Es. Alg Lin 17/12/2014

QUESITI 15/2/2014 II° VER.

① $\begin{pmatrix} 1 & 0 \\ t & \end{pmatrix} = 3 \begin{pmatrix} s \\ -5 \end{pmatrix} - 2 \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ Trovare t, s

$$\begin{cases} 10 = 3s - 2 \\ t = -15 + 6 \end{cases} \quad \begin{cases} s = 4 \\ t = -9 \end{cases}$$

② $V \subset \mathbb{R}^8$

$$\begin{cases} x_1 + 3x_2 + \dots = 0 \\ 4x_1 + 7x_2 + \dots = 0 \end{cases}$$

sono due equazioni indipendenti

$\Rightarrow \dim V = 8 - 2 = 6$

11 generatori $\Rightarrow$ per avere base di V scarto $11 - 6 = 5$ vettori

(4) $$\begin{pmatrix} t+1 & 3t-2 \\ 1-t & t-10 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3-2t \\ 3 \end{pmatrix}$$

$\det \neq 0 \Rightarrow$! soluzione

$$\det = (t^2 - 8t - 10) - (-3t^2 + 5t - 2) =$$
$$= 4t^2 - 14t - 8 = 2(2t+1)(t-4)$$

$t \neq -\frac{1}{2}, 4 \Rightarrow$ unica soluzione

$t = -\frac{1}{2}$ $\begin{pmatrix} 1/2 & -7/2 \\ 3/2 & -21/2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

ma $\begin{pmatrix} 4 \\ 3 \end{pmatrix} \notin \text{Span}\left(\begin{pmatrix} 1/2 \\ 3/2 \end{pmatrix}, \begin{pmatrix} -7/2 \\ -21/2 \end{pmatrix} \right) \Rightarrow \nexists$ S.d.

$t = 4$ $\begin{pmatrix} 5 & 10 \\ -3 & -6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$

$\exists$ almeno $\ker \begin{pmatrix} -1 \\ 0 \end{pmatrix} \Rightarrow \exists$ infinite sol

⑤ $A \in M_{3\times 3}(\mathbb{R})$ $\det(A) = 7$

$$\det(2 \cdot A) = 2^3 \det(A) = 56$$

⑥
$$\begin{pmatrix} 2 & 3 & 7 & 0 \\ 4 & -2 & 0 & -5 \\ -1 & 5 & -3 & 2 \end{pmatrix}$$

det. orlate $1^\circ, 2^\circ, 3^\circ$ col
$$\begin{vmatrix} 2 & 3 & 7 \\ 4 & -2 & 0 \\ -1 & 5 & -3 \end{vmatrix} =$$

$$= 7 \begin{vmatrix} 4 & -2 \\ -1 & 5 \end{vmatrix} - 3 \begin{vmatrix} 2 & 3 \\ 4 & -2 \end{vmatrix} = 7 \cdot 18 + 3 \cdot 16 =$$

$$= 174$$

(I, III, IV) $\begin{vmatrix} 2 & 7 & 0 \\ 4 & 0 & -5 \\ -1 & -3 & 2 \end{vmatrix} = 5 \begin{vmatrix} 2 & 7 \\ -1 & -3 \end{vmatrix} + 2 \begin{vmatrix} 2 & 7 \\ 4 & 0 \end{vmatrix} =$

$= 5 \cdot 1 + 2(-28) = -51$

② $\mathbb{R}^3 = X \oplus Y$ $\quad v = \begin{pmatrix} 3 \\ -4 \\ 7 \end{pmatrix}$

$X = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \underbrace{5x + 6y - 2z = 0}_{f\begin{pmatrix} x \\ y \\ z \end{pmatrix}} \right\}$ $\quad Y = \left\langle \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} \right\rangle$

w

$f(v) = 15 - 24 - 14 = -23$

$$f(w) = 10 - 6 - 10 = -6$$

$$f\left(v - \frac{23}{6} w\right) = 0$$

$$v_x = v - \frac{23}{6} w =$$

$$= \begin{pmatrix} 3 \\ -4 \\ 7 \end{pmatrix} - \frac{23}{6} \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} -28 \\ -1 \\ -73 \end{pmatrix}$$

