Geometria 27/5/15

Quesiti dal libro

$\boxed{11}.3$ Trovare $\cap$ tra $\{[t+3 : t^2-2 : t+4] : t \in \mathbb{R}\}$ e retta per $[2:3:-1]$, $[-4:1:3]$ _

$[t+3 : t^2-2 : t+4] \in$ retta per $\left([\ldots] \text{ e } [\ldots] \right)$

$$\Longleftrightarrow \begin{pmatrix} t+3 \\ t^2-2 \\ t+2 \end{pmatrix} \in \operatorname{Span}\left(\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} \right)$$

$$\Longleftrightarrow \det\begin{pmatrix} 2 & -4 & t+3 \\ 3 & 1 & t^2-2 \\ -1 & 3 & t+2 \end{pmatrix} = 0$$

- calcolo det
- risolvo eq. II grado in t ; trovo $t_{1,2}$
- sostituisco $t_{1,2}$ in $[t+3 : t^2-2 : t+2]$

$$\left(\text{ricordo: } \left[-\frac{19}{3} : -\frac{14}{3} : \frac{2}{3}\right] = [19 : 14 : -2]\right)$$

$\boxed{1}.7$

$$\int_\alpha \left(3x^2y^2\,dx + 2x^3y\,dy\right)$$

$\rightarrow$ funzione $\rightarrow \ldots \|\alpha'(t)\|dt$

$\rightarrow$ forma

$dx = x'(t)\,dt$

$dy = y'(t)\,dt$

$\alpha : [0,1] \to \mathbb{R}^2$

$$\alpha(t) = \begin{pmatrix} \cos\left(\frac{\pi t^2}{4}\right) \\ \cos\left(\frac{\pi t^2}{6}\right) \end{pmatrix}$$

QUI: FORMA

$$\int_0^1 \left(3\cos^2\left(\frac{\pi t^2}{4}\right)\cdot\cos^2\left(\frac{\pi t^2}{6}\right)\cdot\left(-\frac{\pi t}{2}\cdot\sin\left(\frac{\pi t^2}{4}\right)\right) + \ldots\right)dt$$

fattibile ma complicato _ Invece:

$$\omega = 3x^2y^2dx + 2x^3y\,dy$$

Esatta? [Sì: chiaro.]

Siccome è def. su $\mathbb{R}^2$ basta: chiusa?

$$\frac{\partial}{\partial x}\left(2x^3y\right) \overset{?}{=} \frac{\partial}{\partial y}\left(3x^2y^2\right)$$

$$6x^2y \overset{?}{=} 6x^2y$$

Sì

Cerco potenziale $\bar{U}$; voglio:

$$\frac{\partial \bar{U}}{\partial x} = 3x^2y^2 \qquad \frac{\partial \bar{U}}{\partial y} = 2x^3y$$

$$\bar{U} = x^3y^2$$

$$\Longrightarrow \int_\alpha \omega = \int_\alpha d\bar{U} = \bar{U}\Big|_{\alpha(0)}^{\alpha(1)} = x^3y^2\Big|_{(1,1)}^{\left(\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}\right)}$$

$$= \frac{1}{2\sqrt{2}} \cdot \frac{3}{4} - 1 \cdot 1 = \cdots = \frac{3\sqrt{2}}{16} - 1$$

$\boxed{2}$.7 $$\int_\alpha \left(2xy^3\,dx + 3x^2y^2\,dy\right) = x^2y^3 \Big|_{\alpha(0)}^{\alpha(1)} = \dots$$

