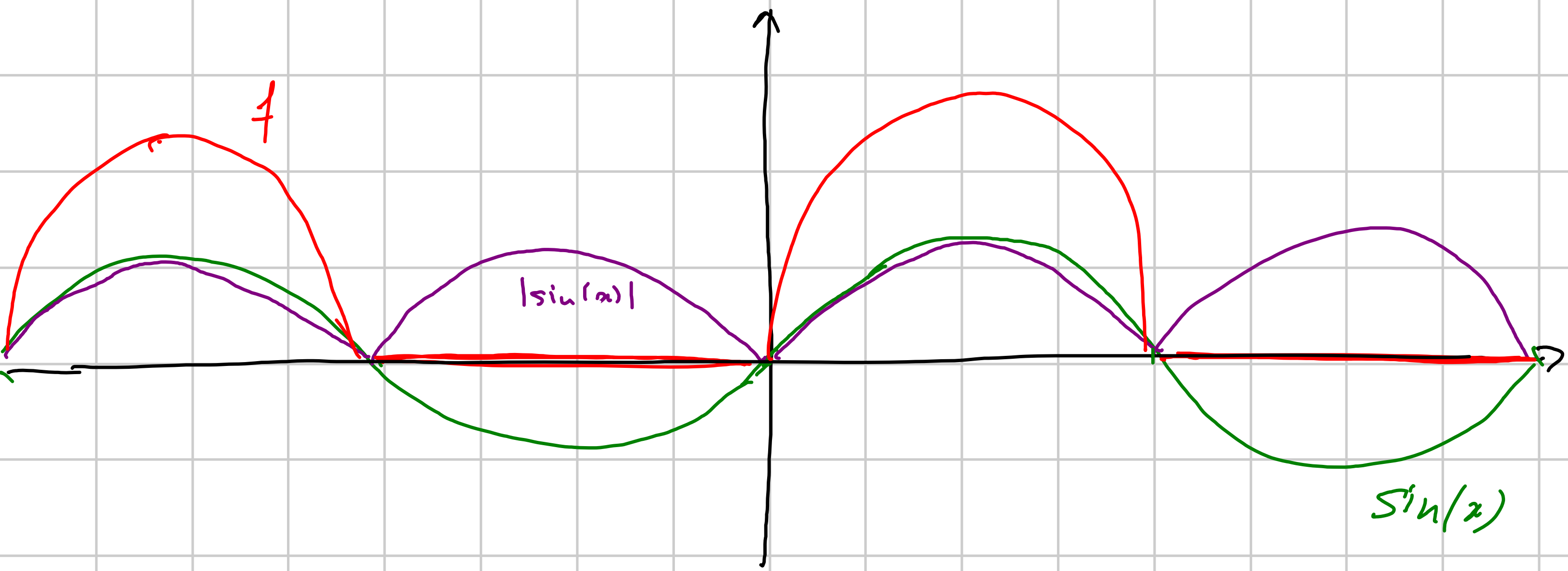
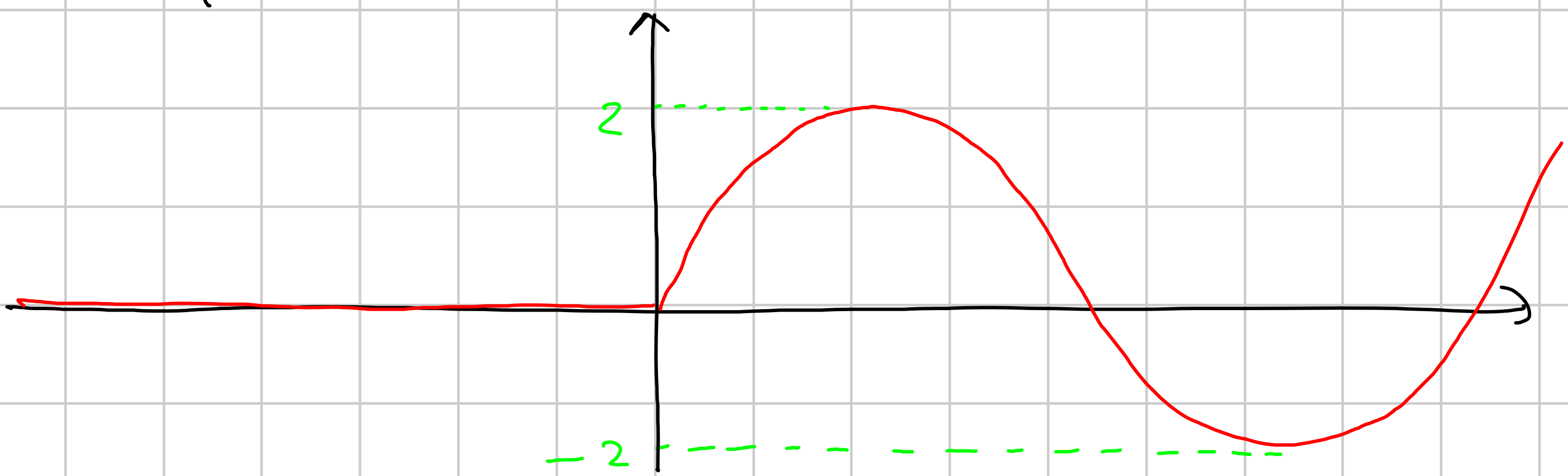
Matematica A (Bio) 9/3/16

