Es Alg Lin 21/10/2014

Foglio $\boxed{2}$

Es 4

$V = \mathcal{F}(\{a, b, c\}, \mathbb{R})$

$f \in V$

$$f : \begin{cases} a \longmapsto f(a) \\ b \longmapsto f(b) \\ c \longmapsto f(c) \end{cases}$$

$W = \{ f : f(c) \geq 2 f(a) \}$

$0 \in W$

$f(c) = 100$

$f(a) = 0$

$f(b) = 0$

$-f \in W$?

$-100 \neq 2 \cdot (-0)$

$\Rightarrow W$ non è sottospazio

$W = \{ f : 3f(a) - 7f(b) = 0 \}$

$0 \in W$ ✓

$f \in W \overset{?}{\Longrightarrow} \lambda f \in W$ per $\lambda \in \mathbb{R}$ sì

$f, g \in W$ $\quad f+g \in W$?

$3f(a) - 7f(b) = 0 \qquad 3g(a) - 7g(b) = 0$

$$3\big(\underbrace{f(a)+g(a)}\big) - 7\big(\underbrace{f(b)+g(b)}\big) = 0$$

$$3\,(f+g)(a) - 7\,(f+g)(b) = 0$$

$\Rightarrow f+g \in W$

$\Rightarrow W$ è ssp di V

$$W = \{ f : f(a)\cdot f(b) + f(a)\cdot f(c) + f(b)\cdot f(c) = 0 \}$$

$0 \in W$ ✓

$f : f(a) = 1,\ f(b) = 0,\ f(c) = 0 \qquad f \in W$

$g : g(a) = 0 \quad g(b) = 1 \ , \ g(c) = 0 \quad g \in W$

$h : h(a) = 0 \quad h(b) = 0 \ , \ h(c) = 1 \quad h \in W$

$$(f+g) \begin{cases} a \longmapsto 1 \\ b \longmapsto 1 \\ c \longmapsto 0 \end{cases} \qquad 1 \cdot 1 + 1 \cdot 0 + 1 \cdot 0 = 1 \neq 0$$

$(f+g) \notin W$

W non è ssp di V

$W = \{ f : |f(b) + f(c)| \leq |f(a)| \}$

$$W = \{ f : f(a)^2 + 2f(a)f(b) + f(b)^2 = 0 \}$$

$$(f(a)+f(b))^2 = 0$$

$$\boxed{f(a) + f(b) = 0}$$

$$f : \begin{cases} a \longmapsto x \in \mathbb{R} \\ b \longmapsto 0 \\ c \longmapsto 0 \end{cases}$$

$$\boxed{f \in W}$$

$$|f(a)| \geq |f(b) + f(c)|$$

$$|\lambda| \geq |0+0| = 0$$

$$\begin{cases} g(a) = 1 \\ g(b) = 1 \\ g(c) = 0 \end{cases} \qquad (g - f_1) \begin{cases} a \longmapsto 0 \\ b \longmapsto 1 \\ c \longmapsto 0 \end{cases}$$

$$g \in W \qquad g - f_1 \notin W$$

⑤ $V = \mathbb{R}^2$

a) $v = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ $\quad w = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

$\boxed{v \in \mathrm{Span}(w) \;?}$ $\quad \mathrm{Span}(w) = \{\lambda \cdot w ,\ \lambda \in \mathbb{R}\}$

ovvero chiedo se ha soluzione il sistema

$v = \lambda w \qquad \lambda \in \mathbb{R}$

$$\begin{cases} 3 = \lambda(-2) \\ -4 = \lambda(5) \end{cases} \longrightarrow \lambda = -\frac{3}{2}$$

ma $-4 \neq -\frac{3}{2} \cdot 5 \Rightarrow \nexists$ sol

$v \notin \operatorname{Span}(w)$

(b) $v = \begin{pmatrix} \sqrt{6} \\ -2 \end{pmatrix}$ $\quad w = \begin{pmatrix} -3 \\ \sqrt{6} \end{pmatrix}$

$$\begin{cases} \sqrt{6} = \lambda \cdot (-3) \\ -2 = \lambda \sqrt{6} \end{cases} \quad \rightsquigarrow \quad \lambda = -\frac{\sqrt{2}}{\sqrt{3}}$$

$$-2 = -\frac{\sqrt{2}}{\sqrt{3}} \cdot \sqrt{6} = -\frac{\sqrt{2}}{\cancel{\sqrt{3}}} \cdot \sqrt{2} \cdot \cancel{\sqrt{3}}$$