$\boxed{3}$.6 Matrice hessiana $\underbrace{}_{\text{in } (0,0)}$ + segni autoval per

$$f(x,y) = \sin(xy) - 2\exp(x^2+3y^2)$$

$$\frac{\partial f}{\partial x} = y \cdot \cos(xy) - 4x \cdot e^{x^2+3y^2}$$

$$\frac{\partial f}{\partial y} = x \cdot \cos(xy) - 12y \cdot e^{x^2+3y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = 0 \cdot \cos + y \cdot (\ldots) - 4 \cdot e^{\cdots} - 4x \cdot (\ldots) \Longrightarrow -4$$

$$\frac{\partial^2 f}{\partial x \partial y} = \cos(xy) + y \cdot (\ldots) + 0 \ldots \Longrightarrow 1$$

$$\frac{\partial^2 f}{\partial y^2} = 0 \cdot \cos + x \cdot (\ldots) - 12 \cdot e^{\cdots} + y(\ldots) \Longrightarrow -12$$

$$(Hf)_{\binom{0}{0}} = \begin{pmatrix} -4 & 1 \\ 1 & -12 \end{pmatrix}$$

$$d_1 < 0$$

$$d_2 > 0 \Longrightarrow - \; -$$

$\boxed{14}$.1 Autovd. di $\begin{pmatrix} -i & 1-i \\ 2 & 1 \end{pmatrix}$ + base diago-

$$\underset{\lambda_1+\lambda_2}{\underset{\shortparallel}{tr}} = 1-i \qquad \underset{\lambda_1\cdot\lambda_2}{\underset{\shortparallel}{det}} = -i-2+2i = -2+i$$

$$\lambda^2-(1-i)\lambda+(-2+i)=0$$

$$\Delta = 1-2i-1+8-4i = 8-6i = (a+ib)^2$$

$$\begin{cases} a^2-b^2=8 \\ ab=-3 \end{cases} \qquad \begin{cases} a=3 \\ b=-1 \end{cases}$$

$$\lambda_{1,2} = \frac{1-i \pm (3-i)}{2} = \begin{cases} \to 2-i \\ \to -1 \end{cases}$$

$$v_1: \begin{pmatrix} -i & 1-i \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = (2-i) \begin{pmatrix} x \\ y \end{pmatrix} \quad \Big\} \text{ equiv.}$$

$$2x + y = (2-i)y \qquad v_1 = \begin{pmatrix} 3-i \\ 2 \end{pmatrix}$$

$$v_2: \quad 2x + y = -y \qquad v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$\boxed{5}.1$ $\quad \int_\alpha y \cdot \sin(xy^2)\,(y\,dx + 2x\,dy)$ $\qquad \alpha(t) = \begin{pmatrix} t \\ \sin t/2 \end{pmatrix}$

$t \in [0, \pi]$

FORMA

$$= \int_\alpha \sin(xy^2)\,(y^2\,dx + 2xy\,dy)$$

$$= -\cos(xy^2)\Big|_{\alpha(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}}^{\alpha(\pi) = \begin{pmatrix} \pi \\ 1 \end{pmatrix}} = -\cos(\pi) + \cos(0)$$

$$= 2$$

$\boxed{5}.2$ $\displaystyle\int_\alpha x$ $\qquad \alpha(t) = \begin{pmatrix} t \\ 2t^2 \end{pmatrix} \qquad t \in [0,1]$

FUNZIONE

$$= \int_0^1 \underset{\underset{X(t)}{\parallel}}{t} \cdot \underbrace{\sqrt{1+(4t)^2}}_{\|\alpha'(t)\|} \, dt$$

$$= \int_0^1 t \sqrt{1+16t^2} \, dt =$$

$$= \frac{2}{3} \cdot \frac{1}{32} \left(1+16t^2\right)^{3/2} \Big|_0^1 = \frac{1}{48}\left(17\sqrt{17}-1\right)$$

$\boxed{6}.7$ Trovare $a>0$ t.c.

$$\underbrace{\begin{pmatrix} 0 & -1 & \sqrt{6} \\ 1 & 0 & -3 \\ -\sqrt{6} & 3 & 0 \end{pmatrix}}_{\text{antisimm. } 3\times 3}$$ sia coniugata e $$\begin{pmatrix} 0 & -a & 0 \\ a & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\Longrightarrow$ ha autoval $\lambda_1 = i \cdot a$ $\quad \lambda_3 = 0$

$\lambda_2 = -i \cdot a$

e $\underline{\underline{\text{so}}}$ che coniugata via ortog. a diag. a blocchi:

$$\left(\begin{array}{cc|c} 0 & -a & 0 \\ a & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right)$$