$f(x) = \sin(x) + |\sin(x)|$



$$f(x) = \sin(x) + \sin(|x|) = \begin{cases} \sin(x) + \sin(x) & \text{se } x \geq 0 \\ \sin(x) + \sin(-x) & \text{se } x \leq 0 \end{cases}$$

$$= \begin{cases} 2\sin x & \text{se } x \geq 0 \\ 0 & \text{se } x \leq 0 \end{cases}$$



$$f(x) = \frac{1}{1-\sin x}$$

$$-1 \leq \sin(x) \leq 1$$

$$\Rightarrow \quad 0 \leq 1-\sin(x) \leq 2$$

Osservo che è periodica di periodo 2π

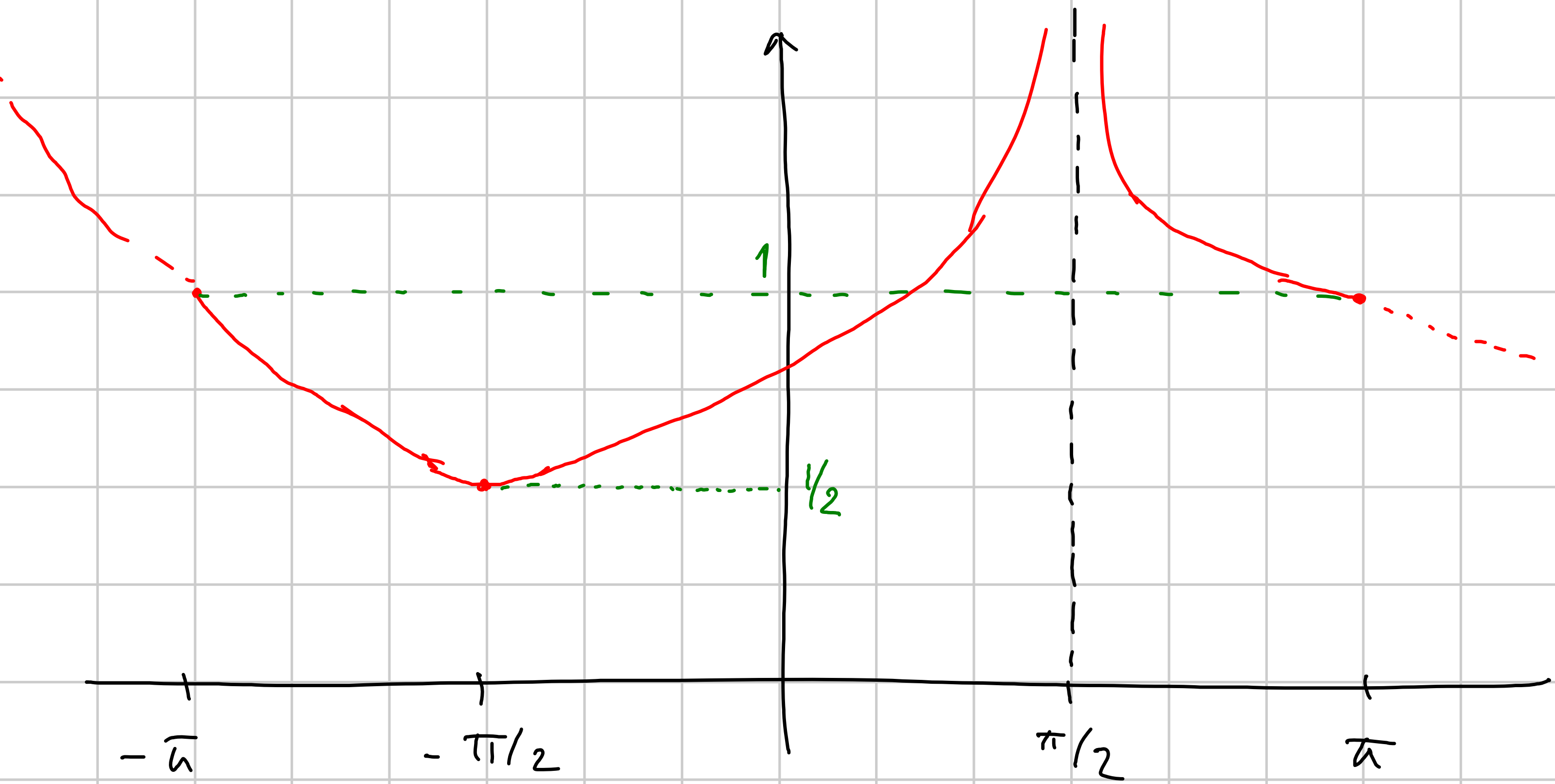
$\Rightarrow$ basta trovare il grafico su $[-\pi,\pi]$

vale l'$=$ precisamente per $x = \frac{\pi}{2}\ (+\,2k\pi)$

$\Rightarrow \quad x = \frac{\pi}{2}$ asintoto verticale e $\lim_{x\to\frac{\pi}{2}^{\pm}} f(x) = +\infty$

