Matematica A - 3/11/2015

(1) <u>con</u>: $\left(2/5\right)^2 + \left(2/5\right)^2 + \left(1/5\right)^2 = \dots$

<u>senza</u>: $2/5 \cdot 1/4 + 2/5 \cdot 1/4 = \dots$

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n eventi successivi con esito pos/neg

prob: p $\quad$ $1-p$

Prob. di k esiti positivi : $\binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}$

2 $= \binom{10}{2} \cdot \left(\frac{1}{4}\right)^2 \cdot \left(\frac{3}{4}\right)^8 = \dots$

$= 1 - \left(\left(\frac{3}{4}\right)^{10} + 10 \cdot \left(\frac{1}{4}\right) \cdot \left(\frac{3}{4}\right)^9 \right) = \dots$

o anche $\sum_{k=2}^{10} \binom{10}{k} \cdot \left(\frac{1}{4}\right)^k \cdot \left(\frac{3}{4}\right)^{10-k}$

$$\left(\frac{3}{4}\right)^{10} + 10\cdot\left(\frac{1}{4}\right)\cdot\left(\frac{3}{4}\right)^{9} + \binom{10}{2}\cdot\left(\frac{1}{4}\right)^{2}\cdot\left(\frac{3}{4}\right)^{8}$$

$$= \sum_{k=0}^{2} \binom{10}{k}\cdot\left(\frac{1}{4}\right)^{k}\cdot\left(\frac{3}{4}\right)^{10-k}$$

3

$$\binom{10}{6}\left(\frac{1}{3}\right)^{6}\cdot\left(\frac{2}{3}\right)^{4} + \binom{10}{7}\left(\frac{1}{3}\right)^{7}\left(\frac{2}{3}\right)^{3}$$

$$+ \ldots$$

$$= \sum_{k=6}^{10} \binom{10}{k}\left(\frac{1}{3}\right)^{k}\cdot\left(\frac{2}{3}\right)^{10-k} = \ldots$$

4. $$1 - \left((0.96)^{75} + 75 \cdot 0.04 \cdot (0.96)^{74} \right) = \dots$$

——— o ———

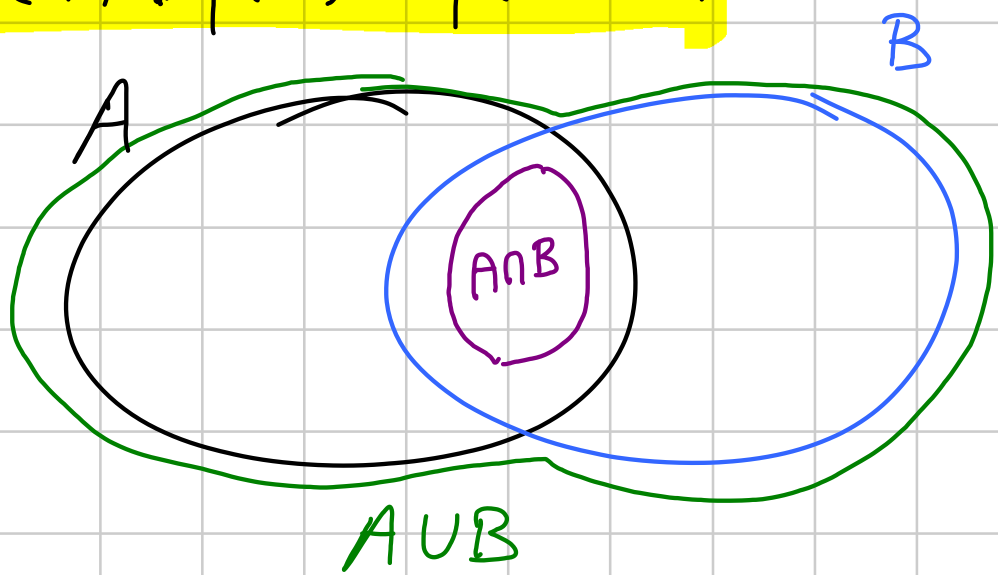
$A, B \subset \Omega$ eventi ; $A \cap B$ = entrambi
$A \cup B$ = almeno uno

Formule essenziali :

- $p(\text{non}(A)) = 1 - p(A)$
- $p(A) = p(A \cap B) + p(A \cap (\text{non}(B)))$
- $p(A|B)$ = "A condizionato da B"
  "prob. che A si realizzi sapendo che B si è realizzato"

$$= p(A\cap B) / p(B)$$

- $p(A\cup B) = p(A) + p(B) - p(A\cap B)$



- A, B indipendenti se $p(A\cap B) = p(A)\cdot p(B)$

$\left(\Longrightarrow p(A|B) = p(A),\ p(B|A) = p(B) \right)$

- A, B incompatibili se $p(A \cap B) = 0$.

