Alg. Lin. 28/10/15

Dire se $v_1, \dots, v_m$ sono lin. indip.

$\boxed{4.2.3}$ (c) $\underset{v_1}{\begin{pmatrix} 1 \\ 1 \\ 0 \\ 3 \end{pmatrix}}$ $\underset{v_2}{\begin{pmatrix} 3 \\ -2 \\ 1 \\ 0 \end{pmatrix}}$ $\underset{v_3}{\begin{pmatrix} 13 \\ -4 \\ 3 \\ -2 \end{pmatrix}}$

v_1, v_2 lin. indip. Ricordo

$v_3 \in \mathrm{Span}(v_1, v_2) \iff v_1, v_2, v_3$ lin. dip.

$v_3 \overset{?}{=} \alpha v_1 + \beta v_2$

$$\begin{cases} 13 = \alpha + 3\beta \\ -4 = \alpha - 2\beta \\ 3 = \beta \quad \checkmark \\ -2 = 3\alpha \quad \checkmark \end{cases} \qquad \begin{cases} \alpha = -2/3 \\ \beta = 3 \\ -\frac{2}{3} + 9 = 13 \quad \underline{\text{No}} \end{cases}$$

$\Longrightarrow$ lin. indip.

(d) $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ -1 \\ 0 \end{pmatrix} \begin{pmatrix} 5 \\ 0 \\ 0 \\ 3 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -2 \\ 1 \end{pmatrix}$

$v_1 \neq 0$; v_2 non mult. di v_1; $v_3 \notin \text{Span}(v_1 v_2)$ –

$v_4 \overset{?}{=} \alpha v_1 + \beta v_2 + \gamma v_3$

$$\begin{cases} \alpha + 5\gamma = 0 \\ \alpha + 2\beta = 0 \\ -\beta = -2 \ \checkmark \\ 3\gamma = 1 \ \checkmark \end{cases} \qquad \begin{cases} \beta = 2 \\ \gamma = 1/3 \\ \alpha = -4 \\ -4 + 5/3 = 0 \end{cases} \quad \underline{\text{No}}$$

$$\Longrightarrow v_1, v_2, v_3, v_4 \text{ lin. indip.}$$

(e) $\begin{pmatrix} 4 \\ 1 \\ 0 \\ -1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 2 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 1 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -1 \\ 5 \end{pmatrix}$

$$\begin{cases} 4\alpha + 2\beta = 0 \ \checkmark \\ \alpha - \beta - \gamma = 0 \ \checkmark \\ 2\beta + \gamma = -1 \ \checkmark \\ -\alpha + 2\gamma = 5 \end{cases} \quad \begin{cases} \beta = -2\alpha \\ \gamma = 3\alpha \\ \underline{-4\alpha + 3\alpha = -1} \end{cases} \quad \begin{cases} \alpha = 1 \\ \beta = -2 \\ \gamma = 3 \\ -1 + 6 = 5 \end{cases} \quad \text{Sì}$$

$\Rightarrow v_1, v_2, v_3, v_4$ lin. dip.

$\boxed{4.2.4}$ (c) $\underset{v_1}{1-t^2+t^5}$ $\underset{v_2}{4+2t^2-3t^5}$ $\underset{v_3}{-1-11t^2+13t^5}$

$v_1 \neq 0$, v_2 non mult. di v_1.

$$-1-11t^2+13t^5 \overset{?}{=} \alpha(1-t^2+t^5)+\beta(4+2t^2-3t^5)$$

$$\begin{cases} \alpha+4\beta=-1 \\ -\alpha+2\beta=-11 \\ \alpha-3\beta=13 \end{cases} \qquad \begin{cases} \beta=-2 \\ \alpha=7 \\ 7+6=13 \end{cases} \quad \text{SI}$$

$\Rightarrow$ lin. dip.

<u>Oss</u>: E' come trattare $\begin{pmatrix}1\\-1\\1\end{pmatrix}\begin{pmatrix}4\\2\\-3\end{pmatrix}\begin{pmatrix}-1\\-11\\13\end{pmatrix} \in \mathbb{R}^3$

poiché $\mathrm{Span}(1, t^2, t^5) \longleftrightarrow \mathbb{R}^3$ con rispetto operaz.

$$a + bt^2 + ct^5 \longleftrightarrow \begin{pmatrix}a\\b\\c\end{pmatrix}$$

Esibire base di V:

$$\bar{V} = \{ x \in \mathbb{R}^3 : 7x_1 + 5x_2 - 4x_3 = 0 \}$$

$$\left(\bar{V} = \{ x \in \mathbb{R}^3 : 0 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 = 0 \} = \mathbb{R}^3 \right)$$

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}$$

Affermo che sono base:

- stanno in $\bar{V}$ ✓
- lin. indip. ✓

· generano (garantito assumendo dim = 2);
lo dimostro.

generico vett. di V è $\begin{pmatrix} x_1 \\ x_2 \\ (7x_1+5x_2)/4 \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}$

$$\begin{cases} \alpha = x_1 \\ \beta = (x_2 - x_1)/4 \\ \dfrac{7x_1+5x_2}{4} = 3x_1 + \dfrac{5x_2-5x_1}{4} \end{cases}$$