QUESITI 7/6/2014

VERSIONE I

① $\dim V = 4$ 7 generatori

base: 4 vettori $\Rightarrow$ ne scarto $7 - 4 = 3$

② $\begin{pmatrix} 1 \\ 6 \end{pmatrix}^3 = 3 \begin{pmatrix} 5 \\ 4 \end{pmatrix} - 2v$

$v = \frac{1}{2} \cdot \left[3 \cdot \begin{pmatrix} 5 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 6 \end{pmatrix}^3 \right] =$

$= \frac{1}{2} \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

③ $\dim V = 7$ $\quad Z, W \subset V$

$\dim W = 3$ $\quad \dim(W + Z) = 7$

$\dim Z$ $\quad \dim(W \cap Z) = ?$

$0 \leq \quad \leq 3$

$$0 \leq \underbrace{\dim W + \dim Z - \dim(W+Z)}_{\dim(W \cap Z)} \leq 3$$

$$0 \leq 3 + \dim Z - 7 \leq 3$$

$$4 \leq \dim Z \leq 7$$

(4)
$$\begin{pmatrix} 1+t & -3t \\ t-4 & 2+t \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1+4t \\ t-8 \end{pmatrix}$$

$$\det = (t^2+3t+2) + (3t^2-12t) =$$
$$= 4t^2 - 9t + 2 = 4\left(t-\tfrac{1}{4}\right)(t-2)$$

$t \neq 1, \frac{1}{4}, 2$ 3! sol.

$t = \frac{1}{4}$ $\begin{pmatrix} 5/4 & -3/4 \\ -15/4 & 9/4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -\frac{31}{4} \end{pmatrix} \notin \operatorname{Span}\begin{pmatrix} 5/4 \\ -15/4 \end{pmatrix}$

$\Rightarrow$ $\nexists$ sol

$t = 2$ $\begin{pmatrix} 3 & -6 \\ -2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ -6 \end{pmatrix} \in \operatorname{Span}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$\Rightarrow$ $\exists\ \infty$ soluzioni

⑤ $A, B \in M_{2\times 2}(\mathbb{R})$ $\det A = 1$, $\det B = 2$

(?)

$\Rightarrow$ $\det(A+B) = 3$ NO

es.: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$

⑥ A 5×3, B sottomatrice 3×3

quante celle?

restano 2 righe, 6 colonne

$\Rightarrow$ $6 \cdot 2 = 12$ celle

⑦ $X = \{ x \in \mathbb{R}^3 : \underbrace{3x_1 + 4x_2 - x_3 = 0}_{f(x)} \}$

$Y = \mathrm{Span}\begin{pmatrix} 7 \\ -2 \\ 3 \end{pmatrix}$ $= w$

$\mathbb{R}^3 = X \oplus Y$

$v = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix}$

$\Pi_X(v) = ?$ proiezione su X di v

$f(v) = 12 + 4 + 2 = 18$

$f(w) = 21 - 8 - 3 = 10$

$f\left(v - \frac{9}{5} w\right) = 0$

$$\Pi_x(v) = v_x = \begin{pmatrix} 4 \\ 1 \\ -2 \end{pmatrix} - \frac{9}{5}\begin{pmatrix} 7 \\ -2 \\ 3 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -43 \\ 23 \\ -37 \end{pmatrix}$$

QUESITI 21/6/2014 (VERSIONE I)

① $\left[\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right]_B = \begin{pmatrix} 7x_1 + 3x_2 \\ 5x_1 + 2x_2 \end{pmatrix}$

chi è B ? $\qquad B = \{v_1, v_2\}$

chi è v_1

$$\begin{pmatrix} 7 & 3 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\det \begin{pmatrix} 7 & 3 \\ 5 & 2 \end{pmatrix} = -1$$

$$x_1 = -1 \cdot 2 = -2$$

$$x_2 = -1(-5) = 5$$

$$v_1 = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} 7 & 3 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow$$

$$x_1 = -1(-3) = 3$$

$$x_2 = -1(7) = -7$$

$$B = \left\{ v_1 = \begin{pmatrix} -2 \\ 5 \end{pmatrix}, v_2 = \begin{pmatrix} 3 \\ -7 \end{pmatrix} \right\}$$