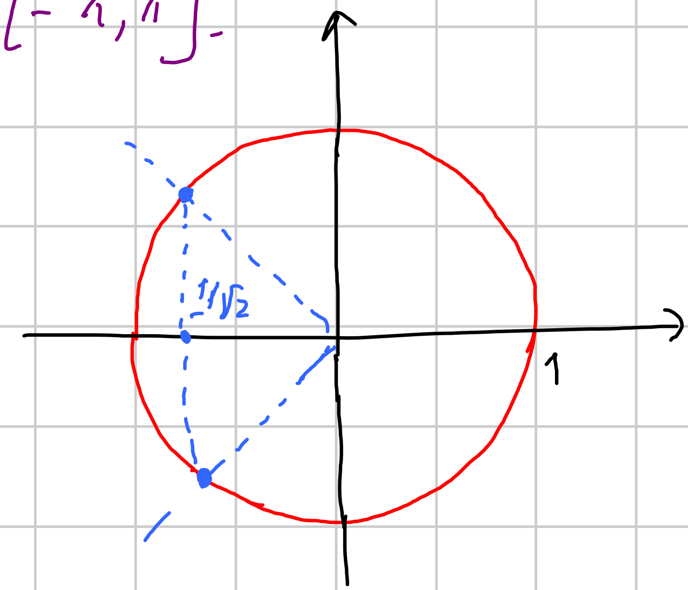
Osservo anche $f(x) \geq \frac{1}{2}$ e l'$=$ vale in $\frac{3}{2}\pi$:

Infine $f(\pm\pi) = 1$


1
1/2
- π
- π/2
π/2
π

$$f(x) = \frac{1}{1+\sqrt{2}\cos x}$$

Dispongo $[-\pi, \pi]$.



