ES 18 $\alpha(t) = \begin{pmatrix} 1-2t \\ 1+t^2 \end{pmatrix}$ $\alpha : [0,1] \to \mathbb{R}^2$

$$\int_\alpha \left(2xy^2\,dx + 3x^2y\,dy\right) =$$

$$= \int_0^1 2(1-2t)(1+t^2)^2(-2)\,dt + 3(1-2t)^2(1+t^2)\,2t\,dt =$$

$$= \int_0^1 \left(-4 + 14t - 32t^2 + 46t^3 - 28t^4 + 32t^5\right) dt =$$

$$= 107/30$$

ES 19 $\alpha : [0, \vartheta] \to \mathbb{R}^2$ $\alpha(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$

$$\int_\alpha (x\,dy - y\,dx) =$$

$dx = -\sin t\,dt$

$dy = \cos t\,dt$

$$= \int_0^\vartheta \cos^2 t\,dt + \sin^2 t\,dt = \vartheta$$

ES 20 $d\left(e^{x+\cos y} \cdot \log \frac{\cos x}{y}\right) =$

$$= e^{x+\cos y}\left(\log \frac{\cos x}{y} + \frac{y}{\cos x}(-\sin x)\right) dx +$$

$$+ e^{x+\cos y}\left(-\sin y \log \frac{\cos x}{y} - \frac{\cancel{\cos x}}{y^2}\frac{y}{\cancel{\cos x}}\right) dy$$

ES 21 $d\left(e^{xy} \cos x\,dx + e^{x-y} \sin(xy)\,dy\right) =$

$$= \left(-x e^{xy} \cos x + e^{x-y}\left(\sin(xy) + y\cos(xy)\right)\right) dx\,dy$$

22 (a) $\omega(x,y) = -5x^2y\,(3y\,dx + 2x\,dy)$

$\Omega = \mathbb{R}^2$ è sempl. connesso

$d\omega = 30x^2y\,dx\,dy - 30x^2y\,dx\,dy = 0$

$U_x = -15x^2y^2$

$U_y = -10x^3y$

$\Rightarrow U(x,y) = -5x^3y^2$

(b) $\omega(x,y) = e^y\left(\frac{1}{x}\,dx + \log x\,dy\right)$

$\Omega = (0,\infty)\times\mathbb{R}$ è sempl. conn.

$d\omega = \left(-\frac{e^y}{x} + \frac{e^y}{x}\right)dx\,dy = 0$

$U_x = e^y/x$

$U_y = e^y\log x$

$U(x,y) = e^y\log(x)$

(c) $\omega(x,y) = (2x + y\sin(xy))\,dx + (x\sin(xy) - 1)\,dy$

$\Omega = \mathbb{R}^2$

$d\omega = 0$

$U_x = 2x + y\sin(xy)$

$U_y = x\sin(xy) - 1$

$\Rightarrow U(x,y) = x^2 - \cos(xy) - y$

(d) $\omega = \dfrac{dx - 2y\,dy}{x - y^2}$

$\Omega = \{(x,y) \mid x \neq y^2\}$ è sempl. conn.

$d\omega = 0$

$U_x = \dfrac{1}{x-y^2}$ $\quad U_y = \dfrac{-2y}{x-y^2}$ $\quad U(x,y) = \log(x-y^2)$

(e) $\omega = y \sin x \, dx + (\cos y - \cos x)\, dy$

$\Omega = \mathbb{R}^2$ s.c.

$d\omega = 0$

$U_x = y \sin x$

$U(x,y) = -y\cos x + \sin y$

$U_y = \cos y - \cos x$

ES. 23 $d\omega = \frac{\partial f}{\partial x} dx\, dy = 0 \iff \frac{\partial f}{\partial x} \equiv 0 \iff$

$\iff f(x,y) = g(y)$

ES. 24 $\omega = (x + Ky^2)dx + (xy - K^2y^2)dy$

è definita in $\mathbb{R}^2 \Rightarrow$ ammette un potenziale

se e solo se è chiusa $\iff d\omega = 0 \iff$

$-2Ky + y = 0 \iff K = \frac{1}{2}$.

ES. 25 $\alpha(t) = (-\sin(\pi t), 2t^2)$ $\alpha : [-1,1] \to \mathbb{R}^2$

$dx = -\pi\cos(\pi t)\, dt$

$dy = 4t\, dt$

$$\int_\alpha \omega = \int_{-1}^{1} \frac{-\sin(\pi t)\, 4t\, dt - (2t^2-1)(-\pi\cos(\pi t))\, dt}{\sin^2(\pi t) + (2t^2-1)^2} = ???$$

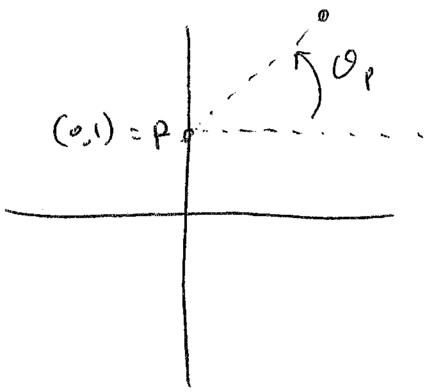
$d\omega = 0$ $\Omega = \mathbb{R}^2 \setminus \{(0,1)\}$ non è s.c.