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(5) M = sono monocoriali

S = hanno stesso sesso

Dati:
$$p(M) = p$$
$$p(S|M) = 1$$
$$p(S|\text{non}(M)) = 1/2$$

$- p(S)$ $\quad = \cancel{p(S|M) + p(S|(\text{non}(M)))}$

No

ignore la prob. di M

$$p(S) = p(S \cap M) + p(S \cap (\text{non } M))$$

$$p(S \cap M) = p(S|M) \cdot p(M)$$

$$p(S \cap (\text{non } M)) = p(S|\text{non}(M)) \cdot p(\text{non}(M))$$

$$1 \cdot p + \frac{1}{2} \cdot (1-p) = \frac{1+p}{2}$$

- $p(M|S) = \dfrac{p(M \cap S)}{p(S)} = \dfrac{p}{\frac{1+p}{2}} = \dfrac{2p}{1+p}$

⑥ V = veloce (laurea entro 5 anni)

G = 100 (alla maturità)

$p(V) = 0.15$ $\quad p(G|V) = 0.6$

$p(G|\text{non } V) = 0.1$

$p(V|G) = ?$

$$p(V \mid G) = \frac{p(V \cap G)}{p(G)} = \ldots$$

So che $\underset{0.6}{p(G \mid V)} = \dfrac{p(G \cap V)}{\underset{0.15}{p(V)}}$ ← uno dei due che mi serve

$\Longrightarrow p(G \cap V) = 0.6 \times 0.15$

$$p(G) = \underset{0.6 \times 0.15}{\underbrace{p(G \cap V)}_{\parallel}} + p(G \cap (\text{non}(V)))$$

So $p(G / \text{non } V) = \dfrac{p(G \cap (\text{non}(V)))}{p(\text{non } V)}$

0.1

0.85

$$\Longrightarrow p(C_1 \cap \text{mo}(V)) = 0.1 \times 0.85$$

$$\Longrightarrow p(\bar{V} \mid C_1) = \frac{0.6 \times 0.15}{0.6 \times 0.15 + 0.1 \times 0.85} = \dots$$

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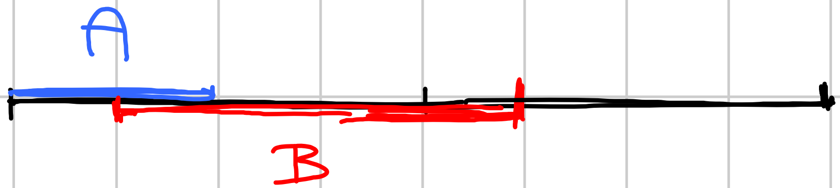
8 $p(A) = \frac{1}{4}$ $p(B \mid A) = \frac{1}{2}$ $p(A \mid B) = \frac{1}{4}$

$$\underbrace{p(B|A)}_{1/2} = \frac{p(B\cap A)}{\underbrace{p(A)}_{1/4}} \Rightarrow p(B\cap A) = \frac{1}{8}$$

$$\underbrace{p(A|B)}_{1/4} = \frac{\overbrace{p(A\cap B)}^{1/8}}{p(B)} \Rightarrow p(B) = \frac{1}{2}$$



$$p(A\cup B) = 5/8 = \underbrace{p(A)}_{1/4} + \underbrace{p(B)}_{1/2} - \underbrace{p(A\cap B)}_{1/8}$$

incompatibili: <u>NO</u>

$A \subseteq B$: <u>NO</u>

$$p(\text{non}(A) \mid \text{non}(B)) = \frac{p(\text{non}(A) \cap \text{non}(B))}{p(\text{non}(B))} = \frac{3/8}{1/2} = \frac{3}{4}$$

$$p(A|B) + p(A|\text{non } B) = \frac{1}{4} + \frac{p(A \cap \text{non } B)}{p(\text{non } B)} = \frac{1}{4} + \frac{1/8}{1/2} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

9 $\quad p(A) = 4/7 \qquad p(B) = 5/7$

$$\Rightarrow p(A \cap B) = p(A) + p(B) - p(A \cup B)$$

$$\geq \frac{4}{7} + \frac{5}{7} - 1 = \frac{2}{7}$$

$\Rightarrow$ non incompatibili

- $p(A \cup B)$ se A, B indipendenti.

indipendenti: $p(A \cap B) = p(A) \cdot p(B) = \frac{4}{7} \cdot \frac{5}{7} = \frac{20}{49}$

$$\Rightarrow p(A \cup B) = p(A) + p(B) - p(A \cap B)$$

$$= \frac{4}{7} + \frac{5}{7} - \frac{20}{49} = \frac{28+35-20}{49} = \frac{43}{49} -$$

(10) $p(A) = \frac{11}{36}$ $\quad p(B) = \frac{19}{36}$ $\quad p(A \cup B) = \frac{2}{3}$

$$p(A \cap B) = \frac{11}{36} + \frac{19}{36} - \frac{2}{3} = \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$$

$$p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{1/6}{19/36} = \frac{6}{19}$$

(11)
$$\begin{aligned} p(A \cup B) &= p(A) + p(B) - p(A \cap B) \\ &\le \frac{1}{4} + \frac{2}{7} - 0 \\ &= \frac{15}{28} < \frac{9}{14} = \frac{18}{28} \end{aligned}$$

No

12 $p(A) = 0.4$ $p(A|B) = 0.3$ $p(B) = 0.1$

$$\left(\underbrace{p(A|B)}_{0.3} = \frac{p(A \cap B)}{\underbrace{p(B)}_{0.1}} \Rightarrow p(A \cap B) = 0.03 \right)$$

- $p(\text{non}(B)) = 0.9$

- $p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$$= 0.4 + 0.1 - 0.03 = 0.47$$

7 $p(R) = 0.15$

$p(M|R) = 0.1$

$p(M|\text{non } R) = 0.005$

$$p(M) = p(M \cap R) + p(M \cap \text{non } R)$$

$$= p(M|R) \cdot p(R) + p(M|\text{non } R) \cdot p(\text{non } R)$$

$$= 0.1 \times 0.15 + 0.005 \times 0.85 = 0.01925$$

$p(R|M) = \ldots$ $\quad$ $p(R|\text{non } M) =$