Ricevimento 7/1/15

27/1/14 Es. 2

(A) $E_s : \begin{cases} (s+2)\,x + 3s\,y + (1-s)\,z = 2s \\ (1+3s)\,x + (5-s)\,y + 4s\,z = 3-s \end{cases}$

$\dim(E_s) = \begin{cases} n_0 & \text{per } s = s_0 \\ n_1 & \text{per } s \neq s_0 \end{cases}$ $\leftarrow$ eccezione: $n_0 = 2$ / $\leftarrow$ caso tipico: tutte $n_1 = 1$

eccezione: rank $\left(\begin{array}{ccc|c} s+2 & 3s & 1-s & 2s \\ 1+3s & 5-s & 4s & 3-s \end{array} \right) = 1$

$$12s^2 - 5 + 6s - s^2 = 11s^2 + 6s - 5 = (11s - 5)(s + 1)$$

possibili s_0: -1, $5/11$

$$s = -1 \qquad \begin{pmatrix} 1 & -3 & 2 & -2 \\ -2 & 6 & -4 & 4 \end{pmatrix} \qquad \text{rank} = 1$$

$$s_0 = -1$$

$$s = 5/11 \qquad \begin{pmatrix} 27/11 & \cdot & & \\ \cdot & \cdot & & \end{pmatrix} \cdot \quad \cdot \qquad \text{rank} = 2$$

(E) E_2: $\begin{cases} 4x + 6y - z = 4 \\ 7x + 3y + 8z = 1 \end{cases}$

$$F_{-1}: \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} + \text{Span} \begin{pmatrix} 1 \\ -3 \\ \cancel{-1}\ 0 \end{pmatrix} \cancel{\begin{pmatrix} 1 \\ -3 \\ -4 \end{pmatrix}} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{cases} 4(2+u)+6(-2-3u) - (+v) = 4 \\ 7(2+u)+3(-2-3u)+8v = 1 \end{cases}$$

$$\begin{cases} v = -14u-4 \\ \ldots \end{cases} \qquad u_0 = \ldots \qquad v_0 = \ldots$$

$$\Rightarrow E_0 \cap F_{-1} = \left\{ \begin{pmatrix} 2+u_0 \\ -2-3u_0 \\ v_0 \end{pmatrix} \right\}$$

(F) $E_1 : \begin{cases} 3x+3y = 2 \\ 4x+4y+4z = 2 \end{cases} \qquad \begin{cases} x+y = 2/3 \\ z = \text{cont} \ldots \end{cases}$

$$F_0 : \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \text{Span} \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ -2 \\ -3 \end{pmatrix}$$

E_1 ha giacitura : $\text{Span} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ che appartiene alla giacitura di F_0

$\Rightarrow$ sono paralleli

Scelto un el. $\overset{w_0}{\downarrow}$ di E_1 si verifica che $w_0 \notin F_0$

$\Rightarrow$ paralleli e disgiunti

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$$F_{-2} : \begin{cases} x + y - z = 1 \\ -x + 2y + 3z = 0 \end{cases} \qquad \begin{cases} x + y - z = 1 \\ 3y + 2z = 1 \end{cases}$$

$$E_k : \begin{cases} x = 2 + 2t + ks \\ y = (k+5)t + 2ks \\ z = (k-4) + 2t - s \end{cases}$$

Valore di k per cui E_k è retta: discuto a parte.
Altri val. di k: sostituisco. Per quasi ogni k avrò soluz unica:

$$\underbrace{\begin{pmatrix} A(k) & B(k) \\ C(k) & D(k) \end{pmatrix}}\begin{pmatrix} s \\ t \end{pmatrix} = \begin{pmatrix} E(k) \\ F(k) \end{pmatrix}$$

se $\det \neq 0 \Longrightarrow$ incidenti in un punto

se $\det = 0$

$\rightarrow \operatorname{rank}\begin{pmatrix} A & B & E \\ C & D & F \end{pmatrix} = 1 \Longrightarrow$ retta giace sul piano

