Matematica A — 2/3/16 — Parte II

(6a) $2^{2\log_3(7-2x)-7} = \frac{1}{8}$

$2\log_3(7-2x) - 7 = -3$

$2\log_3(7-2x) = 4$

$\log_3(7-2x) = 2$

$$3^{\log_3(7-2x)} = 3^2$$

$$7 - 2x = 9$$

$$x = -1$$

(b) $\operatorname{Log}(2^x - 3) + \operatorname{Log}(2^x - 6) = 1$

"

$$\operatorname{Log}\left((2^x - 3)(2^x - 6)\right) = 1$$

$\log_{10}(\dots)$

$$10^{} = 10^1$$

$$(2^x - 3)(2^x - 6) = 10$$

"

$2^x - 3 > 0$

$2^x - 6 > 0$

cioè

$x > \log_2 6$

2. ...

$$(2^x)^2 - 9 \cdot 2^x + 18 = 10$$

$$(2^x)^2 - 9 \cdot 2^x + 8 = 0$$

$$2^x = \frac{9 \pm \sqrt{81-32}}{2} = \frac{9 \pm 7}{2} = \begin{cases} 8 \\ 1 \end{cases}$$

$x = 3$

~~$x = 0$~~

(6c) $\begin{cases} \log_2\left((x+2)^2\right) = \left(\log_2(x+2)\right)^2 \\ 3^{5^{x^2-3}-1} = 81 \end{cases}$

I: $2\cdot\log_2(x+2) = \left(\log_2(x+2)\right)^2$

$\left(t = \log_2(x+2) \quad : \quad 2t = t^2 \quad t=0,\ t=2\right)$

$\log_2(x+2) = 0 \qquad x+2 = 2^0 \qquad x = -1$

$\log_2(x+2) = 2 \qquad x+2 = 2^2 \qquad \boxed{x = 2}$

II: $5^{x^2-3} - 1 = 4 \qquad 5^{x^2-3} = 5 \qquad x^2 - 3 = 1$

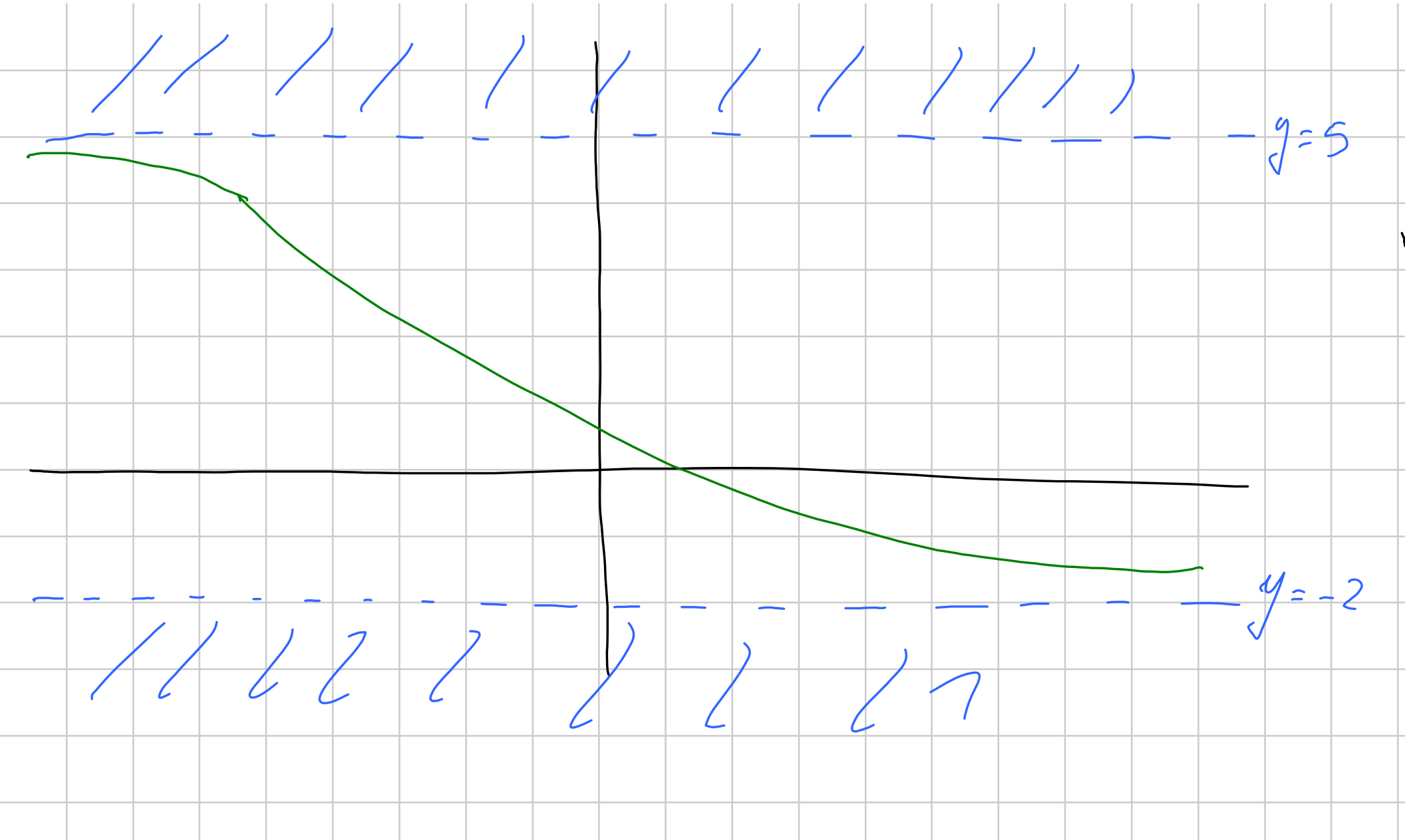
$x^2 = 4$ $\quad$ $x = 2$ $\quad$ $x = -2$

