es 17. e) del 27/3/2015

$$A = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \qquad A^*A = I \qquad \Rightarrow \text{è UNITARIA}$$

$\Rightarrow \exists\, M$ unitaria t.c.

$$M^{-1}AM = \begin{pmatrix} \lambda_1 & \\ & \lambda_2 \end{pmatrix}$$

$\lambda_1 + \lambda_2 = 2/\sqrt{2} = \sqrt{2}$

$\lambda_1 \lambda_2 = 1$

$$\lambda_1 = \frac{1+i}{\sqrt{2}}$$

$$\lambda_2 = \frac{1-i}{\sqrt{2}}$$

f)

$$A = \begin{pmatrix} -i & i+\sqrt{3} \\ i-\sqrt{3} & 2i \end{pmatrix} \qquad A^* = -A$$

$\Rightarrow A^*A = AA^*$ quindi A è normale $\Rightarrow$ diagonalizzabile con M unitaria

$\lambda_1 + \lambda_2 = i$

$\lambda_1 \cdot \lambda_2 = 2 + 4 = 6$

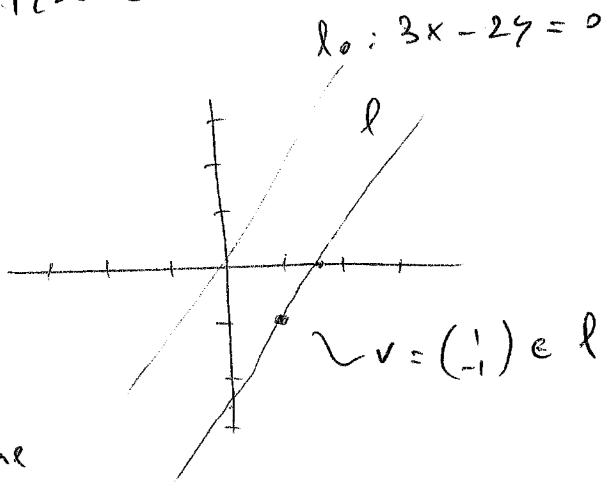
$$\lambda_1 = \frac{i+\sqrt{-25}}{2} = 3i$$

$$\lambda_2 = \frac{i-\sqrt{-25}}{2} = -2i$$

foglio del 27/4/2015

Es (1) a)

$\ell : \quad 3x - 2y = 5$



cerco la riflessione rispetto a ℓ.

$v = \binom{1}{-1} \in \ell$ quindi posso traslare di $-v$, riflettere rispetto a ℓ_0, traslare di v.

$$\sigma_0 \binom{x}{y} = \binom{x}{y} - 2 \frac{\left\langle \binom{x}{y}, \binom{3}{-2} \right\rangle}{\left\| \binom{3}{-2} \right\|^2} \binom{3}{-2} =$$

$$= \binom{x}{y} - 2 \, \frac{3x - 2y}{13} \binom{3}{-2} = \frac{1}{13} \begin{pmatrix} -5 & 12 \\ 12 & 5 \end{pmatrix} \cdot \binom{x}{y}$$

$$\sigma \binom{x}{y} = v + \sigma_0 \left(\binom{x}{y} - v \right) = \begin{pmatrix} -5/13 & 12/13 \\ 12/13 & 5/13 \end{pmatrix} \binom{x}{y} + \frac{10}{13} \binom{3}{-2}$$

Es. (2) a) ellisse di fuochi $\binom{3}{-1}$, $\binom{-4}{5}$ e

parametro $2k = 12$

$$\sqrt{(x-3)^2 + (y+1)^2} + \sqrt{(x+4)^2 + (y-5)^2} = 12$$

$$\cancel{x^2} - 6x + 9 + \cancel{y^2} + 2y + 1 = 144 + \cancel{x^2} + 8x + 16 + \cancel{y^2} - 10y + 25 + \\ - 24 \sqrt{(x+4)^2 + (y-5)^2}$$

$$(-14x + 12y - 175)^2 = 24^2 \left(x^2 + 8x + 16 + y^2 - 10y + 25 \right)$$

$$380x^2 + 336xy + 432y^2 - 292x + 1560y - 7009 = 0$$

ES. 3 1) $x^2+2xy-y^2+2x-\sqrt{3}=0$ 3

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -\sqrt{3} \end{pmatrix}$$

$d_1 = 1 > 0$

$d_2 = -2 < 0$

$d_3 = 1 + 2\sqrt{3} > 0$

$+ \; - \; -$ iperbole

2) $\underline{\underline{2}}x^2 - xy \underline{\underline{-3}}y^2 - 5x + 10y \underline{\underline{-3}} = 0$

$$\begin{pmatrix} 2 & -1/2 & -5/2 \\ -1/2 & -3 & 5 \\ -5/2 & 5 & -3 \end{pmatrix}$$