✓

$$V = \left\{ x \in \mathbb{R}^3 : \begin{array}{l} 3x_1 + 2x_2 - 5x_3 = 0 \\ -4x_1 + x_2 + 7x_3 = 0 \end{array} \right\}$$

(dim attesa = 1)

$$x \in V \iff \begin{cases} \text{—} \\ \text{—} \end{cases} \iff \begin{cases} x_1 + 19x_2 = 0 \\ 5x_3 = 3x_1 + 2x_2 \end{cases}$$

$$\iff \begin{cases} x_1 = -19x_2 \\ x_3 = -11x_2 \end{cases} \iff x = x_2 \cdot \begin{pmatrix} 19 \\ -1 \\ 11 \end{pmatrix}$$

Dunque $V = \mathrm{Span}\begin{pmatrix} 19 \\ -1 \\ 11 \end{pmatrix}$. Lui è base.

$$V = \left\{ x \in \mathbb{R}^3 : \begin{array}{l} -4x_1 + 2x_2 + 5x_3 = 0 \\ 3x_1 + 7x_2 - 2x_3 = 0 \\ x_1 + 27x_2 + 4x_3 = 0 \end{array} \right\}$$

$$x \in V \iff \begin{cases} x_1 = -27x_2 - 4x_3 \\ 110x_2 + 21x_3 = 0 \\ -74x_2 - 14x_3 = 0 \end{cases} \iff x = 0$$

$$V = \left\{ x \in \mathbb{R}^4 : 5x_1 - 8x_2 + 2x_3 - 3x_4 = 0 \right\}$$

$$\begin{pmatrix} 0 \\ 0 \\ 3 \\ 2 \end{pmatrix} \quad \begin{pmatrix} 8 \\ 5 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \\ 4 \\ 0 \end{pmatrix} \quad \text{lin. indip.}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ (5x_1 - 8x_2 + 2x_3)/3 \end{pmatrix} \overset{?}{=} \alpha \begin{pmatrix} 0 \\ 0 \\ 3 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} 8 \\ 5 \\ 0 \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} 0 \\ 1 \\ 4 \\ 0 \end{pmatrix}$$

$$\beta = x_1/8$$

$$\gamma = x_2 - 5x_1/8$$

$$\alpha = (x_3 - 4x_2 - 5x_1/2)/3$$

$$\frac{5x_1 - 8x_2 + 2x_3}{3} = \frac{2x_3 - 8x_2 + 5x_1}{3}$$

$$V = \{ x \in \mathbb{R}^4 : 9x_1 + 4x_3 - 5x_4 = 0 \}$$

$$\underset{v_1}{\begin{pmatrix} 0 \\ 0 \\ 5 \\ 4 \end{pmatrix}} \quad \underset{v_2}{\begin{pmatrix} 4 \\ 0 \\ -9 \\ 0 \end{pmatrix}} \quad \underset{v_3}{\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}}$$

$$\underset{v_3'}{\begin{pmatrix} 5 \\ 0 \\ 0 \\ 9 \end{pmatrix}}$$

v_3' No : $\in \operatorname{Span}(v_1, v_2)$

$$V = \left\{ x \in \mathbb{R}^4 : \begin{array}{l} 5x_1 + 3x_2 + 7x_3 - 2x_4 = 0 \\ 8x_1 - 7x_2 + 4x_3 - 5x_4 = 0 \end{array} \right\}$$

$$(\dim = 2)$$

$$x \in V \iff \begin{cases} 9x_1 + 29x_2 + 27x_3 = 0 \\ 59x_1 + 61x_3 - 29x_4 = 0 \end{cases}$$

$$\iff \begin{cases} x_2 = -\frac{1}{29}(9x_1 + 27x_3) \\ x_4 = \frac{1}{29}(59x_1 + 61x_3) \end{cases}$$

$$\Leftrightarrow x = x_1 \cdot \begin{pmatrix} 1 \\ -9/29 \\ 0 \\ 59/29 \end{pmatrix} + x_3 \cdot \begin{pmatrix} 0 \\ -27/29 \\ 1 \\ 61/29 \end{pmatrix}$$

base

Oppure: $\begin{pmatrix} 29 \\ -9 \\ 0 \\ 59 \end{pmatrix} \begin{pmatrix} 0 \\ -27 \\ 29 \\ 61 \end{pmatrix}$.