Basta trovare autoval:

$$\det\begin{pmatrix} t & 1 & -\sqrt{6} \\ -1 & t & 3 \\ \sqrt{6} & -3 & t \end{pmatrix} = t^3 + 3\sqrt{6} - 3\sqrt{6} + t(9+1+6) = t(t^2+16)$$

$\lambda_{1,2} = \pm 4i \implies a = 4$

$\boxed{7}.3$ Trovare tutti i $v \in \mathbb{C}^2$

(1) unitari

(2) $\perp$ a $\begin{pmatrix} 1+i \\ -i \end{pmatrix}$

(3) $v_2 \in i \cdot \mathbb{R}$

CATTIVA IDEA: imporle tutte insieme

Invece: (3) $v = \begin{pmatrix} z \\ i \end{pmatrix}$ (poi potrò moltiplicare per $k \in \mathbb{R}$)

(2) $(1-i)z + i \cdot i = 0$

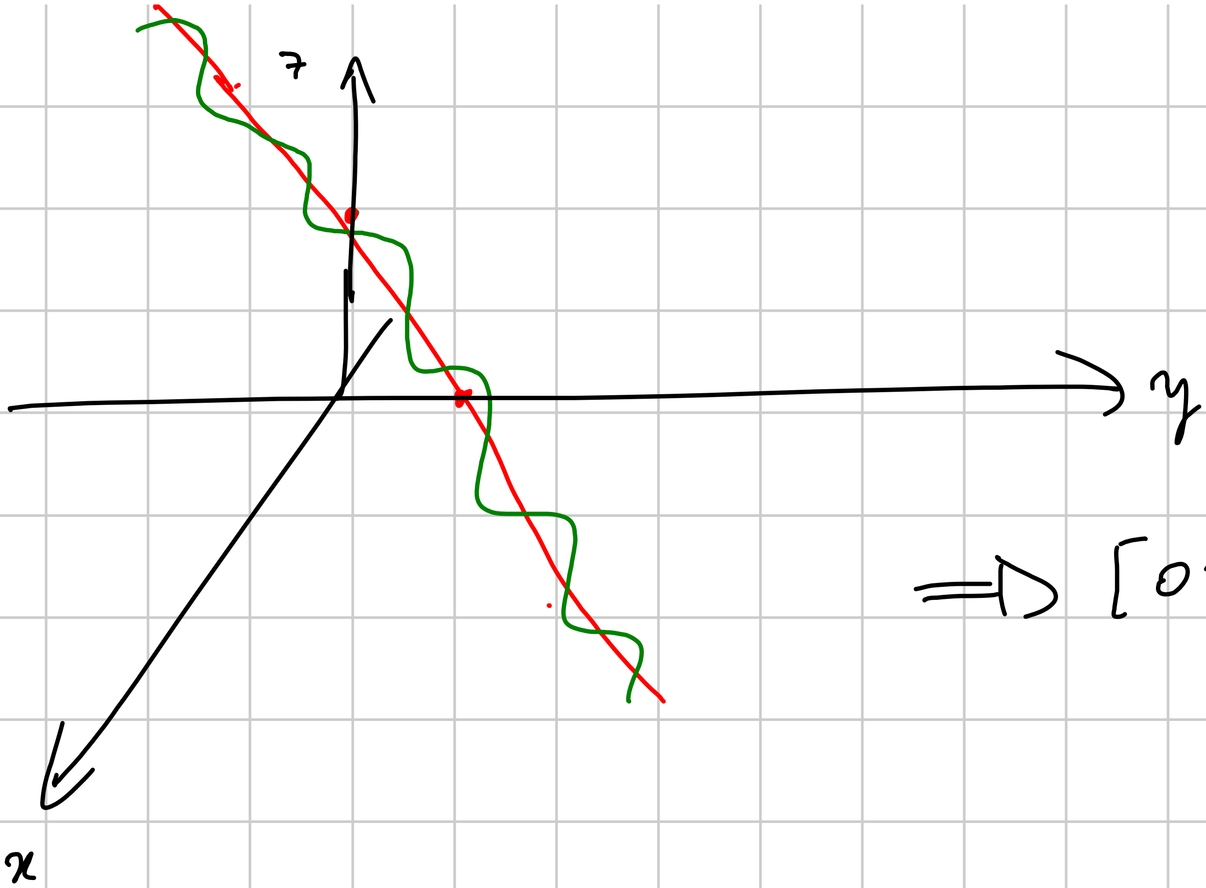
$$z = \frac{1}{1-i} = \frac{1+i}{2}$$

(1) $\pm \frac{1}{\sqrt{6}} \begin{pmatrix} 1+i \\ 2i \end{pmatrix}$

$\boxed{26}$.3 Pti a ∞ di

$$\{ \underline{(\sin t,\ 2t,\ 1-t)} : t \in \mathbb{R} \} .$$

$(0, 2t, 1-t)$



$\Rightarrow [0:2:-1]$

Oppure: $(\sin t, 2t, 1-t)$

$$= [\sin t : 2t : 1-t : 1]$$

$$= \left[\frac{\sin t}{t} : 2 : \frac{1}{t} - 1 : \frac{1}{t}\right]$$

va a 0 se $t \longrightarrow \pm\infty$

$$\lim_{t \to \pm\infty} \left[\frac{\sin t}{t} : 2 : \frac{1}{t} - 1 : \frac{1}{t}\right] = [0 : 2 : -1 : 0]$$

$$= [0 : 2 : -1]$$

$\boxed{30}.1 \quad X = \{ x \in \mathbb{R}^3 : x_1 + x_2 = x_3 \}$

$f : X \to X \qquad f(x) = \begin{pmatrix} 3x_1 + x_2 \\ x_3 - 2x_1 \\ x_1 + x_2 + x_3 \end{pmatrix} \qquad p_f(t) = ?$

Serve $[f]_{\mathcal{B}}^{\mathcal{B}}$ — Scelgo $\mathcal{B} = \left(\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right)$