$\rightarrow \operatorname{rank}\begin{pmatrix} A & B & E \\ C & D & F \end{pmatrix} = 2 \Longrightarrow$ paralleli disgiunti

19/7/14 1 (B)

$$X = \mathrm{Span}\begin{pmatrix} 4 \\ 3 \\ 2 \\ -2 \end{pmatrix} \qquad Y = \mathrm{Span}(e_1, e_2, e_4)$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \frac{x_3}{2}\begin{pmatrix} 4 \\ 3 \\ 2 \\ -2 \end{pmatrix} + \begin{pmatrix} x_1 - 2x_3 \\ x_2 - \frac{3}{2}x_3 \\ 0 \\ x_4 + x_3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & -3/2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

Pag. 79 es. 2 $\quad X: \; x_1 + x_2 + x_3 + x_4 = 0$

$\mathcal{B}_1 = \left(\begin{pmatrix} 3 \\ 2 \\ -3 \\ -2 \end{pmatrix} \ldots \right) \qquad \mathcal{B}_2 = \left(\begin{pmatrix} ? \\ 1 \\ -3 \\ 0 \end{pmatrix} \ldots \right)$

$A = \left(\cdots \right) \qquad [f]^{\mathcal{B}_2}_{\mathcal{B}_1} = A \qquad B = [f]^{\mathcal{B}_1}_{\mathcal{B}_2}$ —

$f \cdot \mathcal{B}_1 = \mathcal{B}_2 \cdot A \qquad f \cdot \mathcal{B}_2 = \mathcal{B}_1 \cdot B$

$B = \mathcal{B}_1^{-1} \cdot f \cdot \mathcal{B}_2 = \mathcal{B}_1^{-1} \cdot \left(\mathcal{B}_2 \cdot A \cdot \mathcal{B}_1^{-1} \right) \cdot \mathcal{B}_2$ } fingendo che $\mathcal{B}_1$ e $\mathcal{B}_2$ siano matrici

$$= (\mathcal{B}_1^{-1} \cdot \mathcal{B}_2) \cdot A \cdot \underbrace{(\mathcal{B}_1^{-1} \cdot \mathcal{B}_2)}$$

ha senso: matrice di cambio da $\mathcal{B}_2$ a $\mathcal{B}_1$:

Cerco la matrice X di cambio da $\mathcal{B}_2$ a $\mathcal{B}_1$:

$$\begin{pmatrix} 2 \\ 1 \\ -3 \\ 0 \end{pmatrix} = x_{11} \begin{pmatrix} 3 \\ +2 \\ -3 \\ -2 \end{pmatrix} + x_{21} \begin{pmatrix} -2 \\ 1 \\ -9 \\ 10 \end{pmatrix} + x_{31} \begin{pmatrix} 3 \\ 1 \\ 3 \\ -7 \end{pmatrix}$$

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$$\rightsquigarrow X = \begin{pmatrix} x_{11} & & \\ x_{21} & \dots & \\ x_{31} & & \end{pmatrix}$$

$\Rightarrow B = X \cdot A \cdot X$

8/1/14 2D

E = esplicito piano $x_0 + \mathrm{Span}(w_1, w_2) \subset \mathbb{R}^4$

F = esplicito piano in forma cart

che trasforma in $y_0 + \mathrm{Span}(u_1, u_2)$

Fatto (dim (C)) : le giaciture hanno intersez. di dimensione 1.

$$F_t = t \cdot e_2 + F$$

$$\dim(E+F_t) = \underbrace{\text{dim somma delle giaciture}}_{2+2-1} + \begin{cases} 0 & \text{se si intersecano} \\ 1 & \text{se sono disgiunti} \end{cases}$$

(D) Per quali t ho $E + F_t \neq \mathbb{R}^4$?