$$1+\sqrt{2}\cos x = 0$$

$$\cos x = -\frac{1}{\sqrt{2}}$$

$$\left(\sin x = \pm\sqrt{1-\frac{1}{2}} = \pm\frac{1}{\sqrt{2}}\right)$$

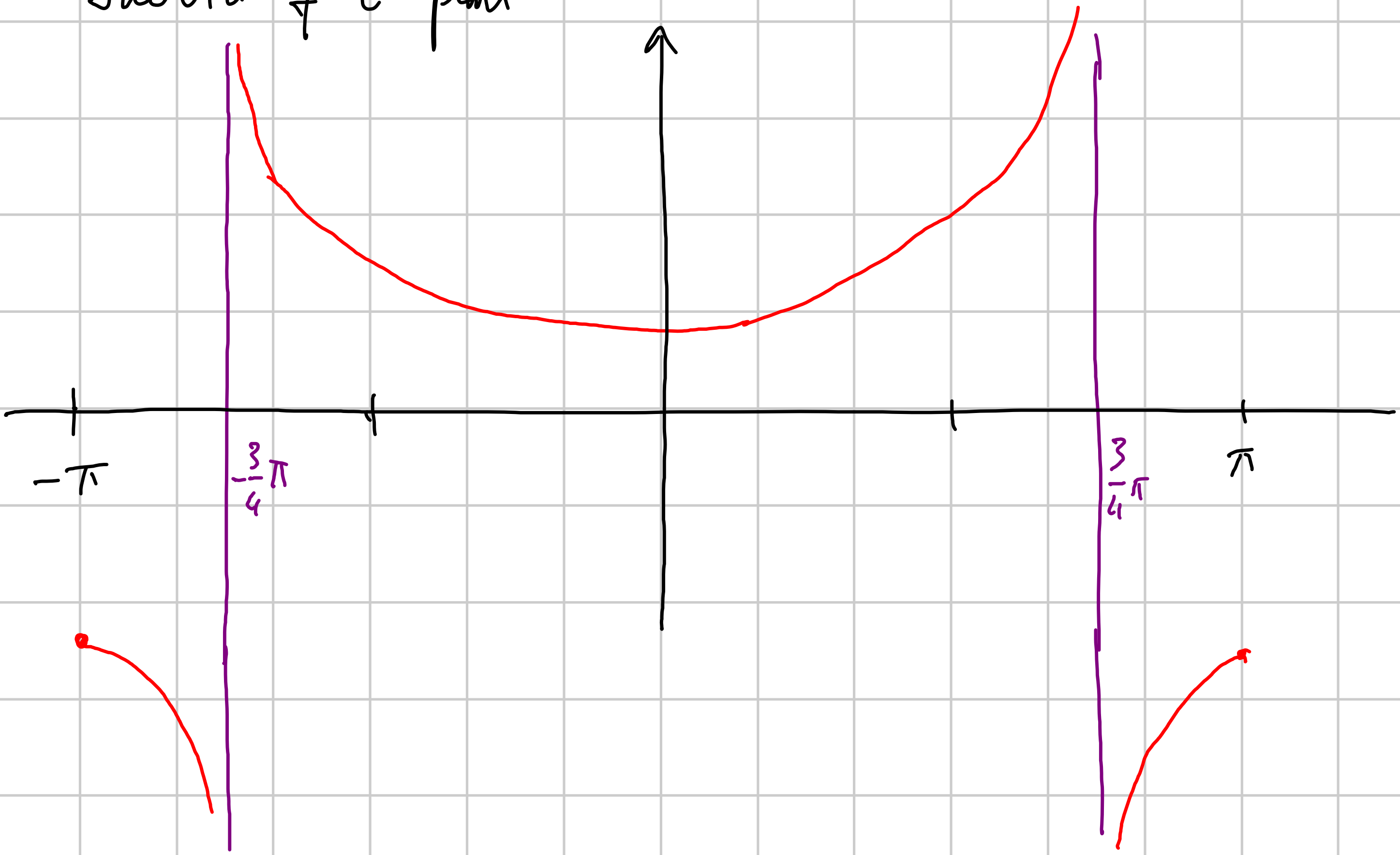
$$x = \pm\frac{3}{4}\pi$$

$1+\sqrt{2}\cos x$ è >0 per $\left(-\frac{3}{4}\pi, \frac{3}{4}\pi\right)$

<0 per $x \in \left[-\pi, -\frac{3}{4}\right) \cup \left(\frac{3}{4}\pi, \pi\right]$

Calcolo qualche valore: $f(\pm\pi) = \frac{1}{1-\sqrt{2}} = -1-\sqrt{2}$, $f(0) = \frac{1}{1+\sqrt{2}}$

Inoltre f è pari



$$f(x) = \frac{1}{\sqrt{3} - \tan(x)}$$

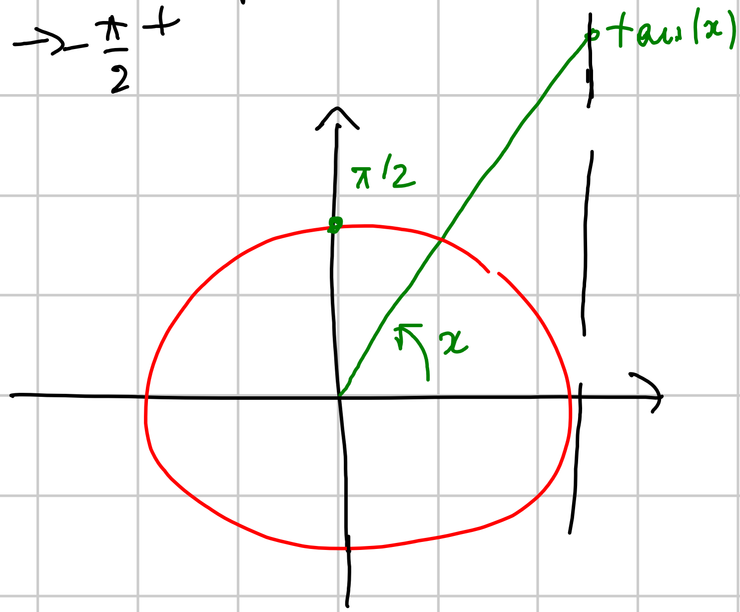
Periodica di periodo π

La disegno su $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Per $x = \pm \frac{\pi}{2}$, $\tan(x)$ non è def ma avrò

$$\lim_{x \to \pm \pi/2} f(x) = 0 \quad \text{anzi:}$$

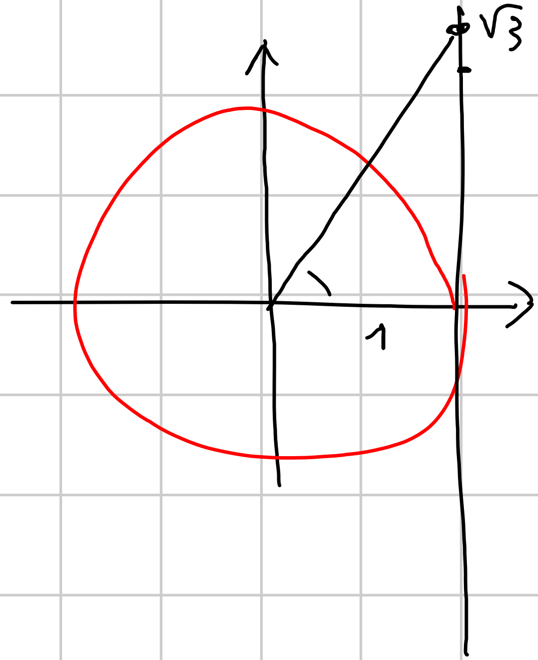
$$\lim_{x \to \frac{\pi}{2}^-} f(x) = 0^- \qquad \lim_{x \to -\frac{\pi}{2}^+} f(x) = 0^+$$



