# Geometria 29/5/15

Esame 28/6/14 - Es. 1.

$$A = \begin{pmatrix} 2 & 3 & 3 \\ -1 & 4 & -1 \\ 1 & -2 & 2 \end{pmatrix}$$

(A) Trovare autoval. e un autovett.

$$p_A(t) = \det \begin{pmatrix} t-2 & -3 & -3 \\ 1 & t-4 & 1 \\ -1 & 2 & t-2 \end{pmatrix}$$

$$= \det \begin{pmatrix} 0 & 2t-7 & t^2-4t+1 \\ 0 & t-2 & t-1 \\ -1 & x & x \end{pmatrix}$$

$$= t^3 - 8t^2 \cdots - D \qquad D = \det A \in \mathbb{Z}$$

Cerco come radice possibile i divisori interi di D

$\lambda_1 = 3$ radice; $\frac{P_A(t)}{t-3} = t^2 + \cdots \quad \lambda_{2,3} = \frac{1}{2}(5 \pm \sqrt{13})$.

$$V_1: \begin{pmatrix} 2 & 3 & 3 \\ -1 & 4 & -1 \\ 1 & -2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3 \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad \begin{cases} -x + 3y + 3z = 0 \\ x - 2y - z = 0 \end{cases}$$

$$v_1 = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$$

(B) $B = A + {}^tA$ provare che ammette base ortonorm. di autovettori

non è "trovare"

Sì: $B$ è simmetrica

(C) $P_B(t)$ + segni autoval

$$B = \begin{pmatrix} 2 & 3 & 3 \\ -1 & 4 & -1 \\ 1 & -2 & 2 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 1 \\ 3 & 4 & -2 \\ 3 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 4 \\ 2 & 8 & -3 \\ 4 & -3 & 4 \end{pmatrix}$$

$$P_B(t) = \det(t \cdot I_3 - B) = \dots$$

$d_1 = 4 > 0 \qquad d_2 = 28 > 0$

$d_3 = \cancel{128} - 24 - 24 - \cancel{128} - 16 - 36 < 0$

$\Longrightarrow + \; + \; -$

(D)

$$C = A - {}^tA = \begin{pmatrix} 2 & 3 & 3 \\ -1 & 4 & -1 \\ 1 & -2 & 2 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 1 \\ 3 & 4 & -2 \\ 3 & -1 & 2 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & 4 & 2 \\ -4 & 0 & 1 \\ -2 & -1 & 0 \end{pmatrix}$$

Trovare $a \in \mathbb{R}$ t.c. sia coniugata via matr. ortog. a

$$\begin{pmatrix} 0 & a & 0 \\ -a & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Antisimm $\Longrightarrow 0, \pm ia$

$p_C(t) = \dots = t(t^2+21)$ $\qquad a = \pm\sqrt{21}$

(E) Trovare tutti gli autovett. reali di $C$, –
Cioè quelli relativi a $0$:

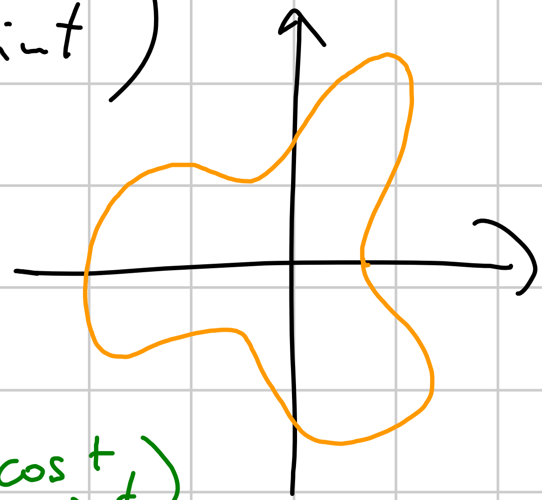
$$\begin{pmatrix} 0 & 4 & 2 \\ -4 & 0 & 1 \\ -2 & -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{aligned} 4y+2z&=0 \\ -4x+z&=0 \end{aligned}$$

$$k\cdot\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}, \quad k\neq 0 .$$

Ese 2:

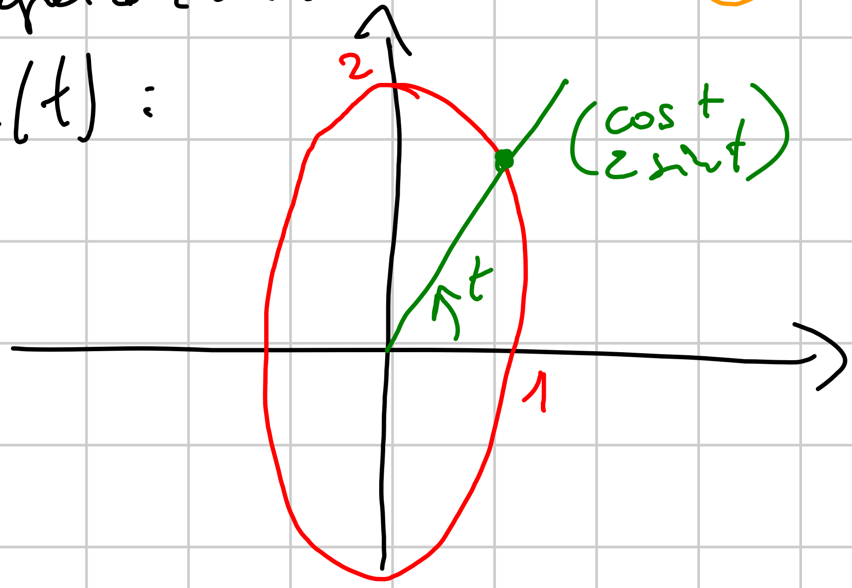
$$\alpha(t) = \underbrace{\left(2 - \cos(3t)\right)}_{r(t)} \begin{pmatrix} \cos t \\ 2 \sin t \end{pmatrix}$$



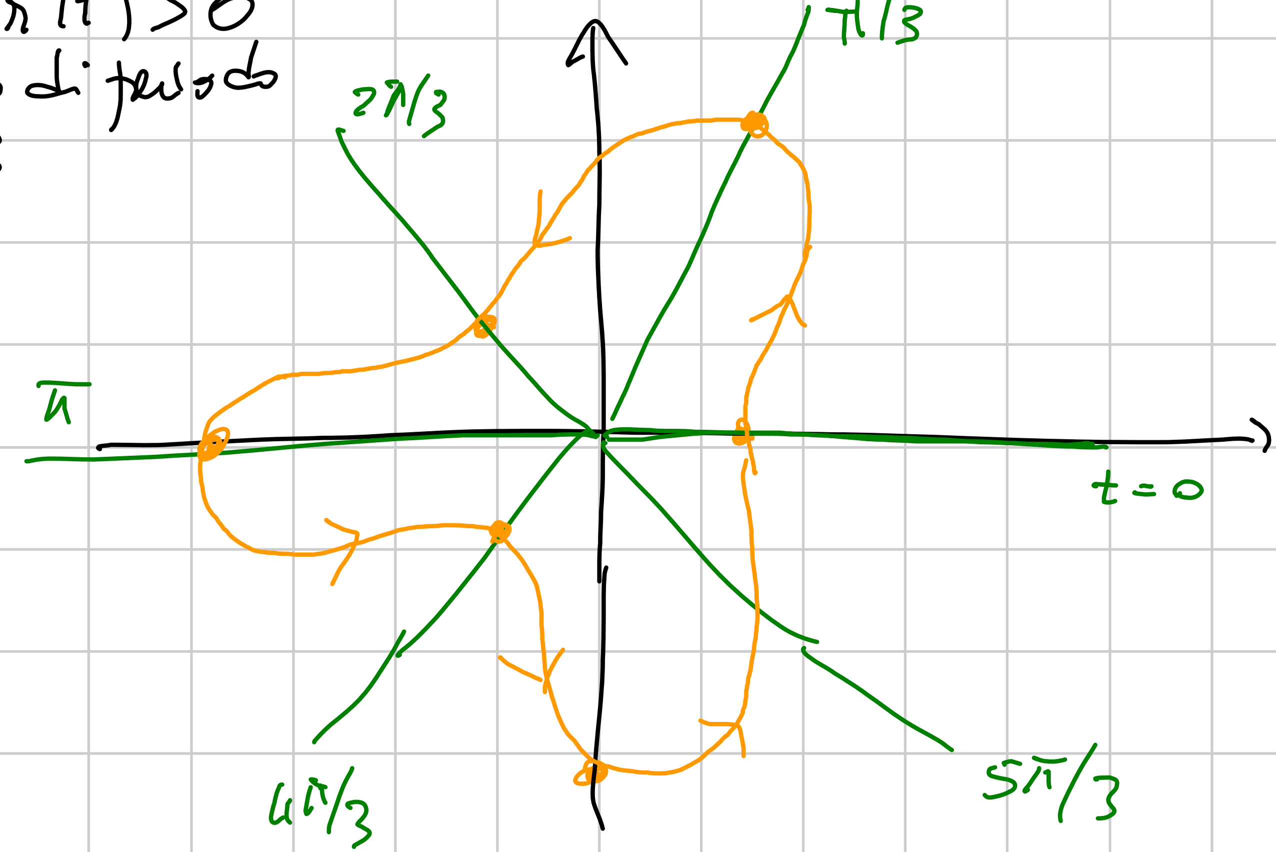
(A) $\alpha$ periodica; periodo _

(B) $\alpha|_{[-T/2, T/2]}$ semplice e chiusa

Dimenticando $r(t)$:



Inoltre $r(t) > 0$
e periodico di periodo
$2\pi/3$



(c) $\kappa$ in $t=0$

$$\underbrace{(2-\cos(3t))\begin{pmatrix}\cos t\\ 2\sin t\end{pmatrix}}_{\alpha(t)} ; \quad \underbrace{-3\sin 3t\begin{pmatrix}\cos t\\ 2\sin t\end{pmatrix}+(2-\cos(3t))\begin{pmatrix}-\sin t\\ 2\cos t\end{pmatrix}}_{\alpha'(t)}$$

$$\alpha''(t) = \dots$$

$$\kappa = \frac{\det(\alpha'(0), \alpha''(0))}{\|\alpha'(0)\|^3} = -2$$

ATTENZIONE

Per $\alpha : [a,b] \to \mathbb{R}^2$ orientata

$k$ ha segno.

Per $\alpha : [a,b] \to \mathbb{R}^3$ orientata

$k \geq 0$, $\tau$ ha segno

(D) Calcolare $\displaystyle\int_\beta \frac{y\,dx - x\,dy}{x^2+y^2}$ $\qquad \beta = \alpha\big|_{[0,2\pi]}$

FORMA

$-d\vartheta$

$$\Rightarrow \int_\gamma (-d\vartheta) = -2\pi \cdot$$ numero di giri fatti da $\gamma$ intorno a $O$ in senso antiorario

$\gamma$ chiusa, $O \notin \gamma$

$$\Longrightarrow \int_{\beta} (-dx) = -2\pi$$

(E) $\displaystyle\int_{\gamma} \underbrace{x\,dx}_{\text{esatta con } U = \frac{1}{2}x^2}$ $\qquad \gamma = \alpha\big|_{[0,\pi]}$

$$\Longrightarrow \frac{1}{2}x^2 \Big|_{\alpha(0)}^{\alpha(\pi)} = \frac{1}{2}x^2 \Big|_{(1,0)}^{(-3,0)} = 4$$

Esame 19/7/14 - Esc 1 -

$$v = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \qquad w_k = \begin{pmatrix} 5-2k \\ k-1 \\ k-2 \end{pmatrix}$$

(A) $k_0$ t.c. $w_{k_0} \perp v$

$$10 - 4k$$
$$-3 + 3k$$
$$+8 - 4k = 0$$

$$15 = 5k$$

$$k = 3$$

$$w = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$$

(B) Matrice proiez. ortog. su $W^{\perp}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \frac{-x+2y+z}{1+4+1} \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} =$$

$$= \underbrace{\frac{1}{6}\begin{pmatrix} 5 & 2 & 1 \\ 2 & 2 & -2 \\ 1 & -2 & 5 \end{pmatrix}}_{\underline{M}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Verificare che $M \cdot M = M$

$$\frac{1}{6}\begin{pmatrix} 5 & 2 & 1 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \cdot \frac{1}{6}\begin{pmatrix} 5 & \cdot & \cdot \\ 2 & \cdot & \cdot \\ 1 & \cdot & \cdot \end{pmatrix} =$$

$$= \frac{1}{36}\begin{pmatrix} 30 & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} = \frac{1}{6}\begin{pmatrix} 5 & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

(C) Matrice $N$ di proiez ortog su $v^{\perp}$

$$N = \frac{1}{29}\begin{pmatrix} 25 & -6 & 8 \\ -6 & 20 & 12 \\ 8 & 12 & 13 \end{pmatrix}$$

(D) $P = 6M + 29N$ ammette base ortonormale di autovettori

Sì: è simmetrico.

(E) Trovare autoval di $P$ e una base ortonormale di autovettori.

$$6M = \begin{pmatrix} 5 & 2 & 1 \\ 2 & 2 & -2 \\ 1 & -2 & 5 \end{pmatrix} \qquad 21N = \begin{pmatrix} 25 & -6 & 8 \\ -6 & 20 & 12 \\ 8 & 12 & 13 \end{pmatrix}$$

$$\underline{P} = \begin{pmatrix} 30 & -4 & 9 \\ -4 & 22 & 10 \\ 9 & 10 & 18 \end{pmatrix}$$

Così ... oppure:


$v \perp w$

$\Longrightarrow v \in W^\perp$

$w \in v^\perp$

$$P \cdot v = (6M + 29N) = 6v + 29 \cdot 0 = 6v$$

$\qquad\qquad\quad\overset{\shortparallel}{\text{proiez su } w^\perp}\qquad \overset{\shortparallel}{\text{proiez. su } v^\perp}$

$$P \cdot w = (6M + 29N) = 6 \cdot 0 + 29 \cdot w = 29w$$

autoval 6 autovett v

autoval 29 autovett w

autovett $v \wedge w$

autovalore $6+29=35$