24/4/2015 Esercitazione di algebra lin. e geometria

1

Es. 10

$$A = \begin{pmatrix} -5 & -1 & -2 \\ -1 & 9 & 5 \\ -2 & 5 & 7 \end{pmatrix}$$

$$p_A(t) = \begin{vmatrix} t+5 & 1 & 2 \\ 1 & t-9 & -5 \\ 2 & -5 & t-7 \end{vmatrix} = \begin{vmatrix} t+5 & 1 & 0 \\ 1 & t-9 & -2t+13 \\ 2 & -5 & t+3 \end{vmatrix} =$$

$$= (t+5)(t^2-16t+38) - (5t-23) =$$

$$= t^3 - 11t^2 - 47t + 213 .$$

Cerco radici intere

± 1 no, , 3 ok

$(27 - 99 - 141 + 213 = 0)$ $\Rightarrow$ divido con Ruffini

$$p_A(t) = (t-3)(t^2-8t-71)$$

$$t_1 = 3$$

$$t_{2,3} = 4 \pm \sqrt{87}$$

$$v_1 : \begin{cases} 8x + y + 2z = 0 \\ x - 6y - 5z = 0 \quad \Rightarrow \quad x = 6y + 5z \quad \Rightarrow \quad \cancel{49}^{\,7}\, y + \cancel{42}^{\,6}\, z = 0 \end{cases}$$

$$v_1 = \begin{pmatrix} 1 \\ 6 \\ -7 \end{pmatrix}$$

$$v_{2,3} : \begin{cases} (9 \pm \sqrt{87})x + y + 2z = 0 \\ x + (-5 \pm \sqrt{87})y - 5z = 0 \quad \Rightarrow \quad x = (5 \mp \sqrt{87})y + 5z \end{cases}$$

$$(-41 - 4\sqrt{87})\,y + (47 + 5\sqrt{87})\,z = 0$$

$$y = 47 \pm 5\sqrt{87} \qquad z = 41 \pm 4\sqrt{87} \qquad x = -200 \mp 22\sqrt{87}$$

11

$$P_A(t) = \begin{vmatrix} t - 3/2 & 3/4 & \frac{3\sqrt{3}}{4} \\ 3/4 & t + 9/8 & \frac{7\sqrt{3}}{8} \\ \frac{3\sqrt{3}}{4} & \frac{7\sqrt{3}}{8} & t - 5/8 \end{vmatrix} = \dots =$$

$$= t^3 - t^2 - 6t = t(t^2 - t - 6) =$$

$$= t(t+2)(t-3)$$

$\lambda_1 = 0 \qquad \lambda_2 = -2 \qquad \lambda_3 = 3$

$$v_1 : \begin{cases} 12x - 6y + 6\sqrt{3}z = 0 \\ -6x - 9y - 7\sqrt{3}z = 0 \\ x^2 + y^2 + z^2 = 1 \end{cases}$$

pongo intanto $y = 1$ e risolvo le prime due eq.:

$$\begin{cases} \overset{2}{\cancel{12}}\,x + \overset{1}{\cancel{6}}\sqrt{3}\,z = \overset{1}{\cancel{6}}\,y \\ -6x - 7\sqrt{3}z = 9 \end{cases}$$

$$x = \frac{\begin{vmatrix} 1 & \sqrt{3} \\ 9 & -7\sqrt{3} \end{vmatrix}}{\underbrace{\begin{vmatrix} 2 & \sqrt{3} \\ -6 & -7\sqrt{3} \end{vmatrix}}_{= -8\sqrt{3}}} = 2 \qquad z = \frac{\begin{vmatrix} 2 & 1 \\ -6 & 9 \end{vmatrix}}{-8\sqrt{3}} = -\sqrt{3}$$

$\Rightarrow (2, 1, -\sqrt{3})$ è 0 - autovettore

Normalizziamo

$$\sqrt{2^2 + 1^2 + 3} = 2\sqrt{2}$$

$$v_1 = \frac{1}{2\sqrt{2}}(2, 1, -\sqrt{3})$$

$\lambda_2 = -2$

$$(-2)I - A = \begin{pmatrix} -7/2 & 3/4 & -\frac{3\sqrt{3}}{4} \\ 3/4 & -7/8 & 7\sqrt{3}/8 \\ -\frac{3\sqrt{3}}{4} & \frac{7\sqrt{3}}{8} & -21/8 \end{pmatrix}$$

$$\begin{vmatrix} -7/2 & 3/4 \\ 3/4 & -7/8 \end{vmatrix} = 5/2 \neq 0$$

Pongo $z = 1$ e risolvo le prime due eq. del sistema $\Rightarrow$ $(0, \sqrt{3}, 1)$

Normalizzo

$$V_2 = \frac{1}{2}(0, \sqrt{3}, 1)$$

Nota $v_1 \cdot v_2 = 0$ (una matrice simmetrica si diagonalizza rispetto ad una base ortogonale e, poiché i tre autovalori sono distinti, $\{v_1, v_2, v_3\}$ è base ortogonale).