$\sqrt{3} - \tan(x) = 0$

$\tan(x) = \sqrt{3}$

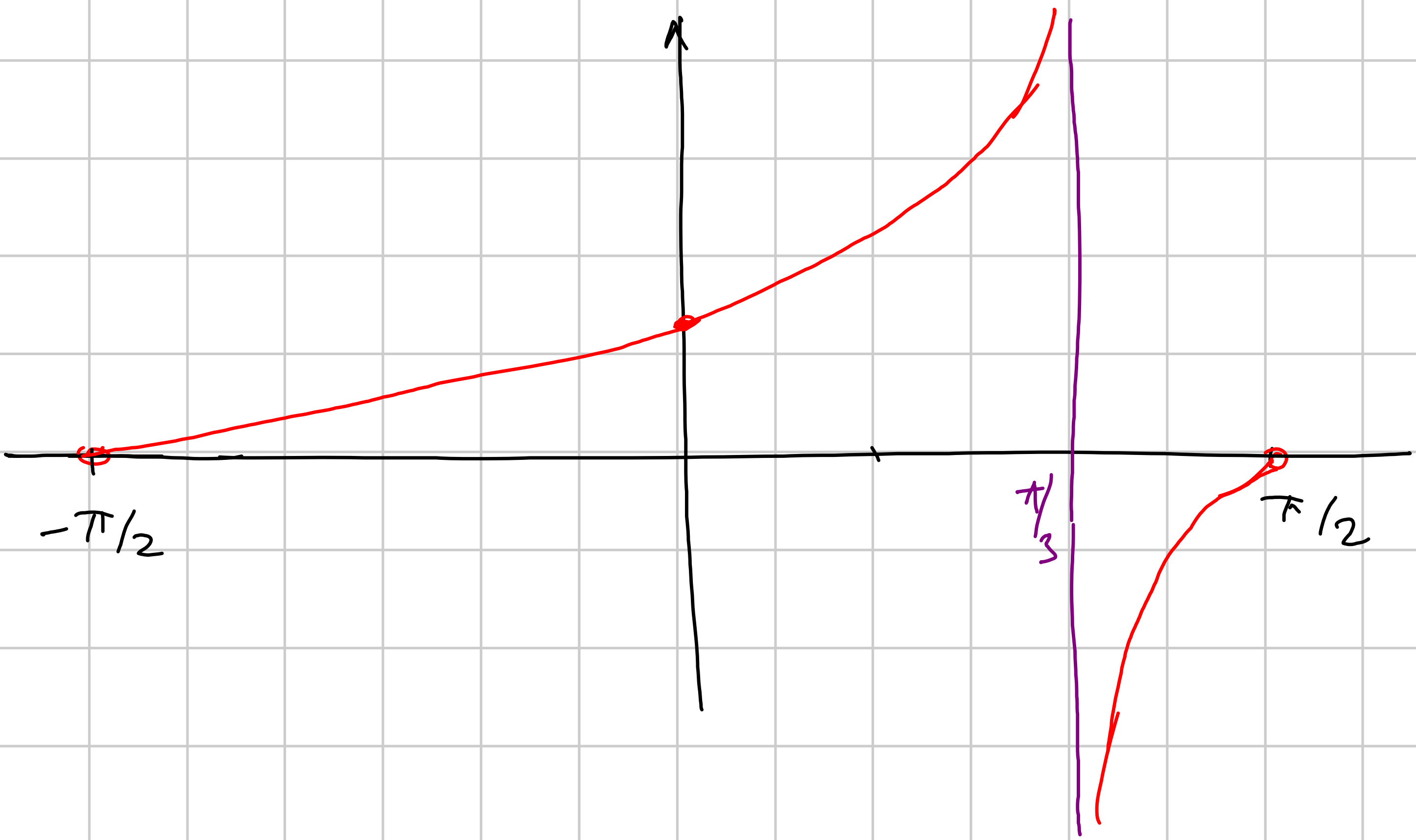
$x = \frac{\pi}{3}$



$\lim_{x \to \frac{\pi}{3}^-} f(x) = +\infty$

$\lim_{x \to \frac{\pi}{3}^+} f(x) = -\infty$

Noto $f(0) = \frac{1}{\sqrt{3}}$



-π/2
π/3
π/2

(2) (a) $\frac{\pi}{2} < \alpha < \pi$, $\sin(\alpha) = \frac{1}{\sqrt{5}}$

- $\operatorname{atan}(\tan(\pi+\alpha))$
  $= \operatorname{atan}(\tan(\alpha))$
  $= \alpha - \pi$

- $\cot(\pi-\alpha) =$
  $= \cot(-\alpha) = -\cot(\alpha)$
  $= -\frac{1}{\tan(\alpha)}$