C1 monotone decr. immagine $(-2, 5)$

Vogliamo $\lim_{x \to -\infty} f(x) = 5$ $\quad$ $\lim_{x \to +\infty} f(x) = -2$

- $$f(x) = a + \frac{b}{1+e^{\pm x}}$$

Oss: $\frac{1}{1+e^{x}}$ decrescente tra 1 e 0 ; $\frac{1}{1+e^{-x}}$ crescente tra 0 e 1


y=5
y=-2

⊕ $a + \frac{b}{1+e^{x}}$

$-\infty$: $a+b=5$

$+\infty$: $a=-2$

$f(x) = -2 + \frac{7}{1+e^{x}}$

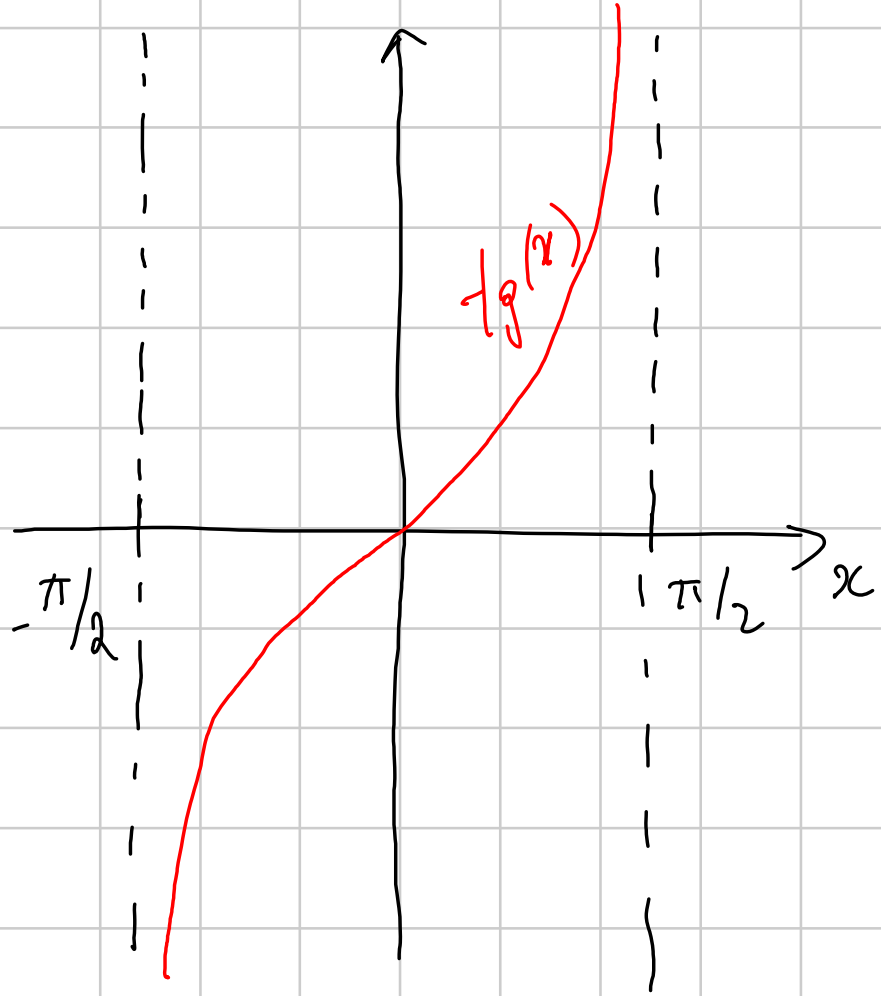
⊖ $a + \frac{b}{1+e^{-x}}$

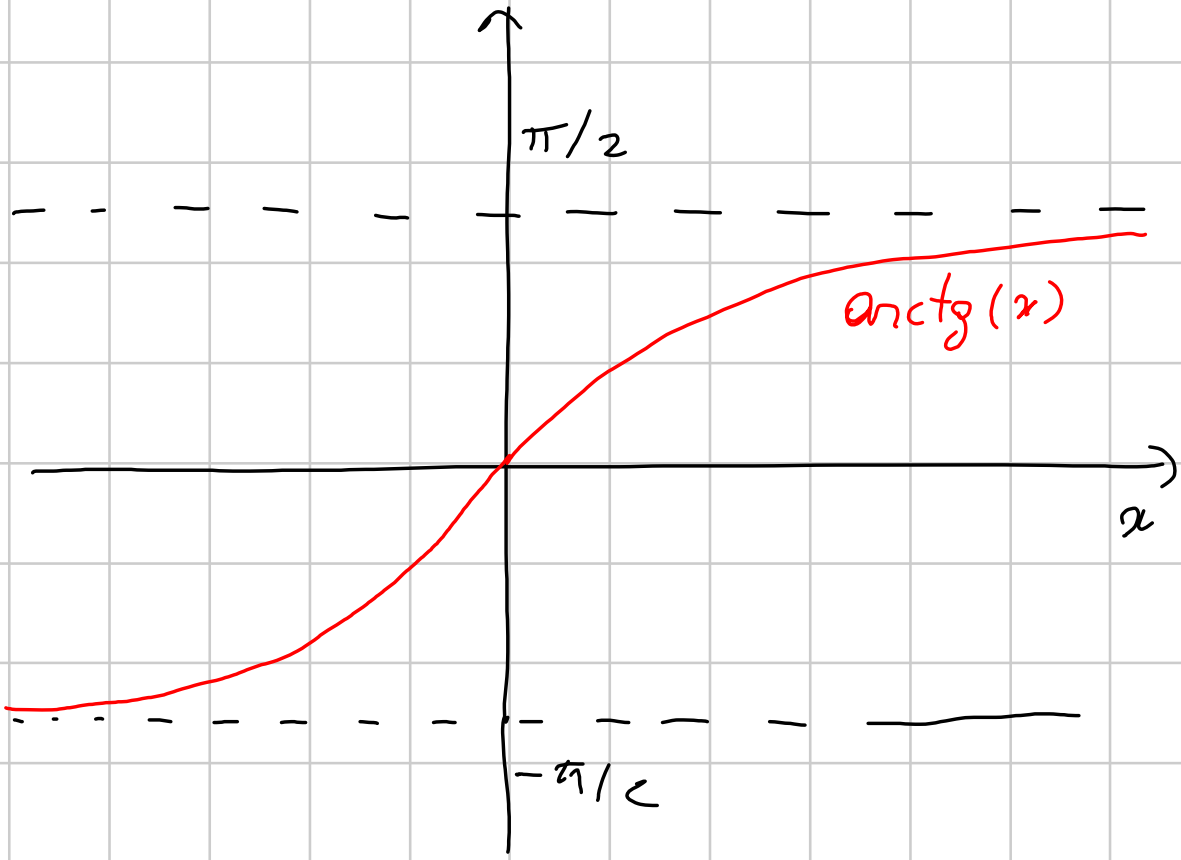
$-\infty$: $a=5$

$+\infty$: $a+b=-2$

$f(x) = 5 - \frac{7}{1+e^{-x}}$

Oppure : Ricordo :





$$f(x) = a + b \cdot \operatorname{arctg} x$$

$$-\infty: \quad a - \frac{\pi}{2} \cdot b = 5$$

$$+\infty: \quad a + \frac{\pi}{2} \cdot b = -2$$

$$\begin{cases} a = \frac{3}{2} \\ b = -\frac{7}{\pi} \end{cases}$$

C2 (a): No: dominio $(-\infty, -1) \cup (-1, 2)$

(b) No: dominio $(2, 3) \cup (3, +\infty)$

(d) No: dominio $(2, 3) \cup (3, +\infty)$

$\Rightarrow$ (c)