Equivale a: per quali t ho $E \cap F_t \neq \emptyset$?

$$\left(x_0+\mathrm{Span}(w_1,w_2)\right)\cap\left(te_2+y_0+\mathrm{Span}(u_1,u_2)\right)\neq\emptyset$$

$$\iff te_2+y_0-x_0\in\underbrace{\mathrm{Span}(w_1,w_2,u_1,u_2)}_{\dim=3}$$

assumendo $u_1\notin\mathrm{Span}(w_1,w_2)$

basta imporre

$$\det\left(te_2+y_0-x_0,\ w_1,\ w_2,\ u_1\right)=0$$

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B $\quad F: \begin{cases} 3 & 1 & -1 & -4 & = -1 \\ 1 & -3 & 2 & 1 & = 18 \end{cases}$

Presentaz. param:

$$\begin{pmatrix} 0 \\ -16 \\ -15 \\ 0 \end{pmatrix} + \operatorname{Span} \begin{pmatrix} +1 \\ +7 \\ 10 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 7 \\ 11 \\ -1 \end{pmatrix}$$

(c) $\begin{cases} 3(-1+2ts)+(3+5t+2s)-(2-t-3s)-4(1+7t+5s)=1 \\ \end{cases}$

$$\begin{cases} \alpha t + \beta s = k \\ \gamma t + \delta s = h \end{cases}$$

Se $\alpha\delta - \beta\gamma \neq 0 \Longrightarrow \cap = 1 \text{ pt}$

$\alpha\delta - \beta\gamma = 0$

$$\operatorname{rank}\begin{pmatrix} \alpha & \beta & k \\ \gamma & \delta & h \end{pmatrix} \begin{cases} \to \underline{1} & \text{si intersecano in una retta} \\ \to 0 & \text{sghembi} \end{cases}$$

13/3/14 Q4

$$f: \mathbb{R}^6 \to \mathbb{R}^9 \qquad f(e_2 - 7e_5) = f(e_2 - e_3 + 4e_6) = 5e_9$$

Cioè: $5e_9 \in \operatorname{Im} f \Rightarrow \dim \operatorname{Im} f \geq 1$

$$\cancel{e_2} - 7e_5 - \cancel{e_2} + e_3 - 4e_6 \in \operatorname{Ker} f \implies \dim \operatorname{Ker} f \geq 1$$

Che dim. può essere Ker f?

$$\implies 1 \leq \cdot \leq 5$$

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$\boxed{10}$ 2

$$U = \begin{pmatrix} 2 & 1 \\ -4 & 1 \end{pmatrix} \qquad V = \begin{pmatrix} 7 & -2 \\ -4 & 1 \end{pmatrix}$$

(B) $$\operatorname{Span}(U, V) = \operatorname{Span}\left(\begin{pmatrix} 2 & 1 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} 5 & -3 \\ 0 & 0 \end{pmatrix} \right)$$

$$\begin{cases} a_{21} + 4a_{22} = 0 \\ -12a_{11} + 20a_{12} - 11a_{21} = 0 \end{cases}$$

$$\begin{array}{cc} 2 & 5 \\ 1 & -3 \\ -4 & 0 \\ 1 & 0 \end{array}$$

(c) Def. di $[f]_{\mathcal{B}}^{\mathcal{B}}$:

$$\underset{A}{\overset{\shortparallel}{f}} \cdot \underset{V}{\overset{\shortparallel}{\mathcal{B}}} = \underset{V}{\overset{\shortparallel}{\mathcal{B}}} \cdot [f]_{\mathcal{B}}^{\mathcal{B}} \Longrightarrow [f]_{\mathcal{B}}^{\mathcal{B}} = V^{-1} \cdot A \cdot V$$

— o —

$$\left(s : \{0,1,2\} \to \mathbb{R} \right) \longleftrightarrow \begin{pmatrix} s(0) \\ s(1) \\ s(2) \end{pmatrix} \in \mathbb{R}^3$$

$$a_0 + a_1 t + \cdots + a_d t^d \longleftrightarrow \begin{pmatrix} a_0 \\ \vdots \\ a_d \end{pmatrix} \in \mathbb{R}^{d+1}$$