$\Rightarrow v \in \operatorname{Span}(w)$

$\exists !\, \lambda : \quad v = \lambda w$

$$-\frac{2}{\sqrt{6}} \qquad \sqrt{6} = \sqrt{2}\cdot\sqrt{3}$$

$$2 = \sqrt{2}\cdot\sqrt{2}$$

$$\parallel$$

$$-\frac{\cancel{\sqrt{2}}\cdot\sqrt{2}}{\cancel{\sqrt{2}}\ \sqrt{3}} = -\frac{\sqrt{2}}{\sqrt{3}}$$

(c) $\quad v = \begin{pmatrix} 3 \\ 7 \end{pmatrix} \qquad w_1 = \begin{pmatrix} 4 \\ -11 \end{pmatrix} \qquad w_2 = \begin{pmatrix} 3 \\ 8 \end{pmatrix}$

$v \in \mathrm{Span}(w_1, w_2) = \{ \lambda_1 w_1 + \lambda_2 w_2 \ , \ \lambda_1, \lambda_2 \in \mathbb{R} \}$

$\exists \ \lambda_1, \lambda_2 :$

$v = \lambda_1 w_1 + \lambda_2 w_2$

$$\begin{cases} 3 = \lambda_1 \cdot 4 + \lambda_2 \, 3 \\ 7 = \lambda_1 (-11) + \lambda_2 \, 8 \end{cases}$$

$\exists$ sol ?

$!$ sol ?

$$\begin{cases} 24 = \lambda_1 \cdot 32 + \lambda_2 \, 24 & (1) \\ 21 = \lambda_1 (-33) + \lambda_2 \, 24 & (2) \end{cases}$$

$$\begin{cases} 24 = \lambda_1 \, 32 + \lambda_2 \cdot 24 & (1) \\ 3 = \lambda_1 (65) & (1) - (2) \end{cases}$$

$$\lambda_1 = \frac{3}{65}$$

$$24 - \frac{3}{65} \cdot 32 = \lambda_2 \cdot 24$$

$$\frac{1}{24}\left(24 - \frac{3}{65} \cdot 32\right) = \lambda_2$$

ho trovato gli unici valori di λ_1, λ_2

che risolvono

(d) $v = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$ $w_1 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $w_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $w_3 = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

$v \in \text{Span}(w_1, w_2, w_3)$?

$$v = \lambda_1 w_1 + \lambda_2 \cdot w_2 + \lambda_3 w_3$$

$$\begin{aligned}
&① \quad \left\{ \begin{array}{l} -2 = \lambda_1 \cdot 4 + \lambda_2\, 2 + \lambda_3\, 5 \\ 7 = \lambda_1 \cdot 1 + \lambda_2 (-1) + \lambda_3 (-3) \end{array} \right. \\
&② \\
&① + 2② \quad \left\{ \begin{array}{l} -2 = \lambda_1\, 4 + \lambda_2\, 2 + \lambda_3 \cdot 5 \\ 12 = \lambda_1 \cdot 6 + \lambda_3 (-1) \end{array} \right.
\end{aligned}$$

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$$\left\{ \begin{array}{l} \lambda_3 = 6\lambda_1 - 12 \\ \lambda_2 = \frac{1}{2}\left(-2 - \lambda_1 \cdot 4 - \lambda_3 \cdot 5\right) \end{array} \right.$$

Quindi $\exists$ infinite soluzioni: per ogni λ_1 troviamo λ_2 e λ_3 che risolvono il sistema $\Rightarrow v \in \operatorname{Span}(w_1, w_2, v_3)$