C3 (a) No: parabola, niente cuspide

(b) No: ha cuspide in $x = 2$

(c+d): cuspide in -2 — ok valore in -2

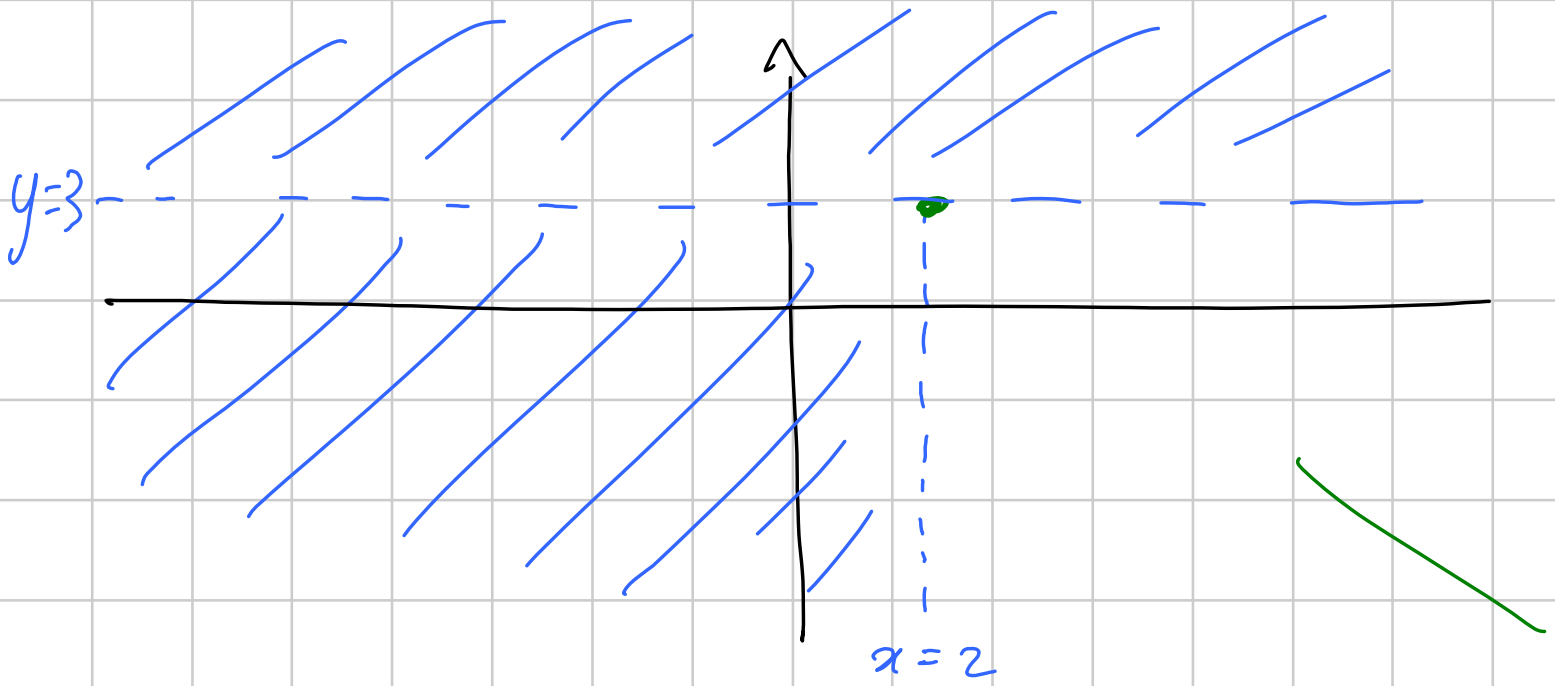
(c) 1 No

(d) 3 Per esclusione: sì

C4 f su $[2,+\infty)$ decrescente

imagine $(-\infty,3]$

Voglio: $f(2)=3$ $\lim_{x\to+\infty} f(x)=-\infty$ decrescente



- $f(x) = 3 + m(x-2)$ $\quad m < 0$
- $f(x) = a + b \cdot e^{x-2}$ $\quad \begin{cases} a+b=3 \\ b<0 \end{cases}$

es $\quad f(x) = 10 - 7 \cdot e^{x-2}$

- $f(x) = a + b \cdot \log(x-1)$ $\quad \begin{cases} a=3 \\ b<0 \end{cases}$