Trovo v_3 ortogonale a v_1, v_2 e unitario

$$v_1 \wedge v_2 = \frac{1}{\sqrt{2}}(+4, -2, 2\sqrt{3})$$

$$|v_1 \wedge v_2| = \frac{1}{\sqrt{2}}\sqrt{16 + 4 + 12} = 4$$

$$v_3 = \frac{1}{2\sqrt{2}}(2, -1, \sqrt{3})$$

Es 12

(a) i determinanti dei minori principali sono :

$$\begin{pmatrix} 3 & -2 & 2 \\ -2 & 1 & 5 \\ 2 & 5 & -3 \end{pmatrix}$$

$d_1 = 3 > 0$, $d_2 = 3 + (-4) = -1 < 0$

$d_3 = \det A = -116 < 0$

i segni degli autovalori sono dati dai cambi di segno e sono

$+ , - , +$.

(b)
$$\begin{pmatrix} -1 & 2 & 7 \\ 2 & -5 & 3 \\ 7 & 3 & -2 \end{pmatrix}$$

$d_1 = -1 < 0$

$d_2 = 1 > 0$

$d_3 = 236 > 0$

segni $- , - , +$

ES. 13

(a) $f(x,y) = x^2 \cos(xy) + 2y^2 - 3xy$

$$\frac{\partial^2 f}{\partial x^2} = 2\cos(xy) - 4xy \operatorname{sen}(xy) - x^2y^2\cos(xy)$$

$$\frac{\partial^2 f}{\partial x \partial y} = -3x^2 \sin(xy) - x^3 y \cos(xy) - 3 = \frac{\partial f}{\partial y \partial x}$$

$$\frac{\partial^2 f}{\partial y^2} = -\cancel{x}^4 \cos(xy) + 4$$

$$H_f(0,0) = \begin{pmatrix} 2 & -3 \\ -3 & 4 \end{pmatrix}$$

$d_1 = 2 > 0$

$d_2 = -1 < 0$

un autovel > 0, uno < 0

$\Rightarrow$ né massimo, né minimo

(b) $f(x,y) = x \ln(1+y) + 3\sin^2 y + x^2$

$$\frac{\partial^2 f}{\partial x^2} = 2$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{1}{1+y} = \frac{\partial^2 f}{\partial y \partial x}$$

$$\frac{\partial^2 f}{\partial y^2} = -\frac{x}{(1+y)^2} - 6\sin^2 \cancel{x} + 6\cos^2 y$$

$$H_f(0,0) = \begin{pmatrix} 2 & 1 \\ 1 & 6 \end{pmatrix}$$

$d_1 = 2 > 0$

$d_2 = 10 > 0$

2 autoval. > 0 $\Rightarrow$ $(0,0)$ è minimo relativo.

(c) $f(x,y) = e^{xy}\cos(2x-y) - 2xy$

$$\frac{\partial^2 f}{\partial x^2} = e^{xy}\left[(y^2-4)\cos(2x-y) - 4y\sin(2x-y)\right]$$

$$\frac{\partial^2 f}{\partial x \partial y} = e^{xy}\left[(xy+3)\cos(2x-y) + (y-2x)\sin(2x-y)\right] - 2 =$$

$$= \frac{\partial^2 f}{\partial y \partial x}$$

$$\frac{\partial^2 f}{\partial y^2} = e^{xy}\left[(x^2-1)\cos(2x-y) + 2x\sin(2x-y)\right]$$

$$H_f(0,0) = \begin{pmatrix} -4 & 1 \\ 1 & -1 \end{pmatrix} \qquad \begin{array}{l} d_1 = -4 < 0 \\ d_2 = 3 > 0 \end{array}$$

due autoval. negativi

$\Rightarrow (0,0)$ è max. relativo.