Morale: trovati i parametri liberi che servono a descrivere un pto di U, ottengo e loro a turno i valori $0, \dots, 0, 1, 0, \dots, 0$ (come nella base canonica di $\mathbb{R}^n$) ottenendo una base di V.

$$V = \left\{ x \in \mathbb{R}^4 : \begin{array}{l} 2x_1 + 4x_2 + 7x_3 - 9x_4 = 0 \\ -5x_1 + 6x_2 + 3x_3 + 2x_4 = 0 \\ 9x_1 + 2x_2 + 11x_3 - 20x_4 = 0 \end{array} \right\}$$

$$\mathrm{III} = 2 \cdot \mathrm{I} - \mathrm{II}$$

$$\Longrightarrow \dim V = 2$$

$$\left\{ x\in\mathbb{R}^n : \sum_{j=1}^{n} (-1)^{j+1} x_j = 0 \right\} = V$$

$$x \in V \iff x_1 - x_2 + x_3 - x_4 \dots (-1)^{n+1} x_n = 0$$

$$\iff x_1 = x_2 - x_3 + x_4 - x_5 \dots (-1)^{n} x_n = 0$$

par. liberi

$\Rightarrow$ base: $\begin{pmatrix} 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ \vdots \end{pmatrix}, \dots, \begin{pmatrix} (-1)^n \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$

$$e_1+e_2,\ -e_1+e_2,\ e_1+e_3,\ -e_1+e_4,\ \dots,\ (-1)^n e_1+e_n$$

$$\left\{x\in\mathbb{R}^n:\ \sum(-1)^j x_j=0\ ,\ \sum\left(1+(-1)^{j+1}\right)x_j=0\right\}$$

$$\begin{cases} -x_1+x_2-x_3+x_4-x_5+x_6\ \cdots &=0\\ x_1\qquad +x_3\qquad +x_5 &=0\end{cases}$$

$$\begin{cases} x_1+x_3+x_5\ \cdots &=0\\ x_2+x_4+x_6\ \cdots &=0\end{cases}\qquad \dim=n-2$$

$$\begin{cases} x_1 = -x_3 - x_5 - \dots \\ x_2 = -x_4 - x_6 - \dots \end{cases}$$

$-e_1+e_3,\ -e_1+e_5,\ -e_1+e_7, \dots$
$-e_2+e_4,\ -e_2+e_6,\ -e_2+e_8, \dots$

Base

$$\left\{ A \in M_{3\times 2}(\mathbb{R}) : \begin{array}{l} a_{11} + a_{32} - 2a_{21} = 0 \\ a_{12} - a_{31} + 5a_{21} = 0 \end{array} \right\}$$

$$\begin{cases} a_{11} = -a_{32} + 2a_{21} \\ a_{12} = a_{31} - 5a_{21} \end{cases}$$

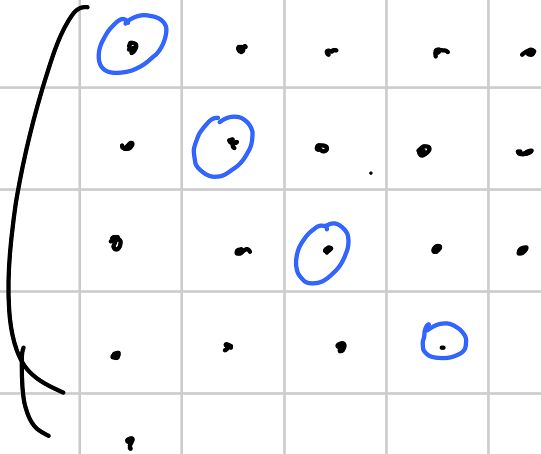
liberi $a_{32}, a_{21}, a_{31}, a_{22}$

Base :

$$\begin{pmatrix} \boxed{2} & \boxed{-5} \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \boxed{0} & \boxed{0} \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \boxed{0} & \boxed{1} \\ 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \boxed{-1} & \boxed{0} \\ 0 & 0 \\ 0 & \underline{1} \end{pmatrix}$$

$$\{A \in M_{n\times n}(\mathbb{R}) \ : \ tr(A) = 0\}$$

$$tr(A) = \sum_{i=1}^{n} (A)_{ii}$$



$$\dim = n^2 - 1$$

E_{ij} : 1 nel posto i,j 0 altrove

$$\left((E_{ij})_{hk} = \delta_{ih} \cdot \delta_{jk} \right)$$

$tr(A) = 0$ $\qquad a_{11} + a_{22} + \dots + a_{nn} = 0$

$$a_{11} = -a_{22} - \dots - a_{nn}$$

Par. liberi $a_{22}, \dots, a_{nn}$ e a_{ij} con $i \neq j$

Base: $-E_{11}+E_{22}$, $-E_{11}+E_{33}$, ..., $-E_{11}+E_{nn}$,
E_{ij} $i \neq j$.

Sono n^2-1? $(n-1)+n(n-1)$
$=(n-1)(n+1)=n^2-1$.

$S_n = \{ A \in M_{n \times n} : {}^tA = A \}$

matrici simmetriche

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & \\ a_{12} & a_{22} & a_{23} & \\ a_{13} & a_{23} & a_{33} & \\ & & & \end{pmatrix}$$

$$\dim = n + (n-1) + \cdots + 2 + 1$$
$$= \frac{1}{2} n(n+1)$$

<u>Base</u>: E_{ii} $\quad i = 1, \ldots, n$

$E_{ij} + E_{ji}$ $\quad 1 \leq i < j \leq n$

Sono $\dfrac{n(n+1)}{2}$?

$$n + \binom{n}{2} = n + \frac{n(n-1)}{2}$$

$$= \frac{n}{2}(2+n-1) = \frac{n(n+1)}{2}.\ \checkmark$$