Ricordo: atan la funzione inversa della tangente su $(-\pi/2, \pi/2)$; quindi

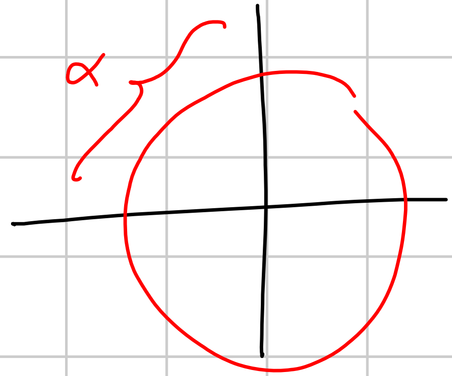
$$\operatorname{atan}(\tan(\theta)) = \theta + k\pi$$

dove k è l'unico intero tale che $\theta + k\pi \in (-\frac{\pi}{2}, \frac{\pi}{2})$

Es: $\operatorname{atan}(\tan(\frac{21}{20}\pi)) = \frac{\pi}{20}$

$\operatorname{atan}(\tan(\frac{37}{20}\pi)) = -\frac{3\pi}{20}$

So: $\sin\alpha = \frac{1}{\sqrt{5}}$



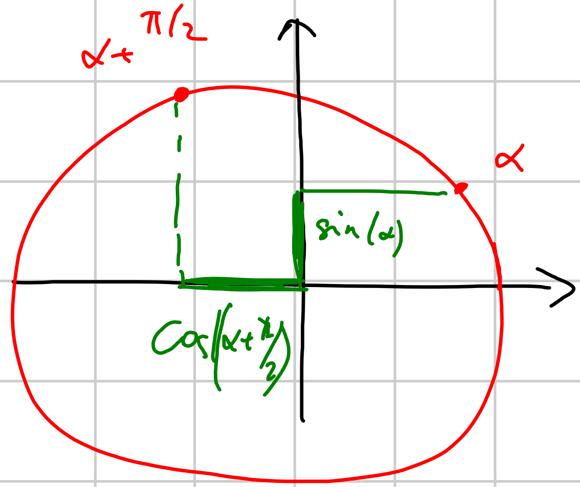
$$\cos\alpha = -\sqrt{1-\frac{1}{5}} = -\frac{2}{\sqrt{5}}$$

$$\Longrightarrow -\frac{1}{\tan\alpha} = 2$$

(d) $\alpha = a\cos\left(\frac{2}{\sqrt{13}}\right)$

• $\cos\left(\frac{3}{2}\pi-\alpha\right) = \cos\left(\alpha-\frac{3}{2}\pi\right) = \cos\left(\alpha+\frac{\pi}{2}\right) = -\sin\alpha$

$$\left(= \cos(\alpha)\cdot\underbrace{\cos\left(\frac{\pi}{2}\right)}_{0} - \sin(\alpha)\cdot\underbrace{\sin\left(\frac{\pi}{2}\right)}_{1}\right)$$



Ricordo: $\operatorname{acos}(y)$

se $|y| \leq 1$ è l'unico angolo in $[0, \pi]$ che ho coseno y; cioè acos è l'inversa di

$$\cos : [0, \pi] \to [-1, 1]$$

$$\sin\left(\operatorname{acos}\left(\frac{2}{\sqrt{13}}\right)\right) = +\sqrt{1 - \frac{4}{13}} = \frac{3}{\sqrt{13}}$$

- $\tan(\pi - \alpha) = -\tan(\alpha) = -\frac{3}{2}$

(3) (a) $\tan\left(3x-\frac{\pi}{4}\right)-\cot\left(2x-\frac{\pi}{3}\right)$

Dominio: $3x-\frac{\pi}{4}\neq\frac{\pi}{2}+k\pi$, $2x-\frac{\pi}{3}\neq k\pi$

$x\neq\frac{\pi}{4}+k\frac{\pi}{3}$ $\qquad x\neq\frac{\pi}{6}+k\frac{\pi}{2}$

in tali valori la f ha asintoti verticali.

Pari o dispari? No perché gli asintoti non sono disposti simmetricamente rispetto a 0:

a dx di 0: $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{5}{12}\pi$, $\frac{2}{3}\pi$, ...