$d_1 > 0$ $\quad d_2 = -\frac{23}{4} < 0$

$d_3 = 0$

$\Rightarrow$ l'eq. si fattorizza

$$(x+ay+b)\left(2x-\frac{3}{a}y-\frac{3}{b}\right)=0$$

e $a=1$, $b=3$ perché risolvo

$$\begin{cases} -\frac{3}{a}+2a=-1 & \Rightarrow a = 1, -\frac{3}{2} \\ 2b-\frac{3}{b}=-5 & \Rightarrow b=-3, \frac{1}{2} \\ -\frac{3a}{b}-\frac{3b}{a}=10 & a=1 \Rightarrow b=-3 \\ & a=-\frac{3}{2} \Rightarrow b=\frac{1}{2} \end{cases}$$

$(x+y-3)(2x-3y+1)=0$ due rette disgiunte incidenti

3). $5x^2 - 4xy + 7y^2 + 4x - 3y - \sqrt{11} = 0$

$$\begin{pmatrix} 5 & -2 & 2 \\ -2 & 7 & -3/2 \\ 2 & -3/2 & -\sqrt{11} \end{pmatrix}$$

$d_1 > 0$

$d_2 > 0$

$d_3 = -\frac{109}{4} - 31\sqrt{11} < 0$ ellisse

4) $3x^2 - 6xy + 3y^2 + 4x - 5y - 19 = 0$

$$\begin{pmatrix} 3 & -3 & 2 \\ -3 & 3 & -5/2 \\ 2 & -5/2 & -19 \end{pmatrix}$$

$d_1 > 0$

$d_2 = 0$

$d_3 = -\frac{3}{4} < 0$

$\Rightarrow$ parabola

10.) $$\begin{pmatrix} 4 & -5/2 & -1 \\ -5/2 & 6 & -1/2 \\ -1 & -1/2 & 22 \end{pmatrix}$$

$d_1 > 0$

$d_2 = \frac{71}{4} > 0$

$d_3 = 4 \cdot 527 - 555 - 19 > 0$

ϕ (insieme vuoto)

11.) $\begin{pmatrix} 1 & 1 & 1/2 \\ 1 & 1 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$ $\quad d_1 > 0$, $d_2 = 0$, $d_3 = 0$

$\Rightarrow$ due rette (il polinomio si fattorizza)

$$(x + ay + b)\left(x + \frac{y}{a}\right) = \underline{\underline{x^2}} + 2xy + \underline{\underline{y^2}} + x + y$$

$$\begin{array}{l} (xy) \\ (x) \\ (y) \end{array} \begin{cases} a + \dfrac{1}{a} = 2 \\ b = 1 & \Rightarrow b = 1 \\ \dfrac{b}{a} = 1 & \Rightarrow a = 1 \end{cases}$$

$$(x + y + 1)(x + y) = 0$$

<u>due rette parallele</u>

33.) $5x^2 - 6xy + 5y^2 - 10x + 6y - K$

$\begin{pmatrix} 5 & -3 & -5 \\ -3 & 5 & 3 \\ -5 & 3 & -K \end{pmatrix}$ $\quad d_1 > 0 \quad d_2 > 0$

$d_3 = -16K - 80 =$
$= -16(K + 5)$

- $K > -5$ <u>ellisse</u>
- $K < -5$ $\emptyset$
- $K = -5$ $+ + 0$ <u>un punto</u> $(x^2 + y^2 = 0)$

34.)
$$\begin{pmatrix} 1 & K/2 & 1 \\ K/2 & -3 & 1/2 \\ 1 & 1/2 & -1 \end{pmatrix}$$

$d_1 = 1 > 0$

$d_2 = -3 - K^2/4 < 0$

$$d_3 = K^2/4 - K/2 + 23/4 = \left(\frac{K}{2} - \frac{1}{2}\right)^2 + \frac{22}{4} > 0$$

$$(\Delta \not< 0)$$

$\Rightarrow$ sempre un'iperbole.