$$f\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} - 1 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$f\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow p_1(t) = p_{\begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}}(t) = t^2 - \mathrm{tr}(\ ) \cdot t + \det(\ )$$

$(-1)^n$ $\quad n = 2$

$$= t^2 - 4t + 4 = (t-2)^2$$

$\boxed{37}$ 6. $H_{(0,0)} f$ e semi antinol $f(x,y) = y^2 + e^{2x + \sin y}$

$$\frac{\partial f}{\partial x} = 2 \cdot e^{2x+\sin y} \qquad \frac{\partial f}{\partial y} = 2y + \cos y \cdot e^{2x+\sin y}$$

$$\frac{\partial^2 f}{\partial x^2} = 4 \cdot e^{\cdots} \qquad \frac{\partial^2 f}{\partial x \partial y} = 2\cos y \cdot e^{\cdots}$$

$$\frac{\partial^2 f}{\partial y^2} = 2 - \sin y \cdot e^{\cdots} + \cos^2 y \cdot e^{\cdots}$$

$$\Rightarrow \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \qquad +, 0$$

28/6/14. ro. 2

Trovare tutti i $v \in \mathbb{C}^2$

(1) unitari

(2) $\perp \begin{pmatrix} 3-i \\ 1+2i \end{pmatrix}$

(3) somma coord $\in \mathbb{R}$.

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(3) $v = \begin{pmatrix} z \\ 1-z \end{pmatrix}$ poi potrò moltiplicare per $k \in \mathbb{R}$ _

(2) $(3+i)z + (1-2i)(1-z) = 0$

$$z = \frac{1-2i}{1-2i-3-i} = \frac{1-2i}{-2-3i} =$$

$$= \frac{(1-2i)(-2+3i)}{4+9} = \frac{4+7i}{13}$$

$$1-z = \frac{9-7i}{13}$$

$$(1)\ \pm \frac{1}{\sqrt{16+49+81+49}} \begin{pmatrix} 4+7i \\ 9-7i \end{pmatrix} -$$