a sx di 0: $-\frac{\pi}{12}$, - - - - -

(b) $\operatorname{acos}(x^2-3) + |\tan(x)|$

Dominio: $x \neq \frac{\pi}{2} + k\pi$ $\qquad -1 \leq x^2-3 \leq 1$

$$2 \leq x^2 \leq 4$$

$$-2 \leq x \leq -\sqrt{2} \qquad \sqrt{2} \leq x \leq 2$$

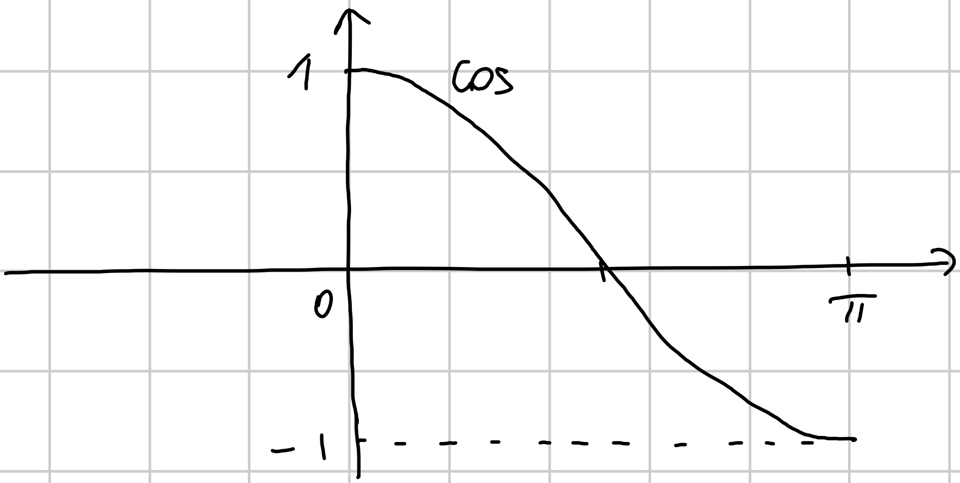
Dominio:

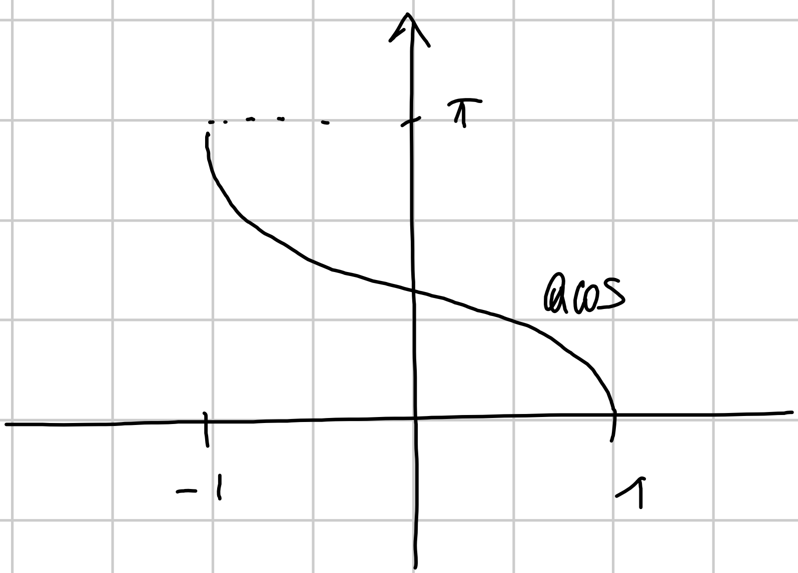
$$[-2, -\pi/2) \cup (-\pi/2, -\sqrt{2}] \cup [\sqrt{2}, \pi/2) \cup (\pi/2, 2]$$

Pari o dispari? Dominio simmetrico (ok).

$\operatorname{tg} x$ dispari $\Rightarrow$ $|\operatorname{tg}(x)|$ pari

x^2 pari $\Rightarrow$ x^2-3 pari





Non è pari

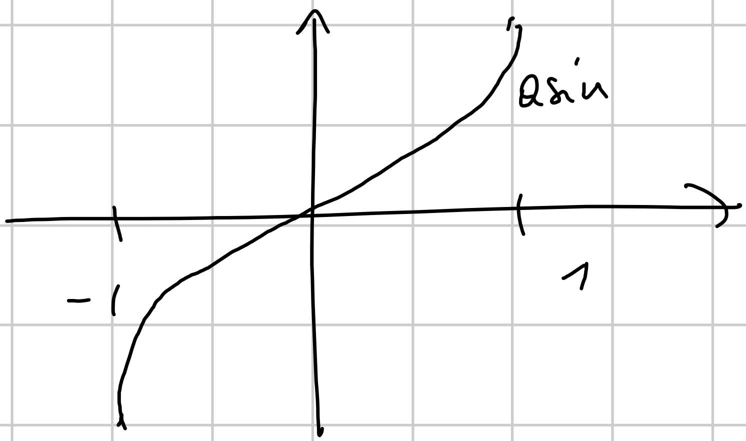
⟹ non pari

(né dispari)

(3)(e) $f(x) = \operatorname{asin}\left(\frac{1-x^2}{1+x^2}\right)$

$-1 < \frac{1-x^2}{1+x^2} < 1 \implies$ dominio $\mathbb{R}$

segno



è il segno di $1-x^2$: pos su $(-1,1)$

nullo in ± 1

neg on $(-\infty, -1), (1, +\infty)$

(4) (a) $\cos(\alpha) = -\frac{1}{4}$, $\sin(\beta) = -\frac{2}{3}$, $\pi < \alpha, \beta < \frac{3}{2}\pi$

$$\sin(2\alpha - \beta) = \underbrace{\sin(2\alpha)}_{2\sin(\alpha)\cos(\alpha)} \cdot \cos(\beta) - \underbrace{\cos(2\alpha)}_{\cos^2(\alpha) - \sin^2(\alpha)} \cdot \sin(\beta)$$