35.)
$$\begin{pmatrix} K & 2 & -2 \\ 2 & 1 & -1 \\ -2 & -1 & 5 \end{pmatrix}$$

Scambio x e y

$d_1 > 1$

$d_2 = K - 4$

$$d_3 = \det \begin{pmatrix} K-4 & 2 & -2 \\ 0 & 1 & -1 \\ 0 & -1 & 5 \end{pmatrix} = (K-4) \cdot 4$$

- $K > 4$ $\quad \emptyset$
- $K < 4$ $\quad$ iperbole
- $K = 4$ $\quad (2x + y - 1)^2 + 4 = 0 \Rightarrow \emptyset$

36.)

$$\begin{pmatrix} 1 & -1 & k \\ -1 & 2k & 1 \\ k & 1 & 1 \end{pmatrix}$$

$d_1 > 0$

$d_2 = 2k - 1$

$d_3 = -2k^3 + 2k - k - k - 1 - 1 =$

$= -2(k^3 + 1)$

- $k > \frac{1}{2}$ $\quad d_2 > 0,\ d_3 < 0 \quad \Rightarrow$ ellisse
- $k = \frac{1}{2}$ $\quad d_2 = 0,\ d_3 \neq 0 \quad \Rightarrow$ parabola
- $k < \frac{1}{2},\ k \neq -1 \quad \Rightarrow d_2 < 0,\ d_3 \neq 0 \quad \Rightarrow$ iperbole
- $k = -1$ $\quad d_2 \neq 0,\ d_3 = 0 \quad \Rightarrow$ fattorizza

$$x^2 - 2xy - 2y^2 - 2x + 2y + 1 = 0$$

$$(x + ay + b)\left(x - \frac{2}{a}y + \frac{1}{b}\right) = 0$$

$$\begin{cases} a - \frac{2}{a} = -2 \\ b + \frac{1}{b} = -2 \quad \Rightarrow \quad b^2 + 2b + 1 = 0 \quad \Rightarrow b = -1 \\ \frac{a}{b} - 2\frac{b}{a} = 2 \end{cases}$$

$a^2 + 2a - 2 = 0 \qquad a = -1 \pm \sqrt{3}$

$\qquad\qquad -\frac{2}{a} = -1 \mp \sqrt{3}$

$$(x + (-1+\sqrt{3})y - 1)(x + (-1-\sqrt{3})y - 1) = 0$$

due rette incidenti

37.)

$$\begin{pmatrix} K+1 & 0 & K \\ 0 & K-1 & 1 \\ K & 1 & -1 \end{pmatrix}$$

$d_1 = K+1$

$d_2 = K^2-1$

$d_3 = (K+1)(-K) + K^2(1-K) =$

$= -K(K^2+1)$

K: -1, 0, 1

$K < -1$: $d_1 < 0$, $d_2 > 0$, $d_3 > 0$ ellisse

$K = -1$: parab

$-1 < K < 0$: iperb

$K = 0$: degenere

$0 < K < 1$: iperb

$K = 1$: parabola

$K > 1$: $d_1 > 0$, $d_2 > 0$, $d_3 < 0$ ellisse

$K = 0$

$x^2 - y^2 + 2y - 1 = 0$

$x^2 = (y-1)^2 \qquad (x - y - 1)(x + y + 1) = 0$

38.)

$$\begin{pmatrix} K & \sqrt{K} & \sqrt{K} \\ \sqrt{K} & 3 & 0 \\ \sqrt{K} & 0 & K \end{pmatrix}$$

$\boxed{K \geq 0}$

($K = 0$ è una retta, $y = 0$. supponiamo $K > 0$)

$d_1 > 0$

$d_2 = 3K - K = 2K > 0$

$d_3 = -3K + 2K^2 = K(2K-3)$

$0 < K < 3/2 \qquad d_3 < 0$ ellisse

$K = 3/2$ un punto (autoval. $+, +, 0$)

$K > 3/2 \qquad d_3 > 0 \qquad \emptyset$

39.)