$U_x = \frac{x}{x^2 + (y-1)^2}$

$U(x,y) = \theta_P$ $\omega = d\theta_P$

θ_P angolo rispetto a $P = (0,1)$

$U_y = \frac{-(y-1)}{x^2 + (y-1)^2}$

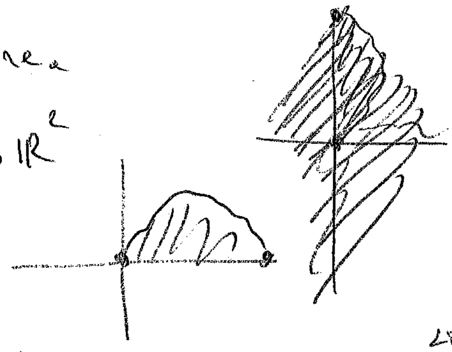


$\alpha(t)$ compie un giro, in senso ~~orario~~ ~~anti~~orario, attorno a $P = (0,1)$

$$\Rightarrow \int_\alpha \omega = -2\pi$$

Es. 26 Area

$\alpha : [-1,1] \to \mathbb{R}^2$



$$\alpha(t) = \begin{pmatrix} t - \sin t \\ 1 - \cos t \end{pmatrix}$$

$t - \sin t \geq 0$

$1 - \cos t \geq 0$

$$\text{Area} = \int_\partial y\,dx = \int_\alpha y\,dx = \int_0^{2\pi} (1-\cos t)(1-\cos t)\,dt =$$

$$= \int_0^{2\pi} (1 + \cos^2 t - 2\cos t)\,dt = \int_0^{2\pi} 1\,dt + \int_0^{2\pi} \cos^2 t\,dt +$$

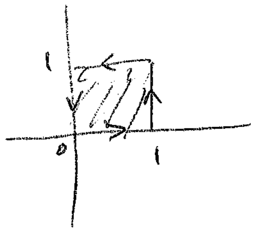
$$+ \int_0^{2\pi} -2\cos t\,dt = 2\pi + \pi + 0 = 3\pi$$

Es 27 ∂A = unione di curve chiuse,

df esatta $\Rightarrow \int_{\partial A} df = 0$

(vale per ogni curva semplice chiusa con ω esatta).

ES. 28 Q :



$$\omega = (x^2+y^2)\,dx + (2xy + e^y)\,dy$$

$$d\omega = 0$$

$$\Rightarrow \int_{\partial Q} \omega = 0$$

ES 29 $d\omega_0 = d\omega_1$

$$d(\omega_0 - \omega_1) = 0 \quad \text{in } \mathbb{R}^2 \text{ e } \mathbb{R}^2 \text{ s.c.}$$

$$\Rightarrow \omega_0 - \omega_1 = dU \qquad U : \mathbb{R}^2 \to \mathbb{R}$$

ES. 30 $\displaystyle\int_\alpha \cos(y)\,(dx+dz) - (x+z)\sin y\,dy =$

$$= \int_\alpha d\big((x+z)\cos y\big) = (x+z)\cos(y)\Big|_{\alpha(0)}^{\alpha(\pi)} =$$

$$= (x+z)\cos(y)\Big|_{(0,0,0)}^{(\pi,0,0)} = \pi$$

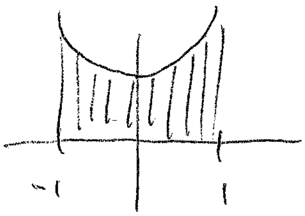
ES 31 $\displaystyle\int_{\partial\Delta} \cos(e^y)\,dy - y\,dx \overset{\text{GAUSS GREEN}}{=} \int_\Delta d\big(\cos(e^y)\,dy - y\,dx\big) =$

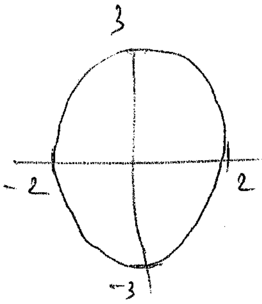
Δ =

$$= \int_\Delta -\,dy\,dx = \int_\Delta dx\,dy = \text{Area}(\Delta) = \pi.$$

(6)

(32) $\displaystyle\int_{\partial A} x\,dy = \int_A dx\,dy = \text{Area}(A) = \int_{-1}^{1} (1+x^2)\,dx = \frac{8}{3}$





(33) $A = \{ (x,y) : 9x^2 + 4y^2 \leq 36 \}$.

$\alpha(t) = (2\cos t, 3\sin t) \qquad t \in [0, 2\pi]$