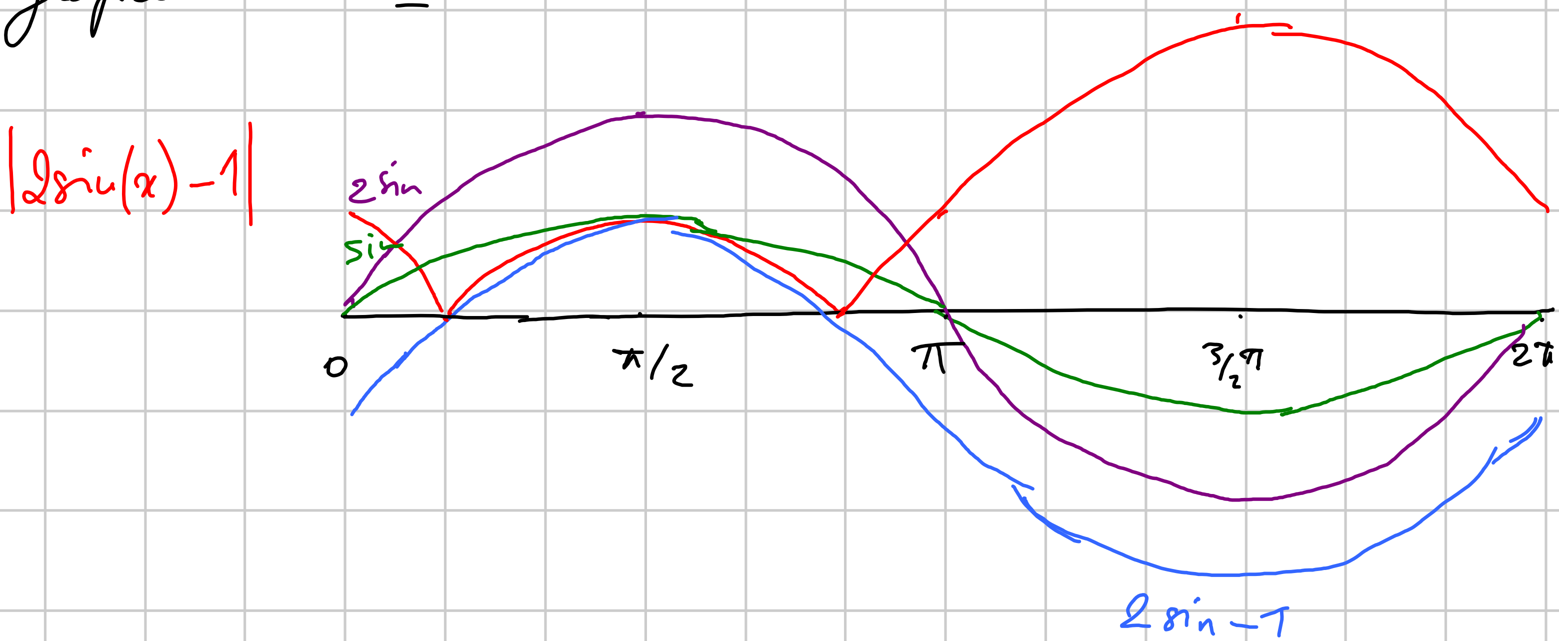
$$\sin(\alpha) = -\sqrt{1 - \frac{1}{16}} = -\frac{\sqrt{15}}{4}$$

$$\cos(\beta) = -\sqrt{1 - \frac{4}{9}} = -\frac{\sqrt{5}}{3}$$

⑥ (d) $|2\sin(x)-1| \geq |2\cos(x)-1|$

Lo risolviamo su $[0, 2\pi]$ (poi si estende per periodicità)

graficamente –



Per concludere: disegno $|2\cos x - 1|$

(traslato del prec. di $\pi/2$)

Ascisse punti intersezione:

$$|2\sin x - 1| = |2\cos x - 1|$$

$$2\sin x - 1 = 2\cos x - 1 \qquad \sin x = \cos x \qquad x = \frac{\pi}{4} + k\pi$$

$$2\sin x - 1 = -2\cos x + 1$$

$$\sin x + \cos x = 1$$

$$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$$

$$\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$x + \frac{\pi}{4} = \frac{\pi}{4} + 2k\pi \qquad x = 2k\pi$$

$$x + \frac{\pi}{4} = \frac{3}{4}\pi + 2k\pi \qquad x = \frac{\pi}{2} + 2k\pi$$

Intersezioni dei grafici in $x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{5}{4}\pi, 2\pi$