Scritto 7/6/14 . v0

1. Data $A \in M_{3\times 3}(\mathbb{R})$ trovare $p_A(t)$ seguendo:

$$tr(A) = -4, \quad det(A) = 11, \quad p_A(-1) = -3.$$

$$p_A(t) = t^3 + 4t^2 + c_1 \cdot t - 11$$

$$-1 + 4 - c_1 - 11 = -3$$

$$\Longrightarrow c_1 = -5$$

$\Longrightarrow t^3+4t^2-5t-11$

2. Autoval di $\begin{pmatrix} 4 & 9 \\ 2 & -3 \end{pmatrix}$ + base diago.

$\lambda_1+\lambda_2 = 1$ $\qquad$ $\lambda_1 = 6$

$\lambda_1 \cdot \lambda_2 = -30$ $\qquad$ $\lambda_2 = -5$

$r_1: \quad 2x-3y=6y \qquad \begin{pmatrix} 9 \\ 2 \end{pmatrix}$

$r_2: \quad 2x-3y=-5y \qquad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

3. Trovare tutti i vett. di $\mathbb{R}^3$ unitari e $\perp$ a

$$\begin{pmatrix} 4 \\ 5 \\ -3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} - \pm \frac{1}{\sqrt{17^2+10^2+6^2}} \begin{pmatrix} +17 \\ -10 \\ +6 \end{pmatrix} -$$

4. Per quali $t \in \mathbb{R}$ la $\begin{pmatrix} t^2 & t^2+t-3 \\ 5-t & 3t \end{pmatrix}$ ammette base ortonorm. di autovettori?

Per quali tali t gli autoval. sono pos?

è simmetrica?

$t^2 + t - 3 = 5 - t$ $\qquad t^2 + 2t - 8 = 0$

$(t+4)(t-2) = 0$

$t = -4, \quad t = 2$

$t = -4 \qquad \begin{pmatrix} 16 & 9 \\ 9 & -12 \end{pmatrix} \qquad + \; -$

$t = 2 \qquad \begin{pmatrix} 4 & 3 \\ 3 & 6 \end{pmatrix} \qquad + \; +$

Att: $\begin{pmatrix} 4 & 3 \\ 3 & 1 \end{pmatrix}$ $+ \, - \quad -$

5. Tipo affine di $5z^2+2xy-2xz-4yz+2x-2z=0$

$$\left(\begin{array}{ccc|c} 0 & 1 & -1 & 1 \\ 1 & 0 & -2 & 0 \\ -1 & -2 & 5 & -1 \\ \hline 1 & 0 & -1 & 0 \end{array}\right)$$

$d_2 < 0$

$d_3 = 2+2-5 < 0$

$$d_4 = \det \begin{pmatrix} 0 & 1 & -1 & 1 \\ 1 & 0 & -2 & 0 \\ -1 & -1 & 4 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

$$= -(1) \cdot (-1) \cdot \det \begin{pmatrix} 1 & -2 \\ 1 & -1 \end{pmatrix} = 1 > 0 .$$

$$d_2 < 0 \qquad d_3 < 0 \qquad d_4 > 0$$

$$\Rightarrow \left(+ \quad - \quad + \right) \quad - .$$

$$x^2 - y^2 + z^2 - 1 = 0$$
$$x^2 + z^2 = 1 + y^2$$
$$x^2 + y^2 = 1 + z^2 \quad \text{(cambiando nomi)}$$

iperboloide 1 falde (iperbolico)

6. Trovare $\cap$ tra $\{[t+1 : -t : t-1] : t \in \mathbb{R}\} \subset \mathbb{P}^2(\mathbb{R})$ e pti a ∞ di $x^2 - 5z^2 + 2xy + yz + 5x + 7 = 0$

equaz. pti a ∞

$$(t+1)^2-5(t-1)^2+2(t+1)(-t)+(-1)(t-1)=0$$

$$7t^2-11t+4=0$$

$$(7t-4)(t-1)=0$$

$$t=1 \longrightarrow [2:-1:0]$$

$$t=\frac{4}{7} \longrightarrow \left[\frac{4}{7}+1 : -\frac{4}{7} : \frac{4}{7}-1\right] = [-11:4:3]$$