$$\begin{pmatrix} 1 & -K & -2 \\ -K & 4 & 3 \\ -2 & 3 & 13/3 \end{pmatrix}$$

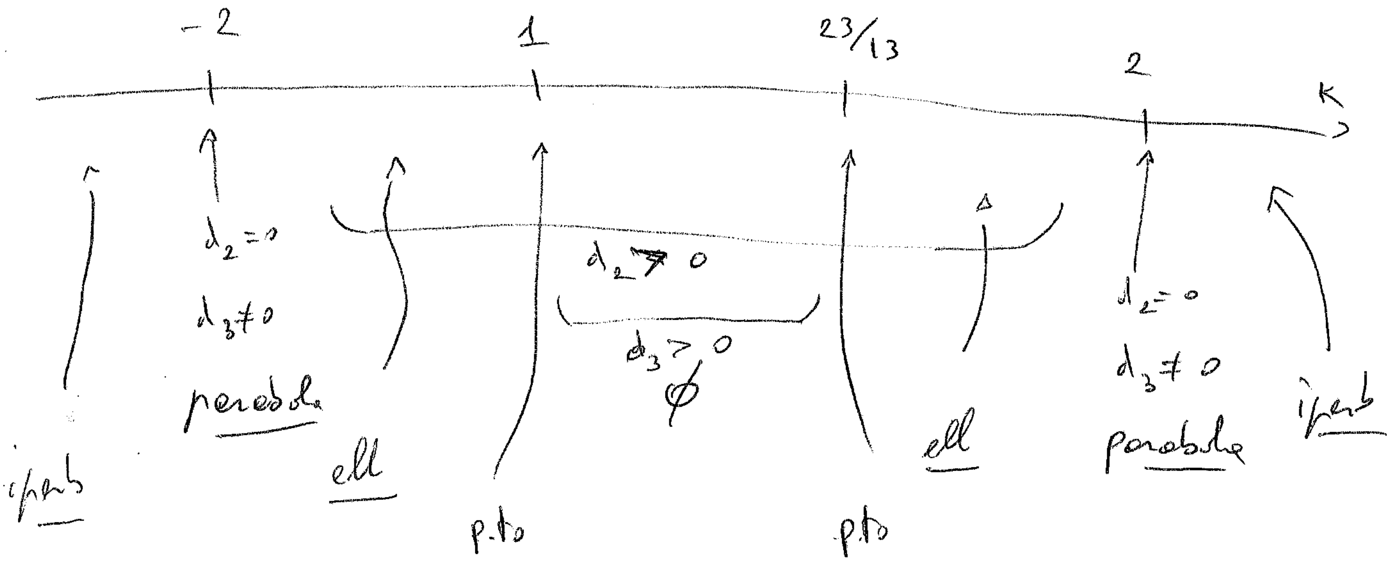
$d_1 > 0$

$d_2 = \cancel{K}4 - K^2$

$d_3 = -\frac{13}{3}K^2 + 12K - \frac{23}{3} =$

$= -\frac{1}{3}\left(13K^2 - 36K + 23\right) =$

$= -\frac{1}{3}(K-1)(13K-23)$



Es 4 $\mathcal{C}: \; 8x^2 - 12xy + 17y^2 + 4x + 12y + 4 = 0$

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 17 & 6 \\ 2 & 6 & 4 \end{pmatrix}$$

$d_1 > 0 \qquad d_2 > 0$

$d_3 = \det \begin{pmatrix} 8 & -6 & 2 \\ -30 & 35 & 0 \\ -14 & 18 & 0 \end{pmatrix} = -540 + 490 = -50 < 0$

$\Rightarrow$ <u>ellisse</u>

cerco di scriverla nella forma:

$(2x + ay + b)^2 + (2x + cy + d)^2 = e^2$

$$
\begin{array}{ll}
(y^2) & e^2 + c^2 = 17 \\
(xy) & a + c = -3 \\
(x) & b + d = 1 \\
(y) & ab + cd = 6 \\
(\text{cost}) & b^2 + d^2 - e^2 = 4
\end{array}
$$

$\Rightarrow$ (a meno di scambiarli) $\underline{a = 1 \quad b = -4}$

$\Rightarrow b = 2 \quad d = -1$

$\Rightarrow e = 1$

$$(2x + y + 2)^2 + (2x - 4y - 1)^2 = 1$$

Trasf. affine :

$$\begin{cases} X = 2x + y + 2 \\ Y = 2x - 4y - 1 \end{cases} \qquad \begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 2 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

Trasf. isometrica :

$$\begin{cases} Z = \alpha (2x + y + 2) \\ W = \beta (2x - 4y - 1) \end{cases} \qquad \begin{pmatrix} 2\alpha & \alpha \\ 2\beta & -4\beta \end{pmatrix} \text{ ortogonale}$$

$$\Rightarrow \begin{cases} \alpha\beta = \pm 1/10 & (\det = \pm 1) \\ 2\alpha^2 - 4\alpha\beta = 0 & 2\alpha = \beta \\ \ldots \end{cases}$$

← relazioni da $A^T A = Id$

$$\alpha = \frac{1}{\sqrt{5}} \qquad \beta = \frac{1}{2\sqrt{5}}$$

$$\begin{pmatrix} Z \\ W \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} +2 & +1 \\ +1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2/\sqrt{5} \\ -1/2\sqrt{5} \end{pmatrix}$$

↑ ortogonale