7. $\displaystyle\int_\alpha \sin(x^2-2y)\,(x\,dx - dy) \qquad \alpha(t) = \begin{pmatrix} t^3-t \\ \dfrac{\pi t}{t+1} \end{pmatrix}$

$t \in [0,1]$

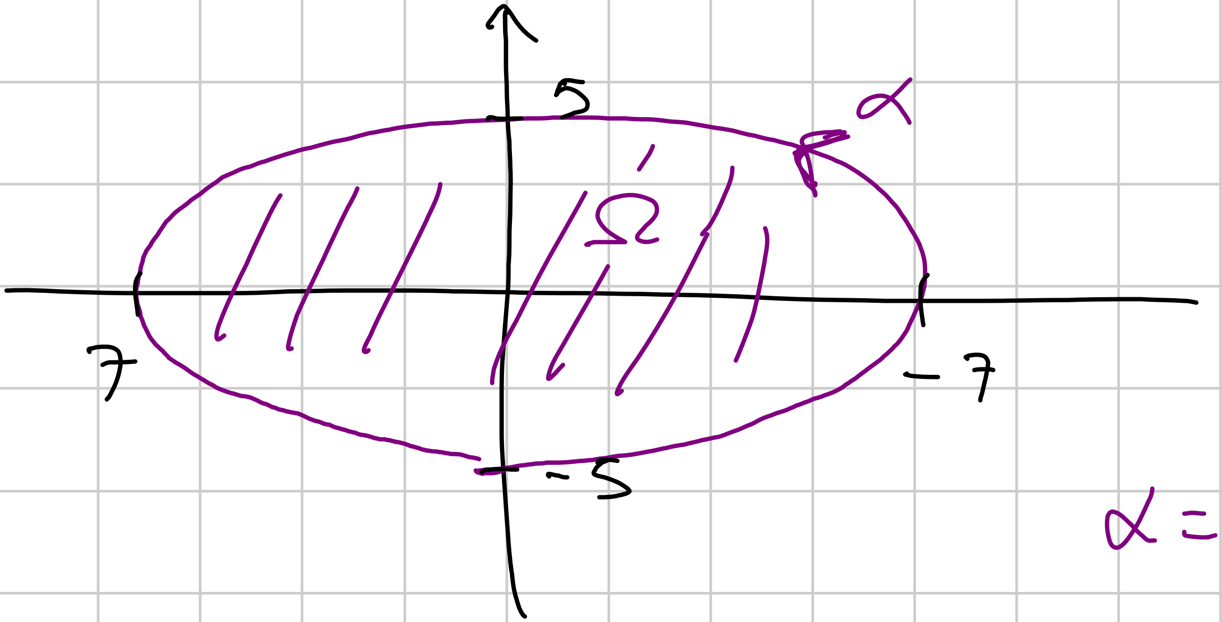
$$= -\frac{1}{2}\cos(x^2-2y)\Bigg|_{\alpha(0)=\begin{pmatrix}0\\0\end{pmatrix}}^{\alpha(1)=\begin{pmatrix}0\\ \pi/2\end{pmatrix}}$$

$$= -\frac{1}{2}\cos(-\pi) + \frac{1}{2}\cos(0) = 1.$$

$\underline{\underline{Q}}$: calcolare $\displaystyle\int_\alpha \left(e^{1+\log(1+\cos^2 x)} + y\right) dx + \left(\log\left(1+\cos^2\left(7y^{99}\right)\right) - x\right) dy$

$\alpha : [0, 2\pi) \to \mathbb{R}^2 \qquad \alpha(t) = \begin{pmatrix} 7\cos t \\ 5\sin t \end{pmatrix}$.

Non è esatto . Però :



$\alpha = \partial\Omega$

G-G: $\displaystyle\int_\alpha \omega = \int_\Omega d\omega = \int_\Omega (0 + dy)\cdot dx + (0 - dx)\cdot dy$

$$= -2 \underbrace{\int_{\Omega} dx\,dy}_{\text{area} \,=\, \pi \cdot 7 \cdot 5}$$

$$= -70 \cdot \pi$$